

# 310-2202 โครงสร้างของระบบคอมพิวเตอร์ (Computer Organization)

Topic 3: Digital logic

Damrongrit Setsirichok

# Topic

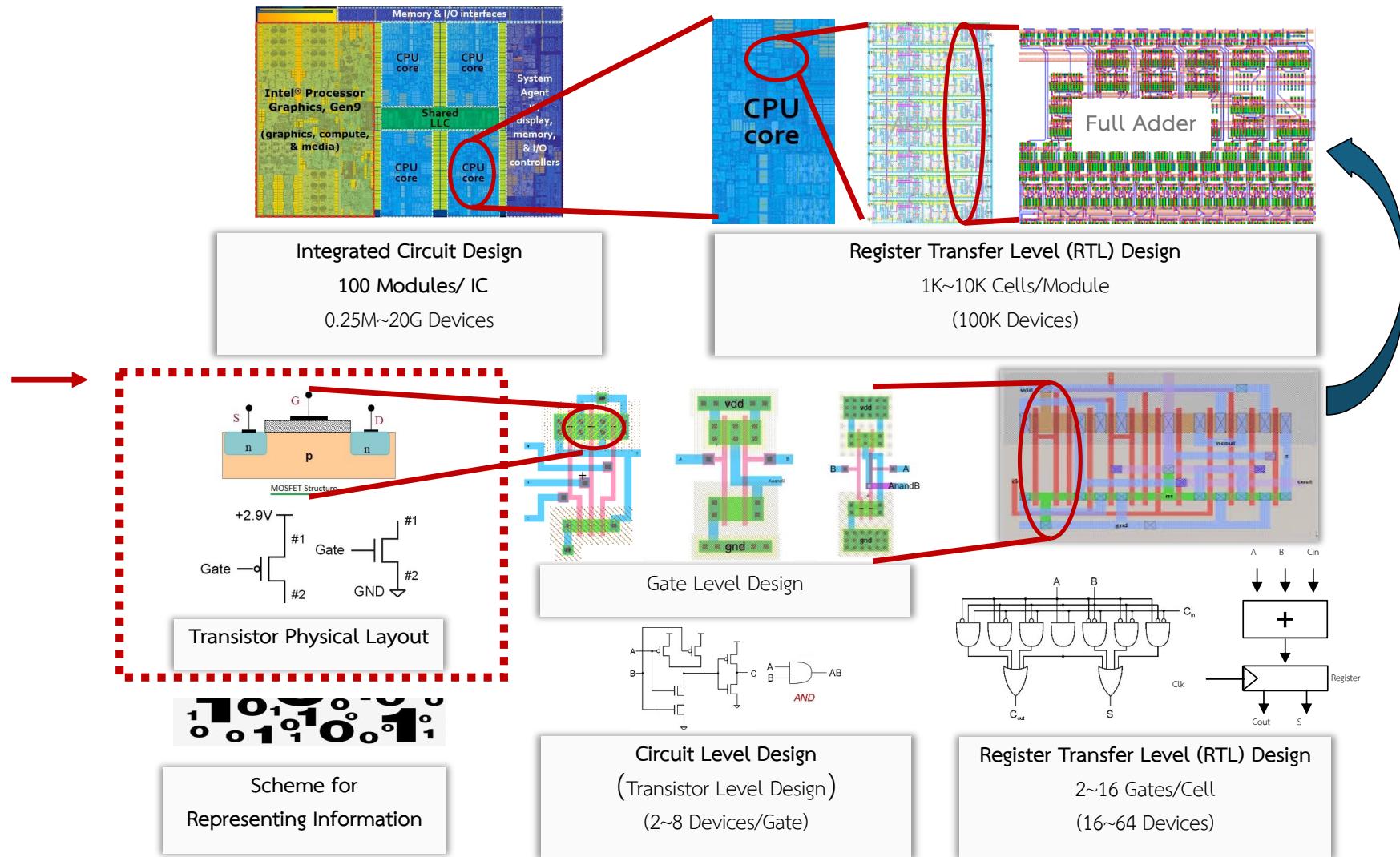
- Transistors & Logic Gates
- Combinational Logic Circuits
- Basic Storage Elements
- Sequential Logic Circuits

# Transistor Count

- Microprocessors contain millions of transistors
  - AMD Instinct: 146 Billion – 5nm technology
  - Apple M3 Max: 97 Billion – 3nm technology
  - Intel 7 Sapphire Rapids – 10nm technology
- Transistor: Building Block of Computers
  - Logically, each transistor acts as a switch
- Combined to implement logic functions
  - AND, OR, NOT

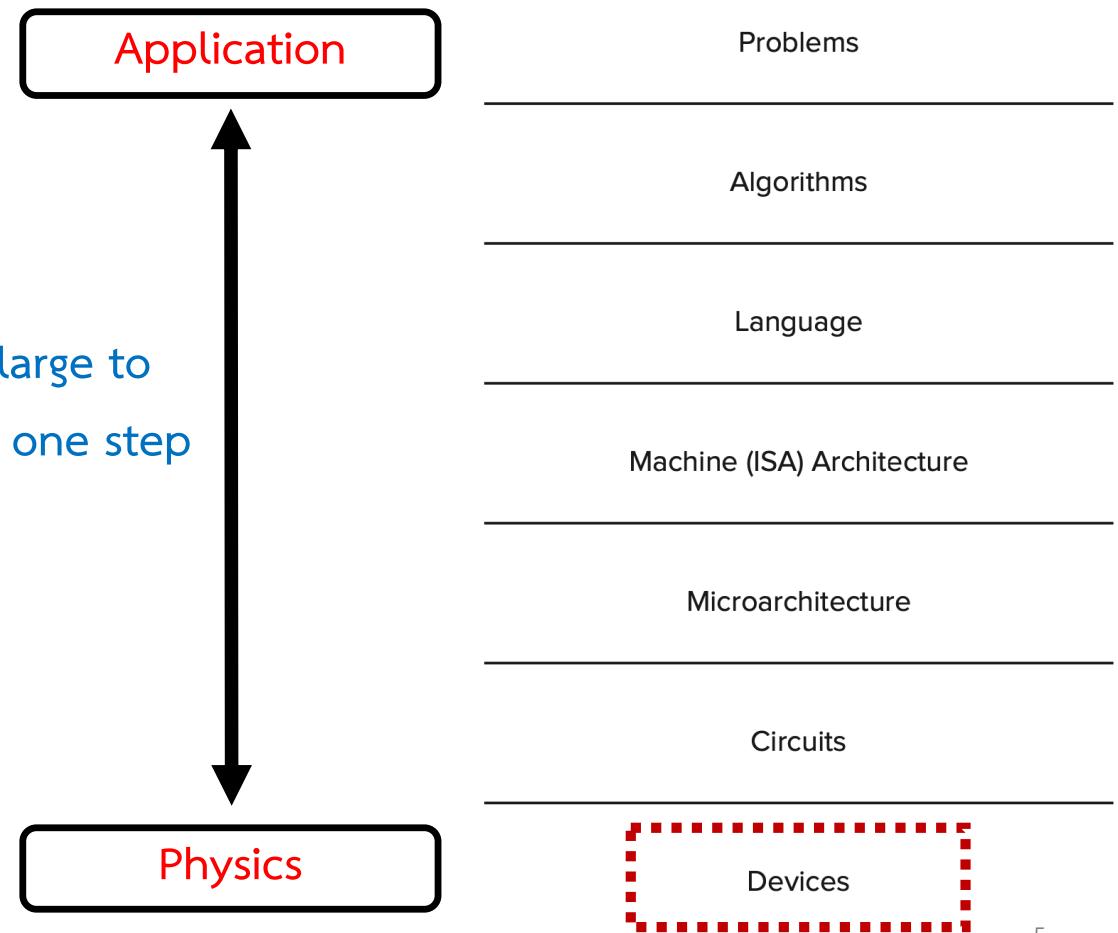
[https://en.wikipedia.org/wiki/Transistor\\_count](https://en.wikipedia.org/wiki/Transistor_count)

# Approach: Bottom Up

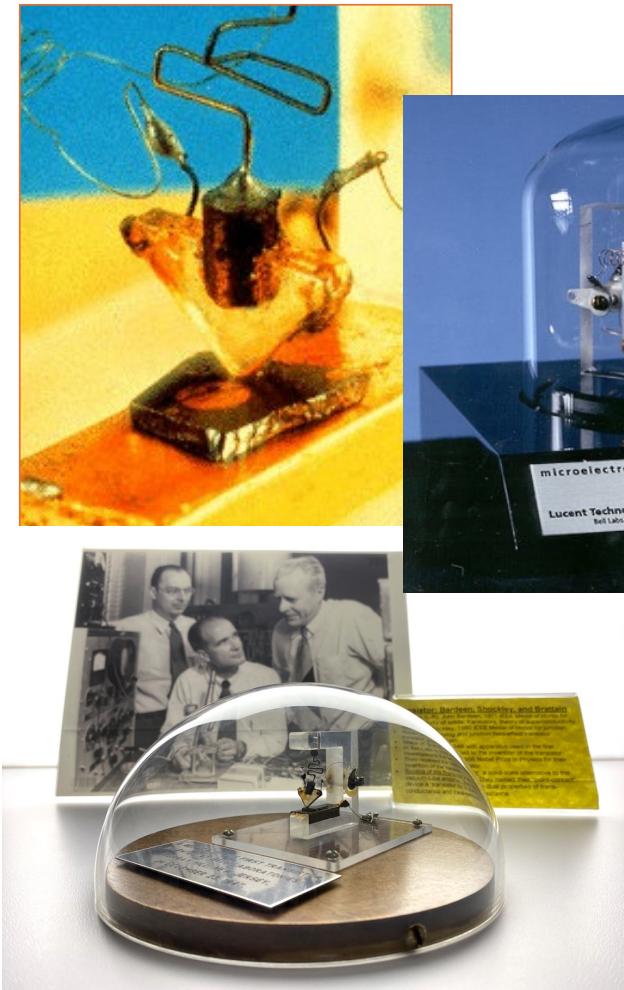


# How Do We Get the Electrons to Do the Work?

- Statement of the Problem
- Algorithm
- Program
- ISA
- Microarchitecture
- Logic Circuit
- Devices

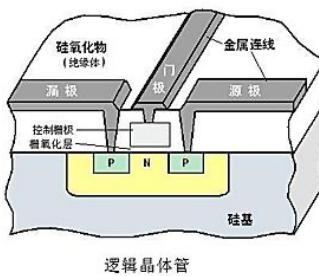
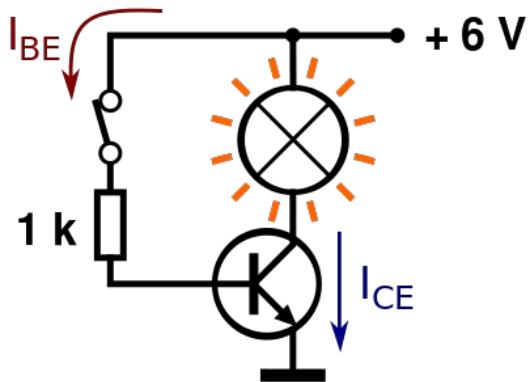


## First use: Vacuum tubes as switches



- **1945:** Bell sets up lab in the hopes of developing “solid state” components to replace existing electromechanical systems.
- **1947:** The Invention of the First Transistor—the point-contact transistor
- **1951:** Shockley develops junction transistor which can be manufactured in quantity.

## Transistor as a switch



- Switch open:

- No current through circuit
- Light is **off**
- $V_{out}$  is +2.9 V

- Switch closed:

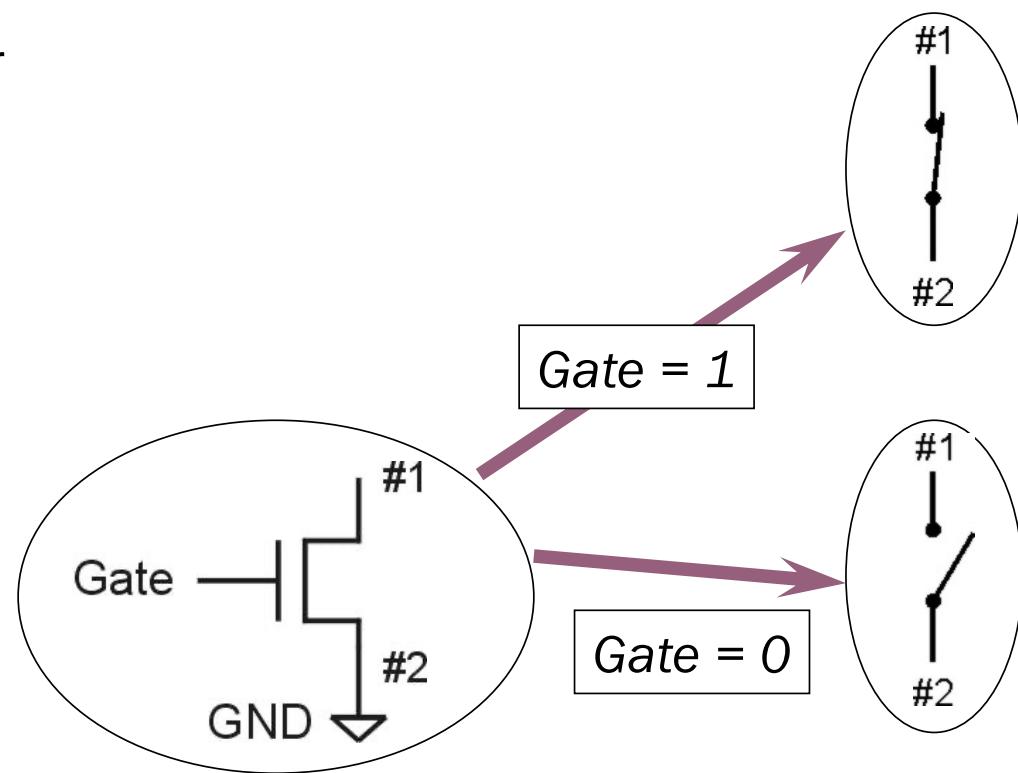
- Short circuit across switch
- Current flows
- Light is **on**
- $V_{out}$  is 0 V

*Switch-based circuits* can easily represent two states:

on/off, open/closed, voltage/no voltage.

# N-type MOS Transistor

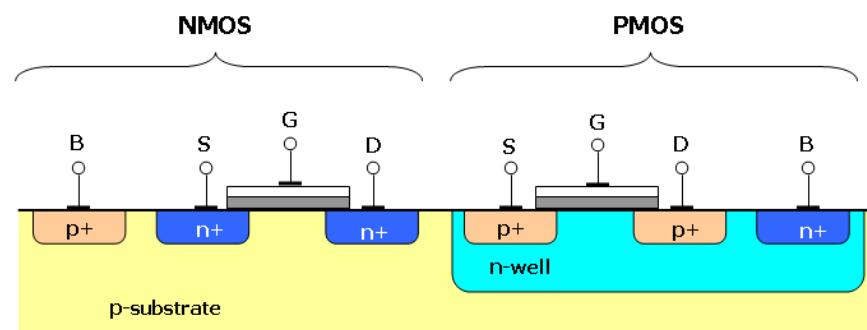
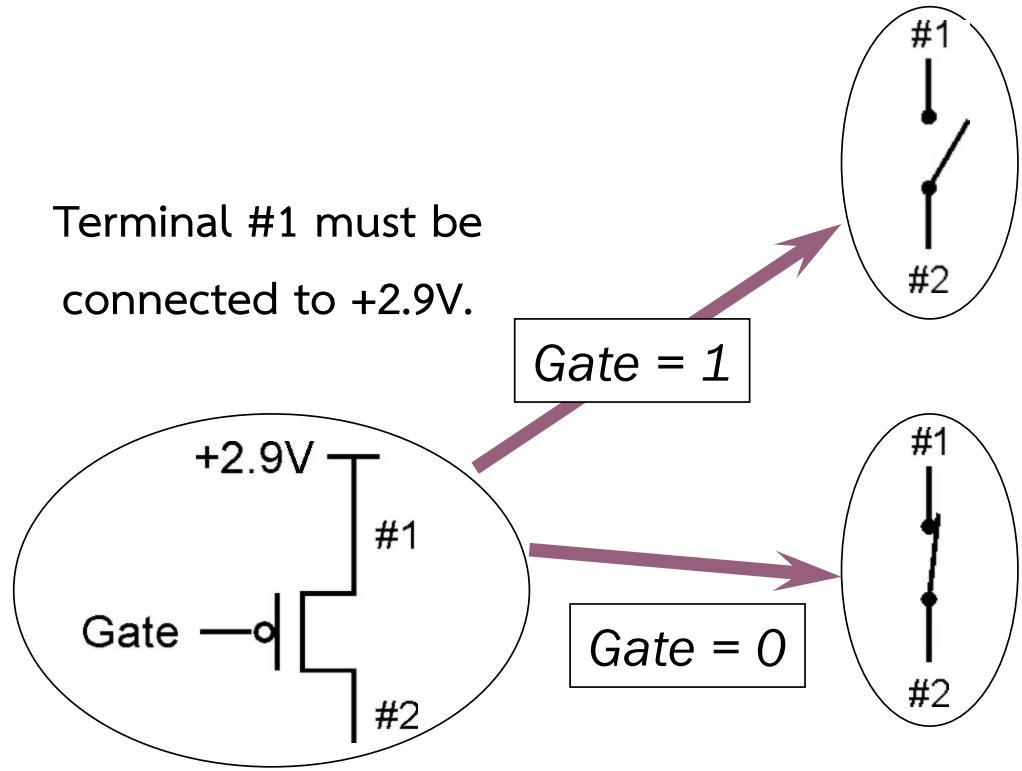
- MOS: Metal–Oxide–Semiconductor
  - two types: N-type and P-type
- N-type
  - when Gate has **positive** voltage, short circuit between #1 and #2 (switch **closed**)
  - when Gate has **zero** voltage, open circuit between #1 and #2 (switch **open**)



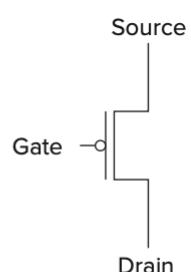
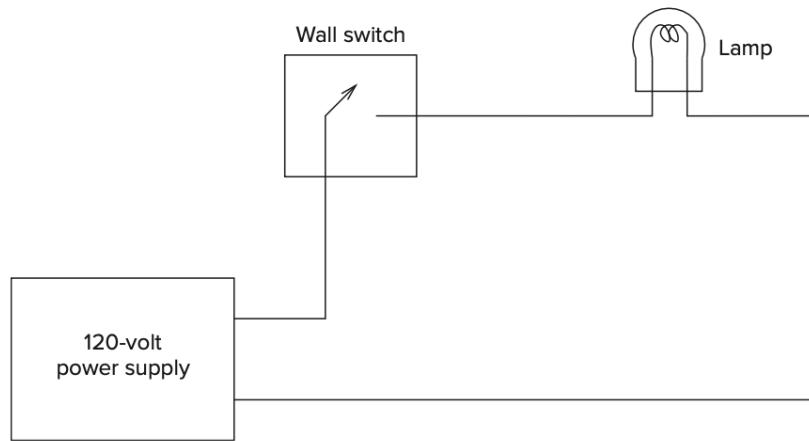
Terminal #2 must be connected to GND (0V).

# P-type MOS Transistor

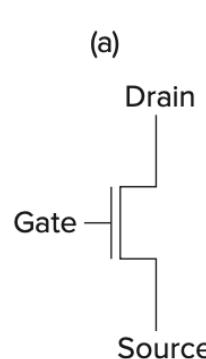
- P-type is complementary to N-type
  - when Gate has positive voltage, open circuit between #1 and #2 (switch open)
  - when Gate has zero voltage, short circuit between #1 and #2 (switch closed)



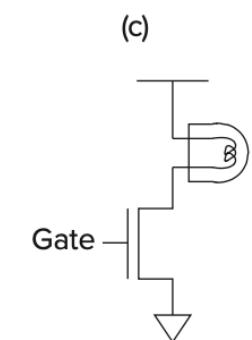
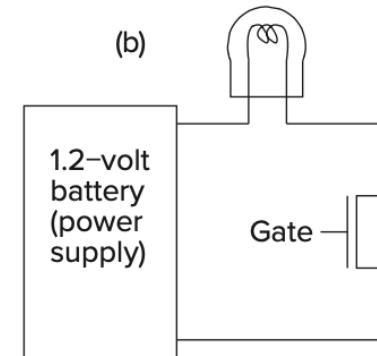
# Transistor as a switch



A P-type MOS transistor.

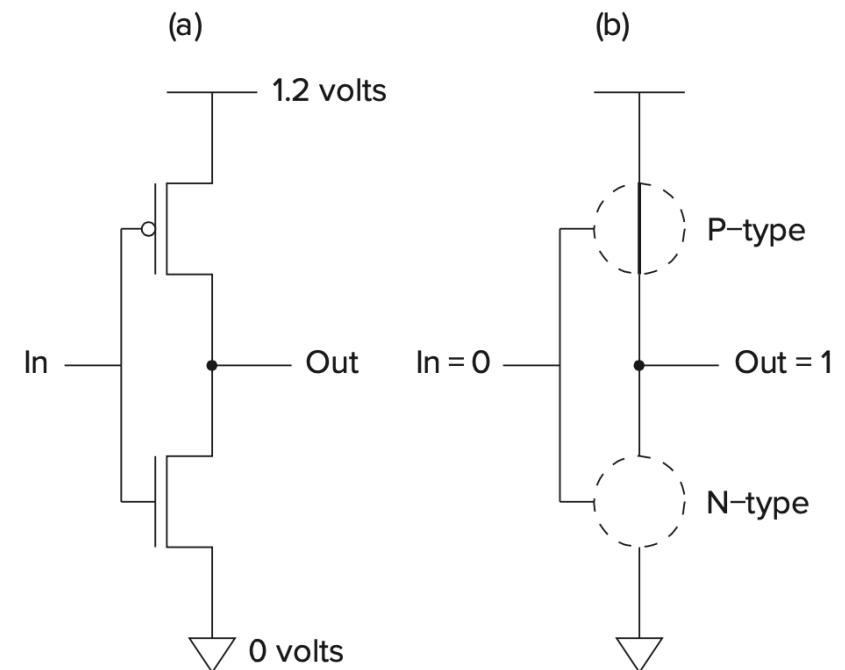


The N-type MOS transistor.

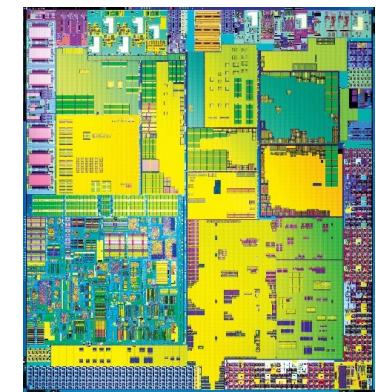
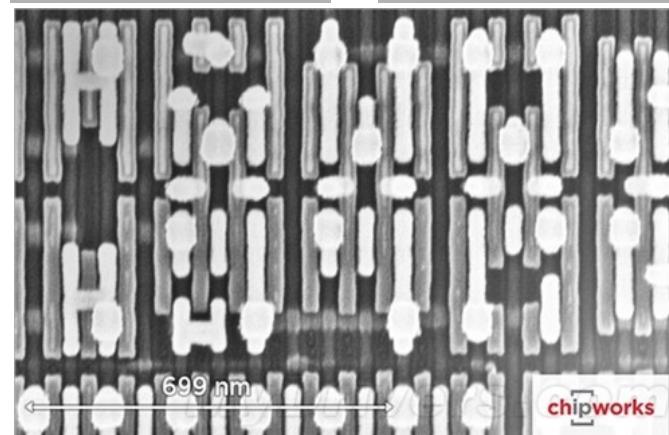
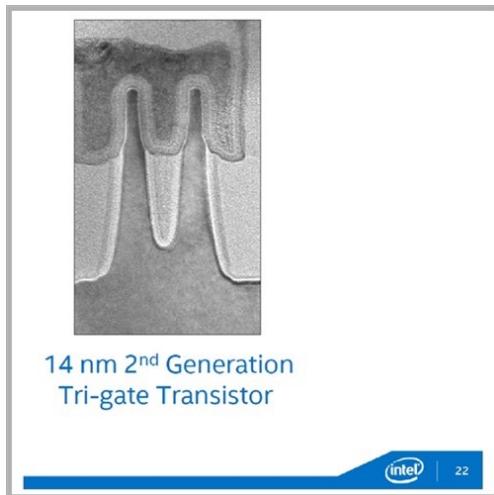
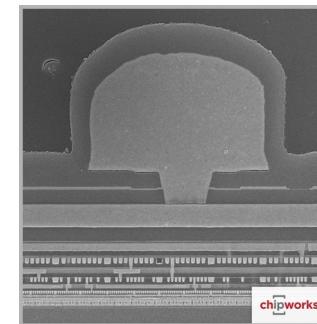
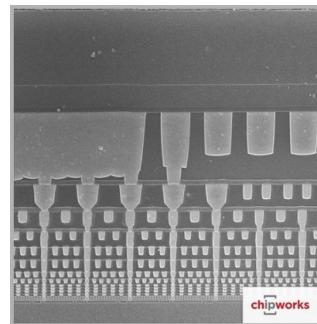
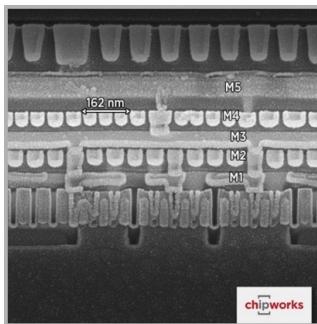


# CMOS Circuit

- CMOS: Complementary Metal-Oxide Semiconductor
- Contain both N-type and P-type MOS transistors
  - For all inputs, make sure that output is either connected to GND or +, not both



# A transistor under a microscope

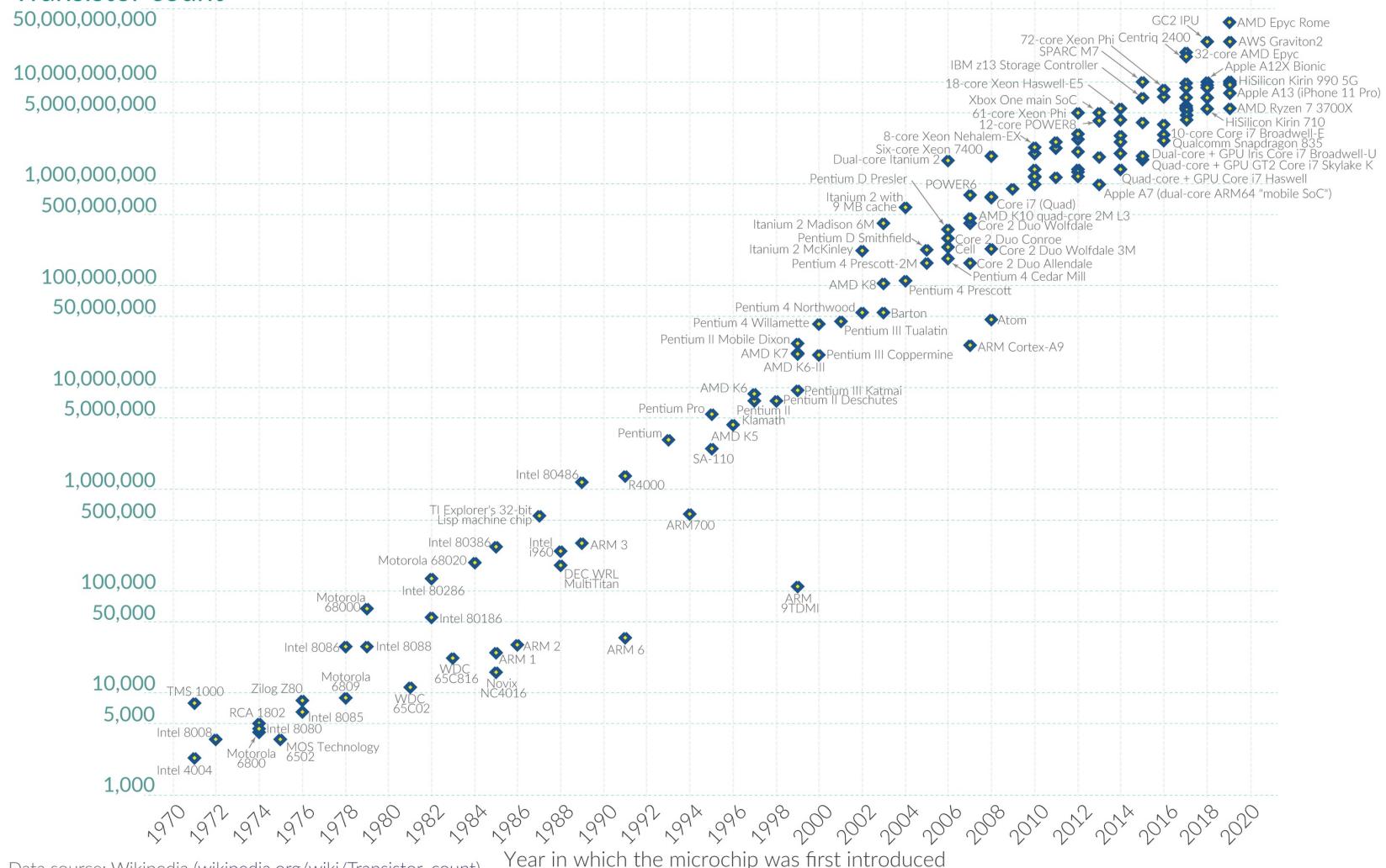


# Moore's Law: The number of transistors on microchips doubles every two years

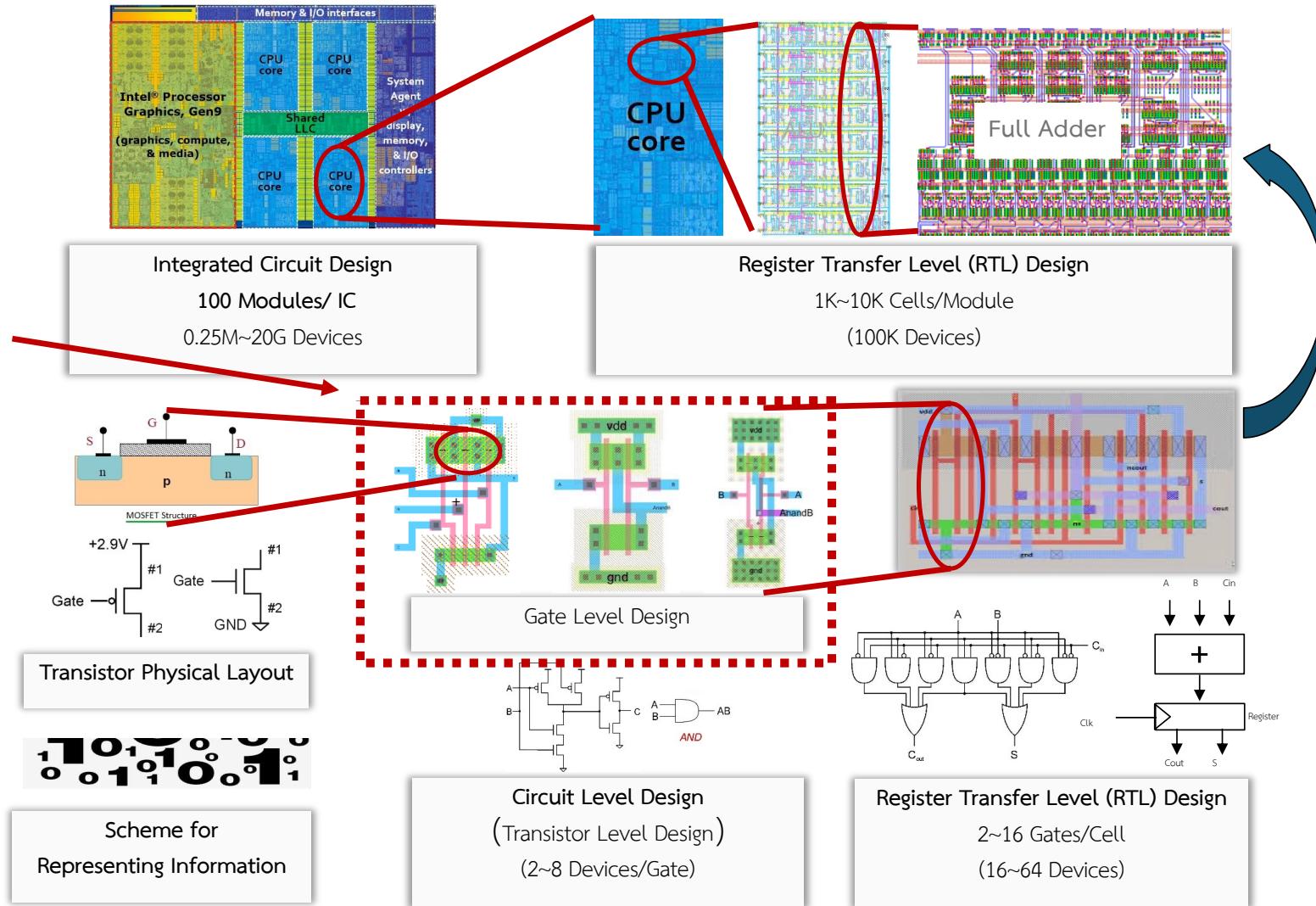
Moore's law describes the empirical regularity that the number of transistors on integrated circuits doubles approximately every two years. This advancement is important for other aspects of technological progress in computing – such as processing speed or the price of computers.

Our World  
in Data

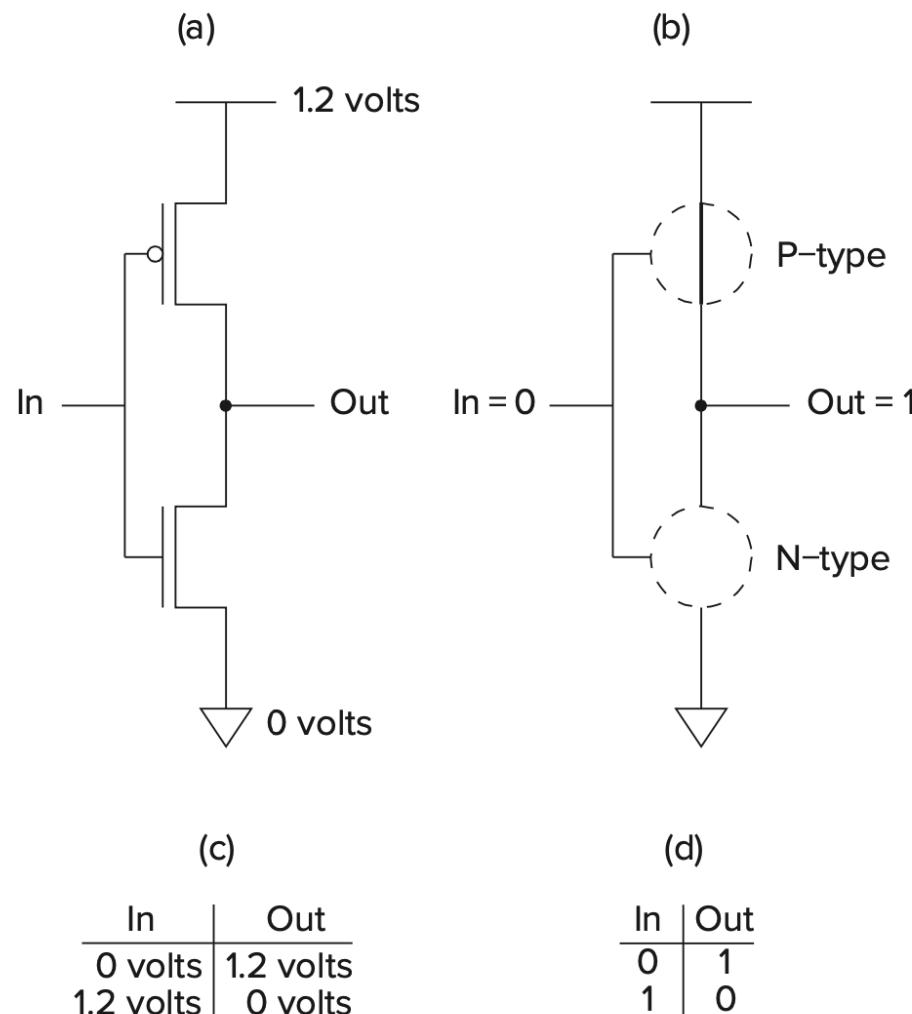
## Transistor count



# Approach: Bottom Up



## Inverter (NOT Gate)



A CMOS inverter.

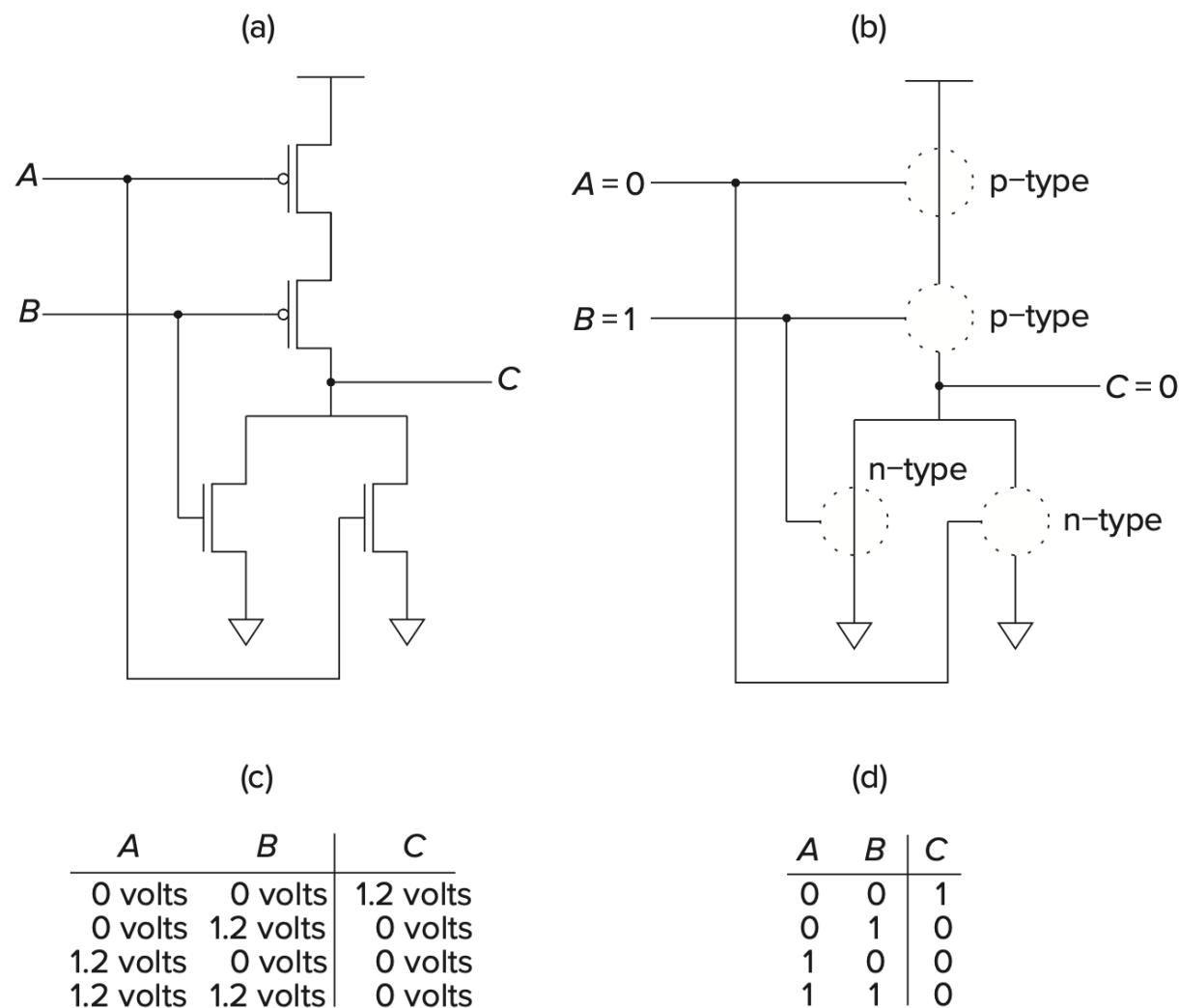


Figure 3.5 The NOR gate.

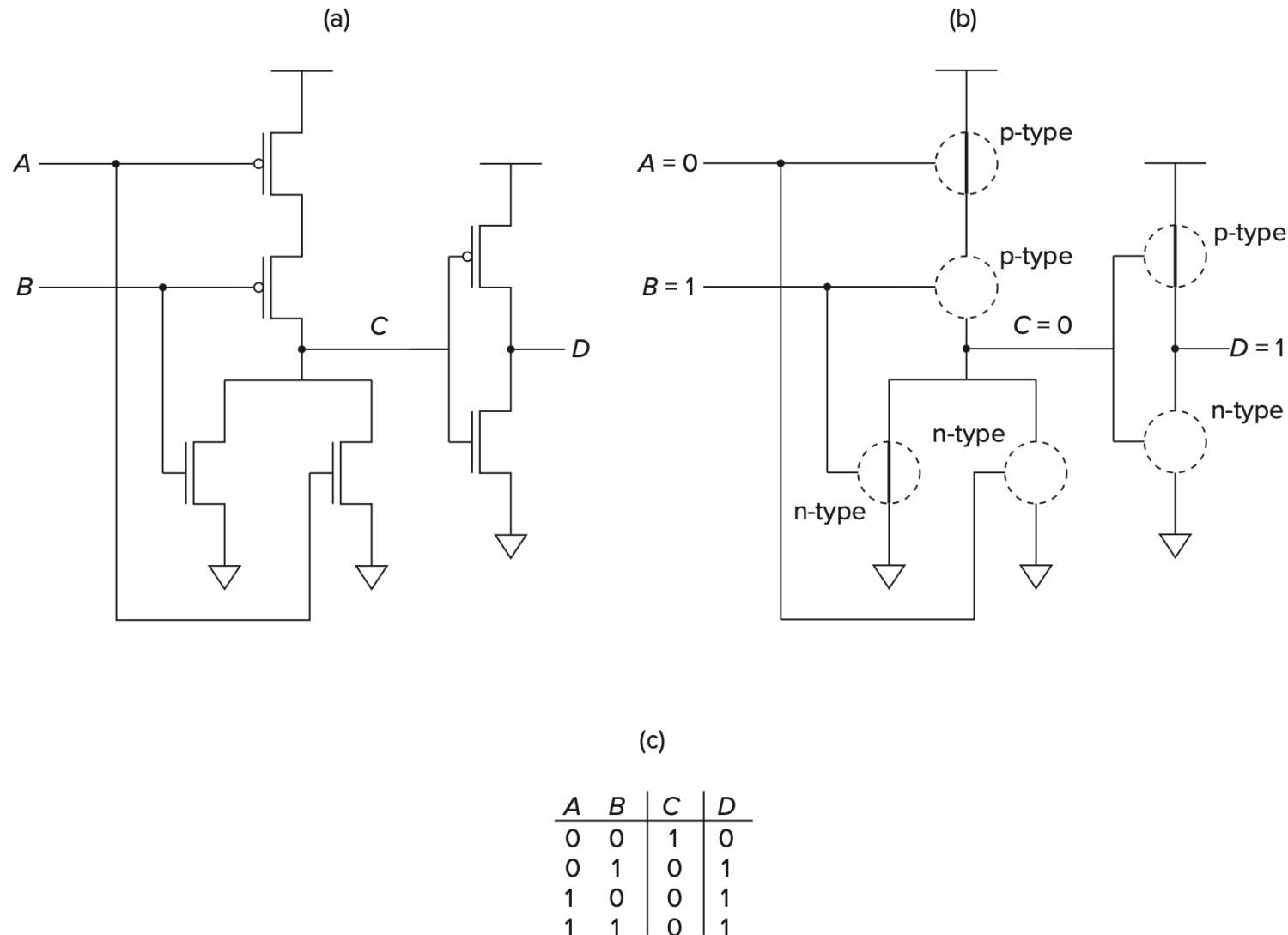
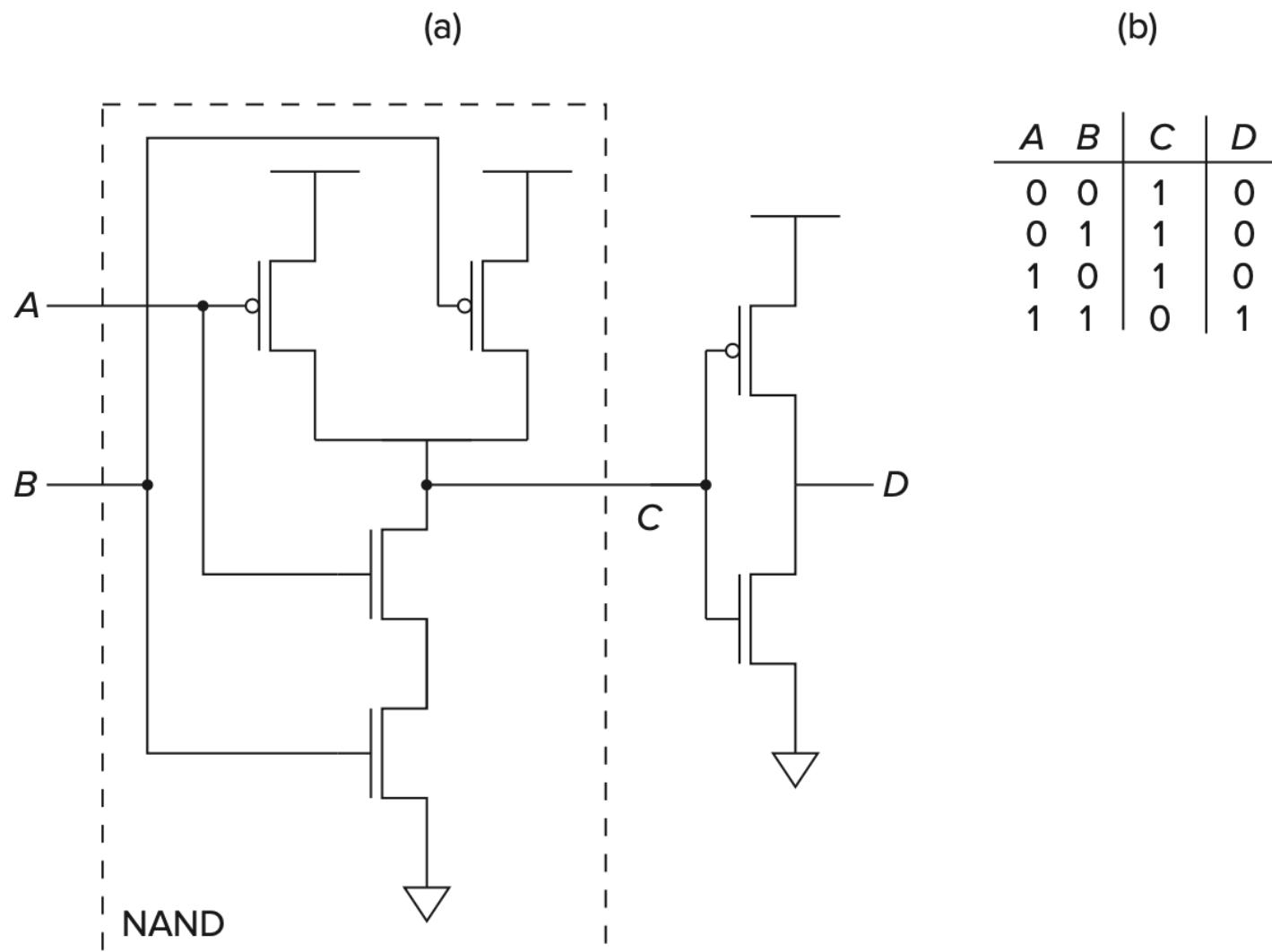
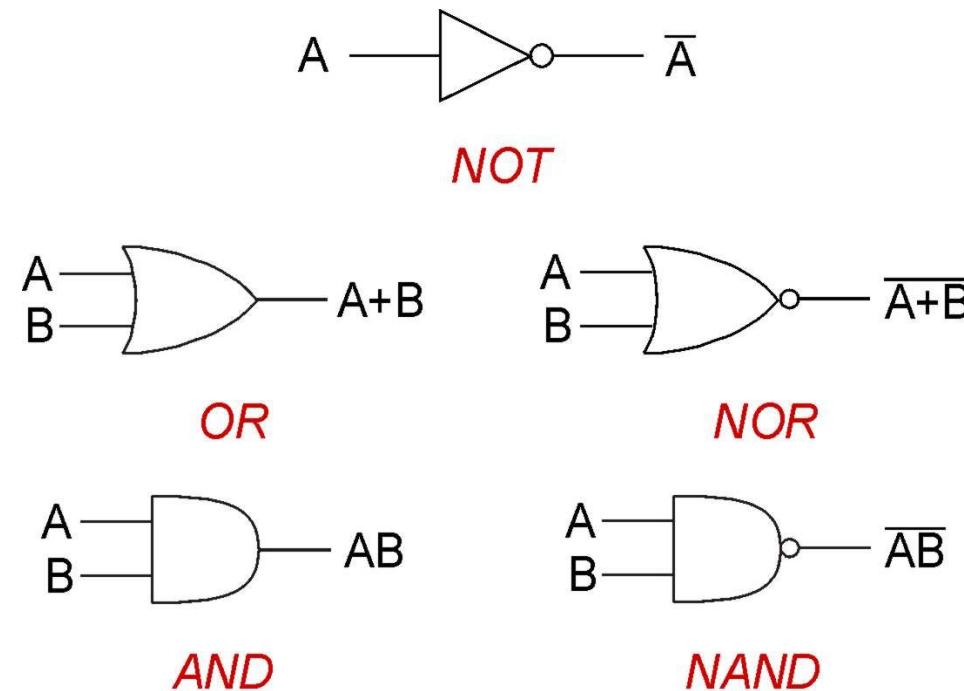


Figure 3.6 The OR gate.



**Figure 3.8** The AND gate.

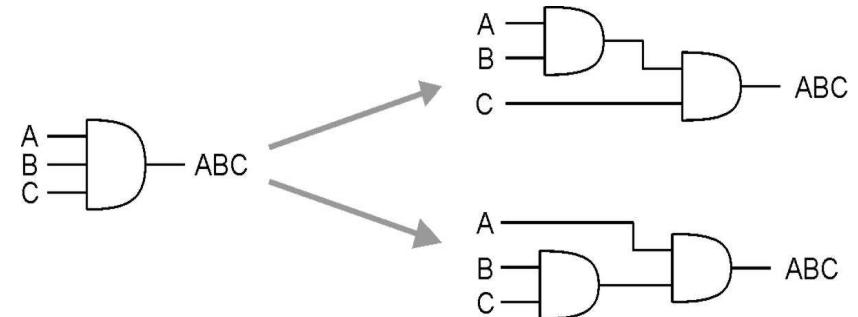
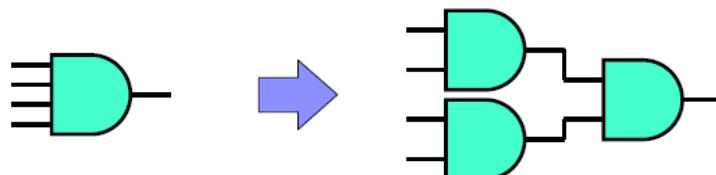
## Basic logic gates



- More complicated gates from transistors possible ex. XOR

## More than 2 Inputs?

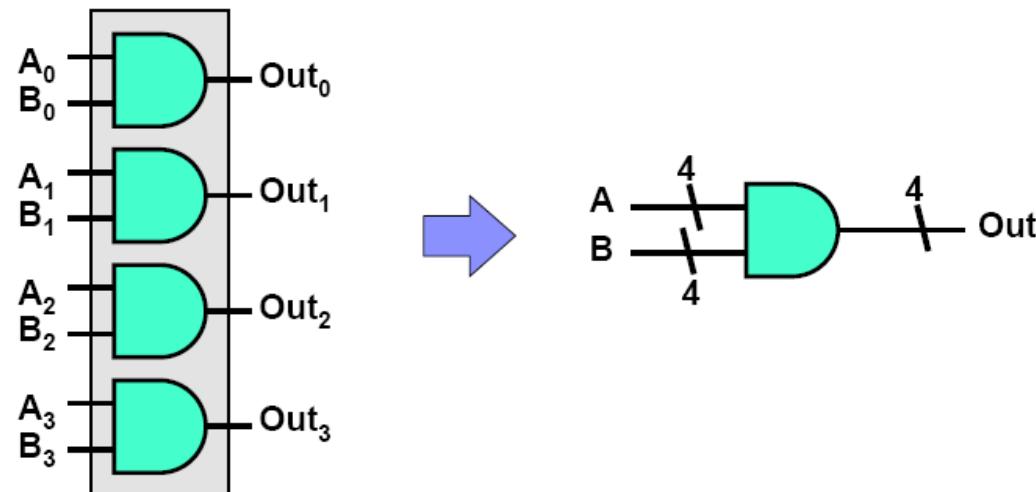
- AND/OR can take any number of inputs.
  - AND = 1 if all inputs are 1.
  - OR = 1 if any input is 1.
  - Similar for NAND/NOR.
- Can implement with multiple two-input gates, or with single CMOS circuit.



A	B	C	OUT
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	0
1	0	0	0
1	0	1	0
1	1	0	0
1	1	1	1

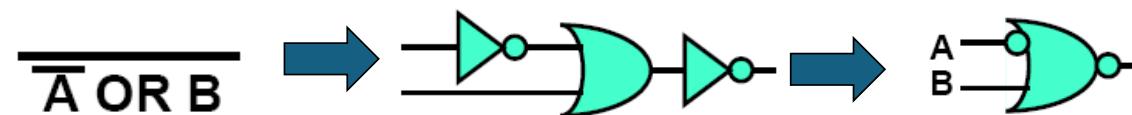
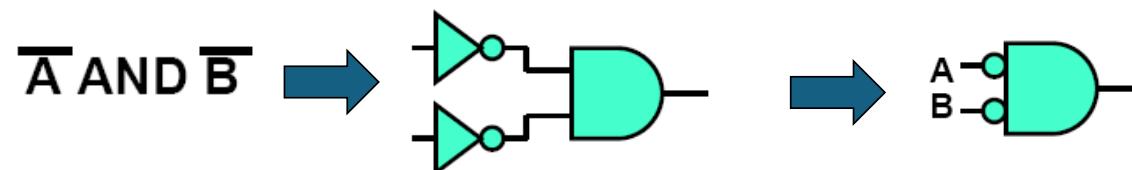
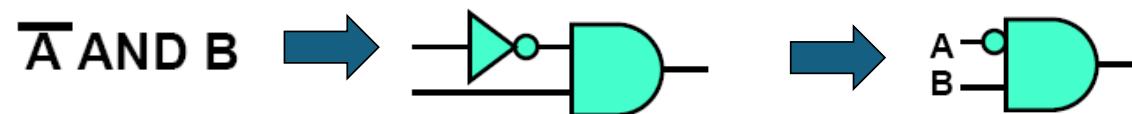
## Visual Shorthand for Multi-bit Gates

- Use a cross-hatch mark to group wires
  - Example: calculate the AND of a pair of 4-bit numbers
  - A<sub>3</sub> is “high-order” or “most-significant” bit
  - If “A” is 1000, then A<sub>3</sub> = 1, A<sub>2</sub> = 0, A<sub>1</sub> = 0, A<sub>0</sub> = 0



## Shorthand for Inverting Signals

- Invert a signal by adding either
  - A before/after a gate
  - A “**bar**” over letter



# Combinational Logic Circuits

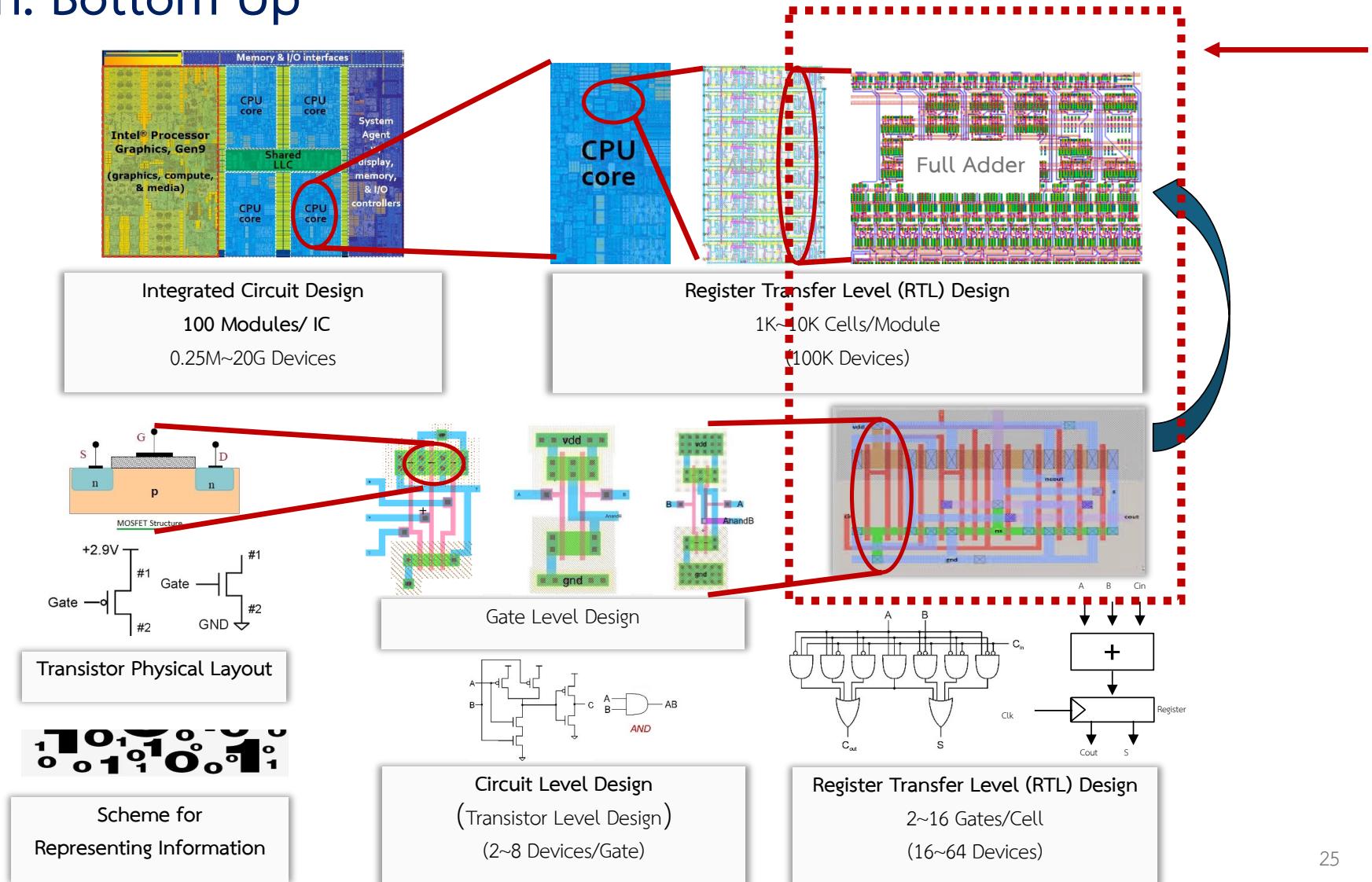
## Combinational Logic Circuits

- Outputs are strictly dependent on the combination of input values
- Stateless
- Building Functions from Logic Gates
- Decoder
- Mux: Multiplexer
- A One-Bit Adder

## Sequential Logic Circuits

- Output depends on the sequence of inputs (past and present)
- Stores information (state) from past inputs

# Approach: Bottom Up



# Decoder

- $n$  inputs,  $2^n$  outputs
  - Exactly one output is 1 for each possible input pattern

*2-bit  
decoder*

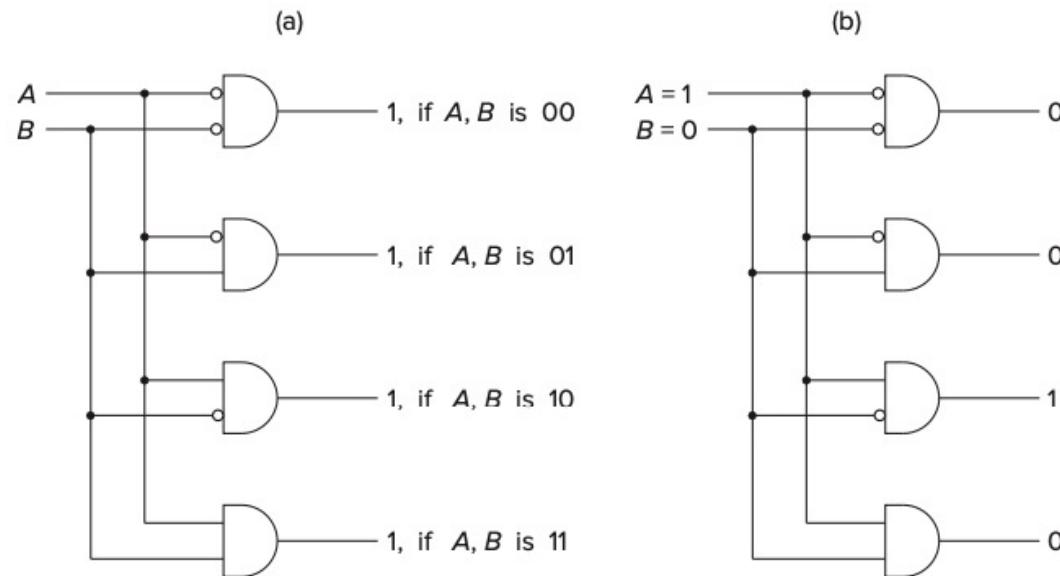


Figure 3.11 A two-input decoder.

## Multiplexer (MUX)

- $n$ -bit selector and  $2^n$  inputs, one output
  - output equals one of the inputs, depending on selector

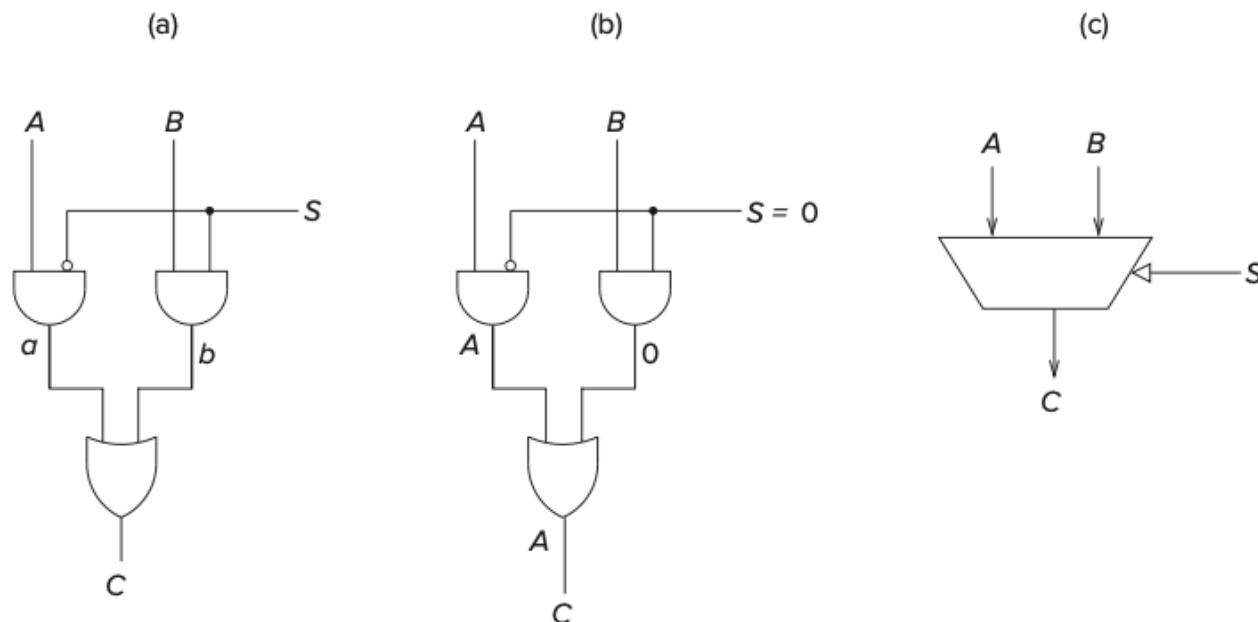


Figure 3.12 A 2-to-1 mux.

## 4-Input Multiplexer

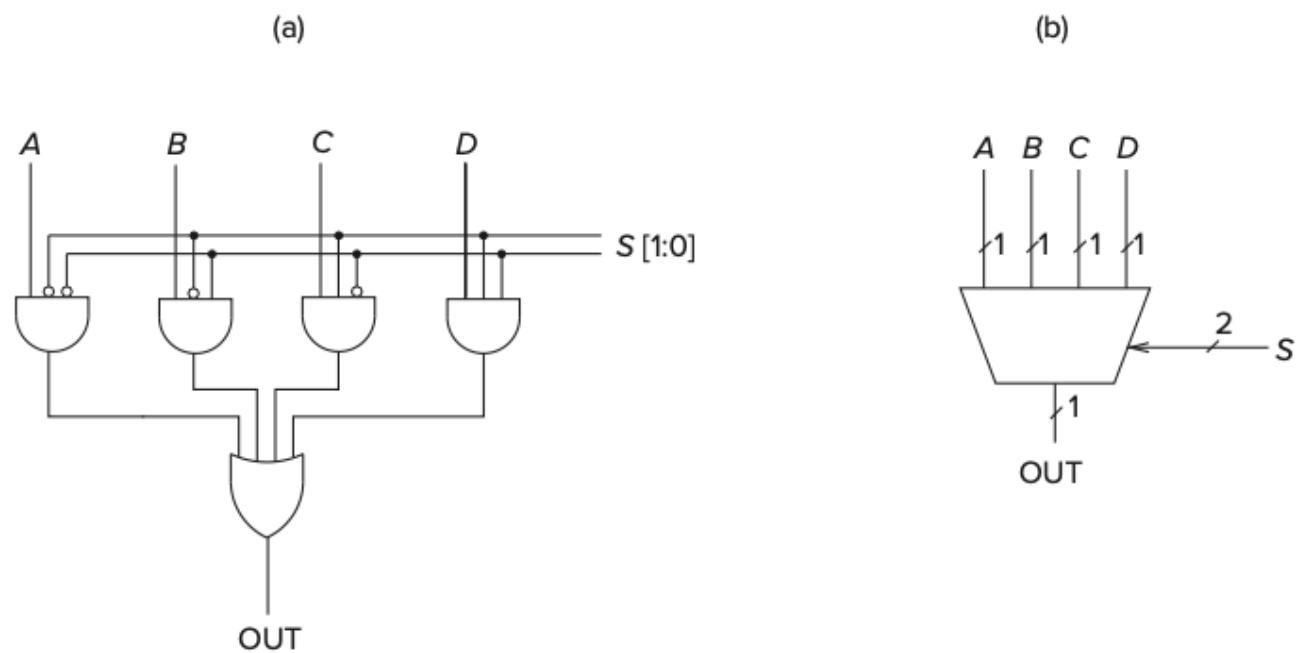
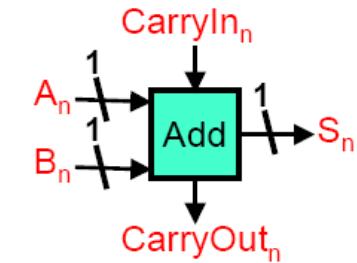
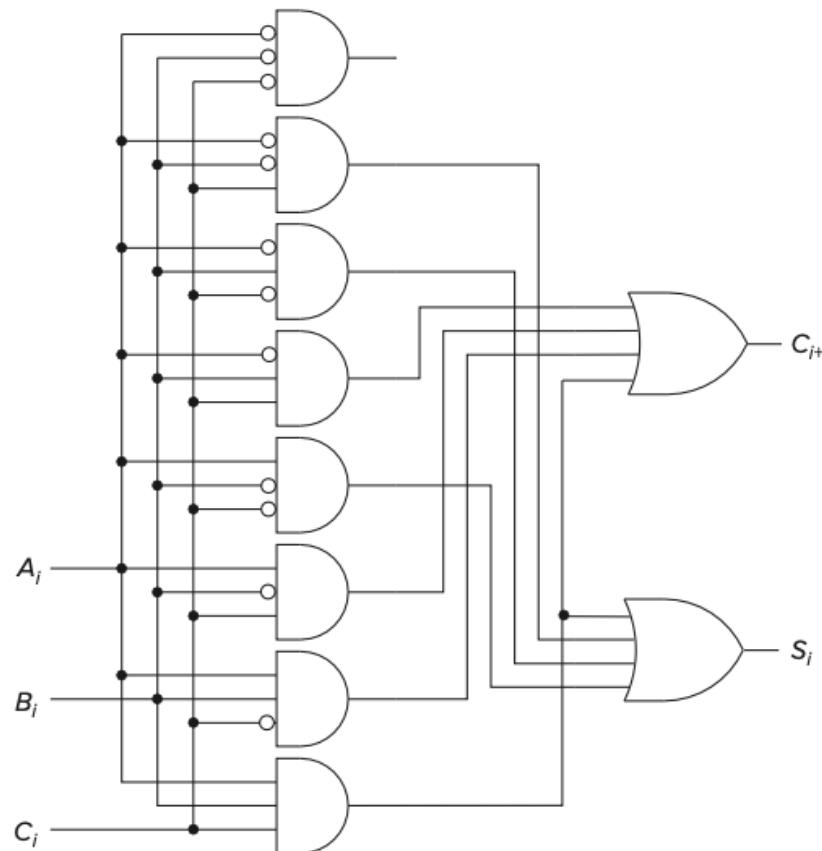


Figure 3.13 A four-input mux.

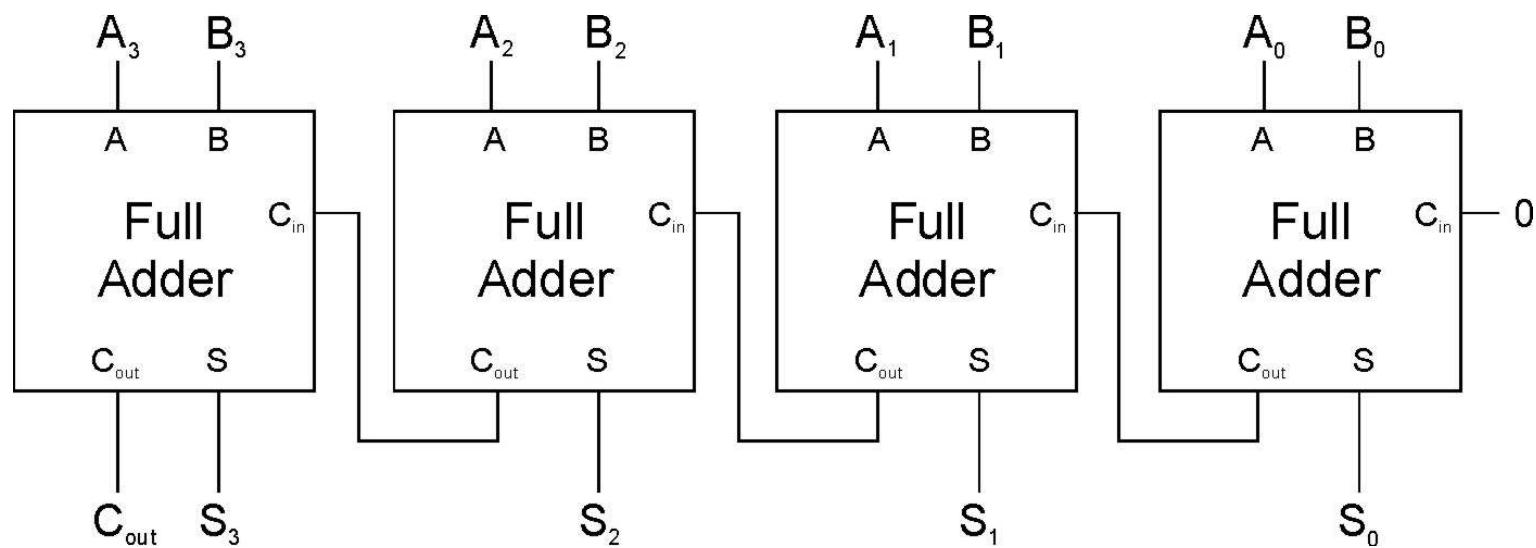
## 1-Bit Adder (Full Adder)

- Add 2-bit and carry-in, produce one-bit sum and carry-out.

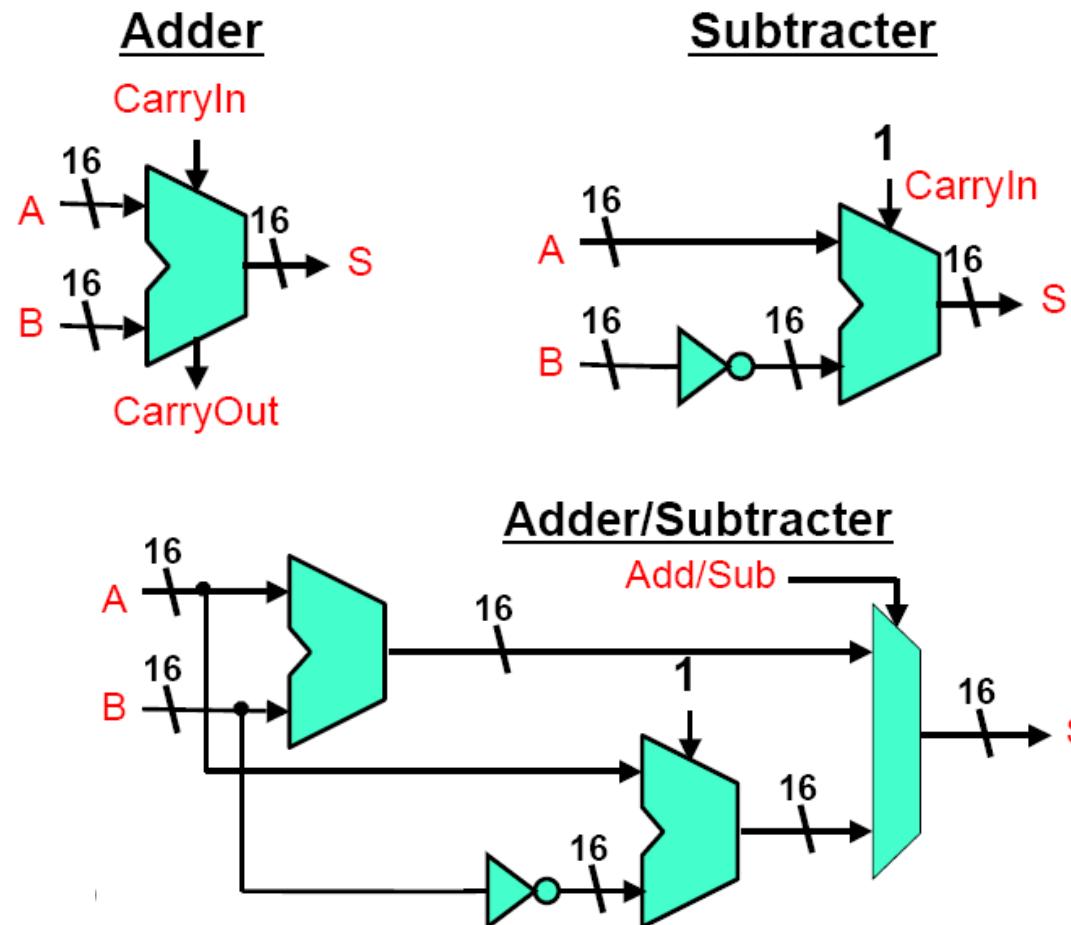


A	B	$C_{in}$	S	$C_{out}$
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	1
1	0	0	1	0
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1

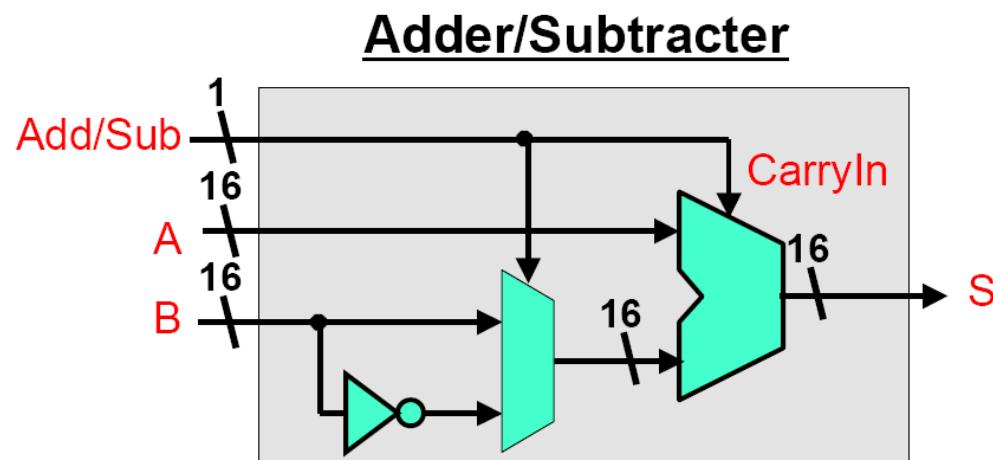
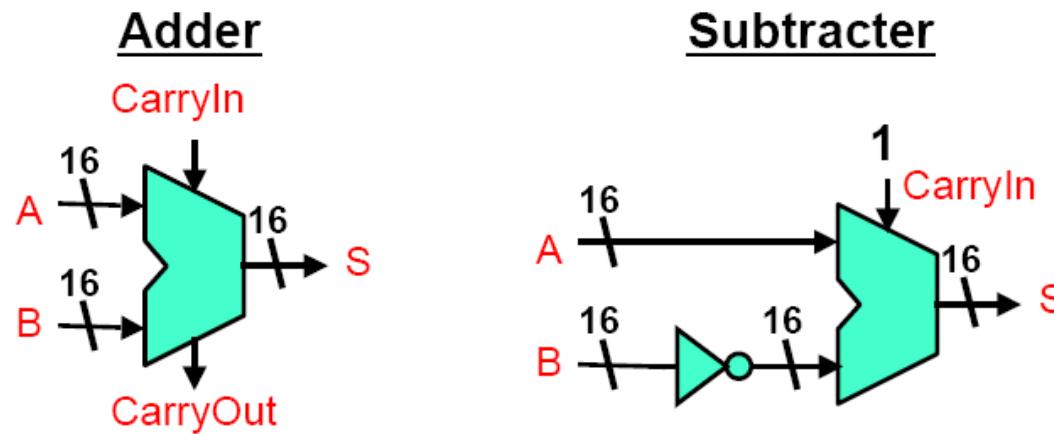
## 4-bit Adder



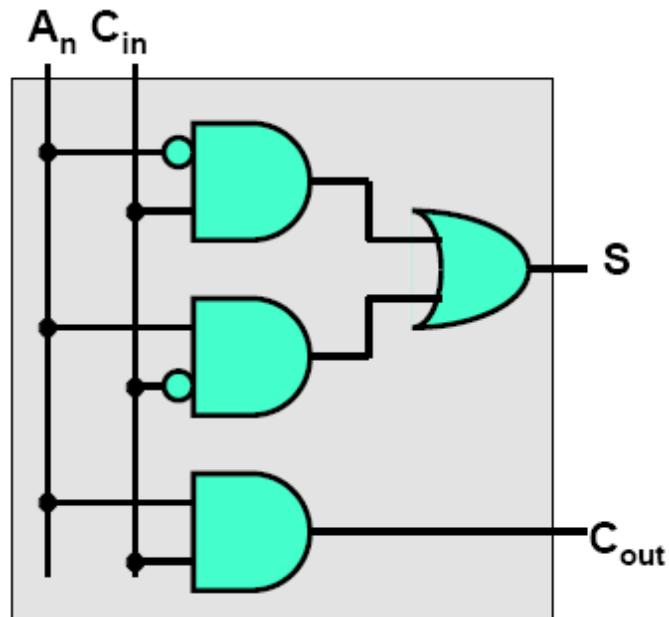
## Adder/Subtractor - Approach #1



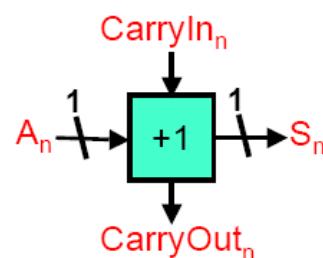
## Adder/Subtractor - Approach #2



## One-bit Incrementor



$A$	$C_{in}$	$S$	$C_{out}$
0	0	0	0
0	1	1	0
1	0	1	0
1	1	0	1



# Basic Storage Elements

## Combinational Logic Circuits

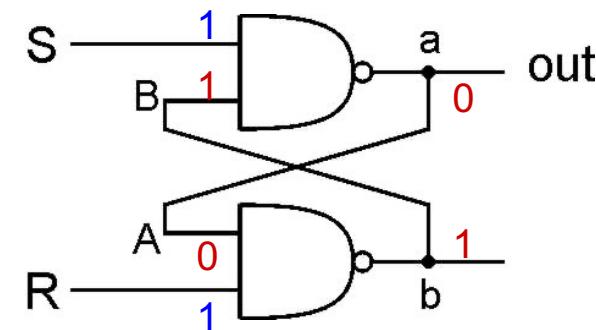
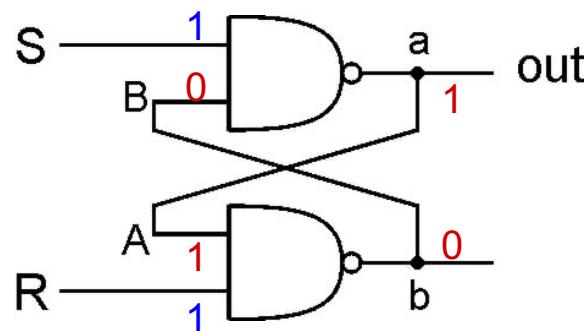
- Always gives the same output for a given set of inputs
  - Ex: adder always generates sum and carry, regardless of previous inputs

## Sequential Logic Circuits

- Output **depends on the sequence of inputs** (past and present)
- Stores information (state) from past inputs
  - A given input might produce different outputs, depending on the stored information
  - Ex: ATM
- Useful for building “memory” elements and “state machines”

## The R-S Latch: A simple example of a storage element

- Store 1-bit of information (0 or 1)
- **R** is used to “**reset**” or “**clear**” the element – set it to **0**
- **S** is used to “**set**” the element – set it to **1**

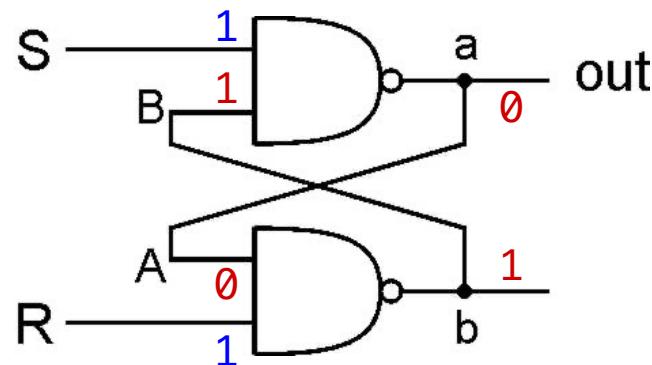


\*\* If both R and S are one, out could be either zero or one

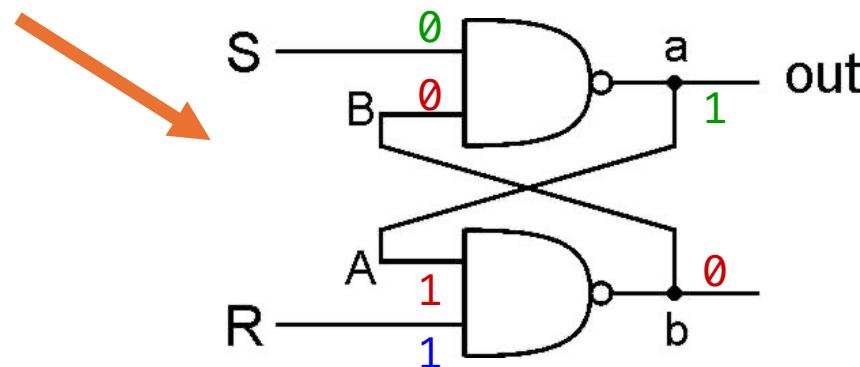
- The Quiescent State holds its previous value

## Setting the R-S Latch

- Suppose we start with output = 0, then change S to 0



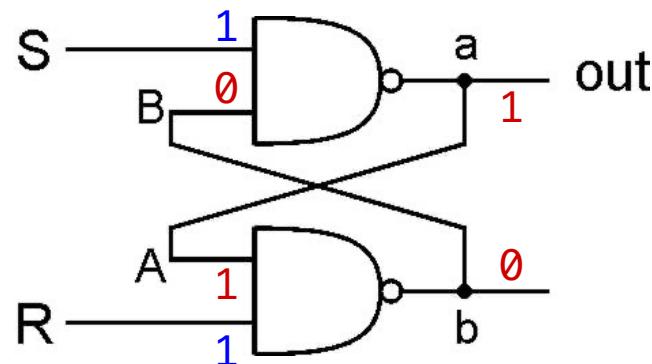
Output changes to one.



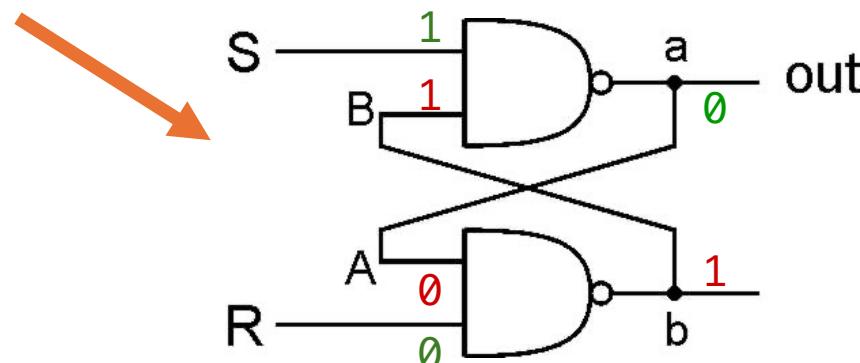
\*\* Then set S=1 to “store” value in quiescent state.

## Clearing the R-S Latch

- Suppose we start with output = 1, then change R to 0



Output changes to one.



\*\* Then set R=1 to “store” value in quiescent state.

## R-S Latch Summary

$R = S = 1$

- hold current value in latch

$S=1$  and  $R = \text{change from } 1 \rightarrow 0$

- set value to 0

$R=1$  and  $S = \text{change from } 1 \rightarrow 0$

- set value to 1

$R = S = 0$

both outputs equal one

final state determined by electrical properties of gates

## Gated D-Latch

- Two inputs: D (data) and WE (write enable)
  - when WE = 1, latch is **set to value of D** ; S = NOT(D), R = D
  - when WE = 0, latch **holds previous value** ; S = R = 1

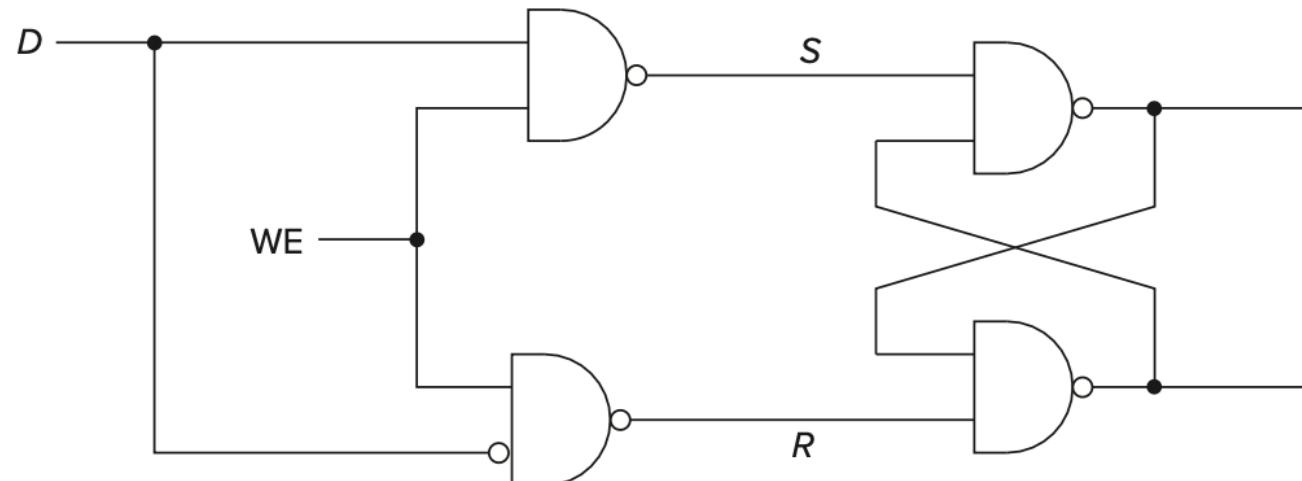
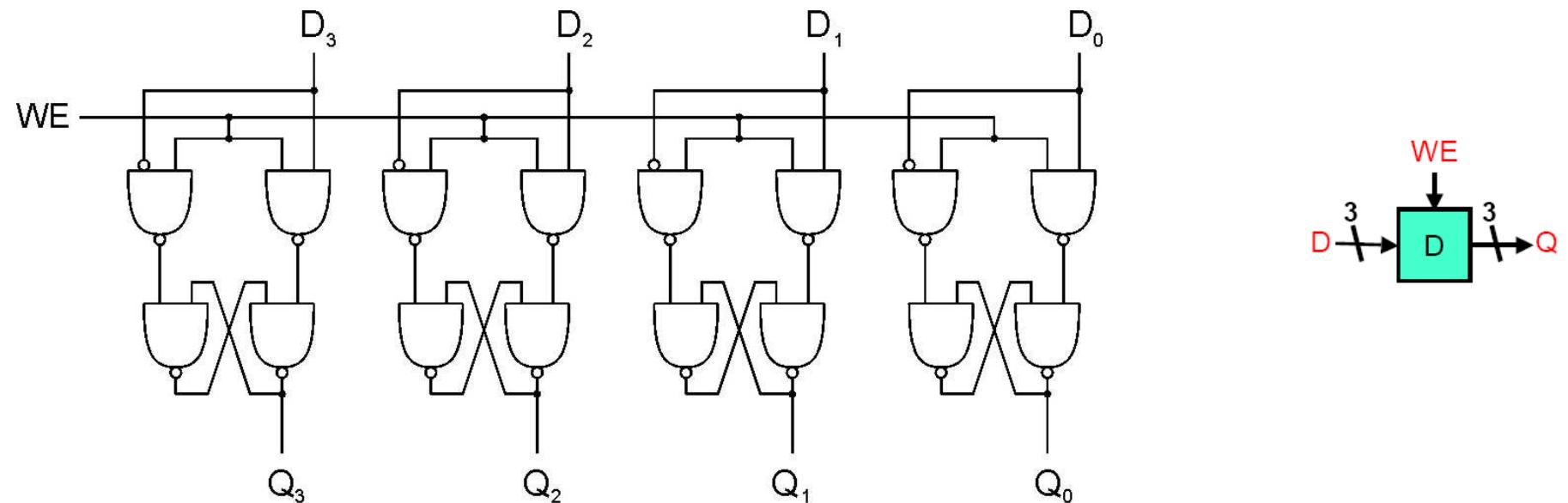


Figure 3.19 A gated D latch.

# Register

- Store a multi-bit value.
  - Collection of D-latches, all controlled by a common WE.
  - When  $WE=1$ ,  $n$ -bit value  $D$  is written to register



## Representing Multi-bit Values

- Number bits from right (0) to left (n-1)
- Use brackets to denote range:  $D[ l : r ]$  denotes bit  $l$  to bit  $r$ , from *left* to *right*

$$A = \begin{matrix} & 15 \\ 0 & 1 & 0 & 1 & 0 & 0 & 1 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 0 \\ & 0 \end{matrix}$$

$A[14:9] = 101001$        $A[2:0] = 101$

The diagram illustrates the extraction of two multi-bit values from a larger bit vector A. Bit vector A is shown as a horizontal sequence of 16 bits, indexed from 15 down to 0. An orange arrow points from the label A[14:9] to the bits at indices 14, 13, 12, 11, 10, and 9. Another orange arrow points from the label A[2:0] to the bits at indices 2, 1, 0.

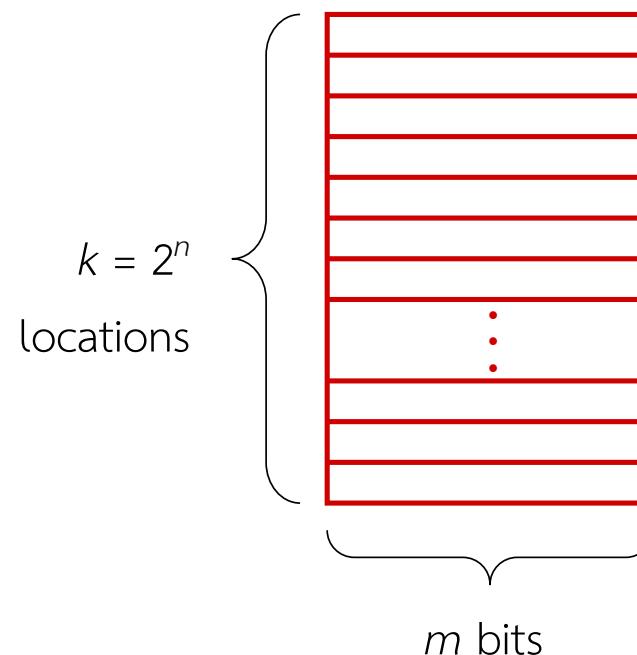
# Memory

- A memory can be built – a logical  $k \times m$  array of stored bits.

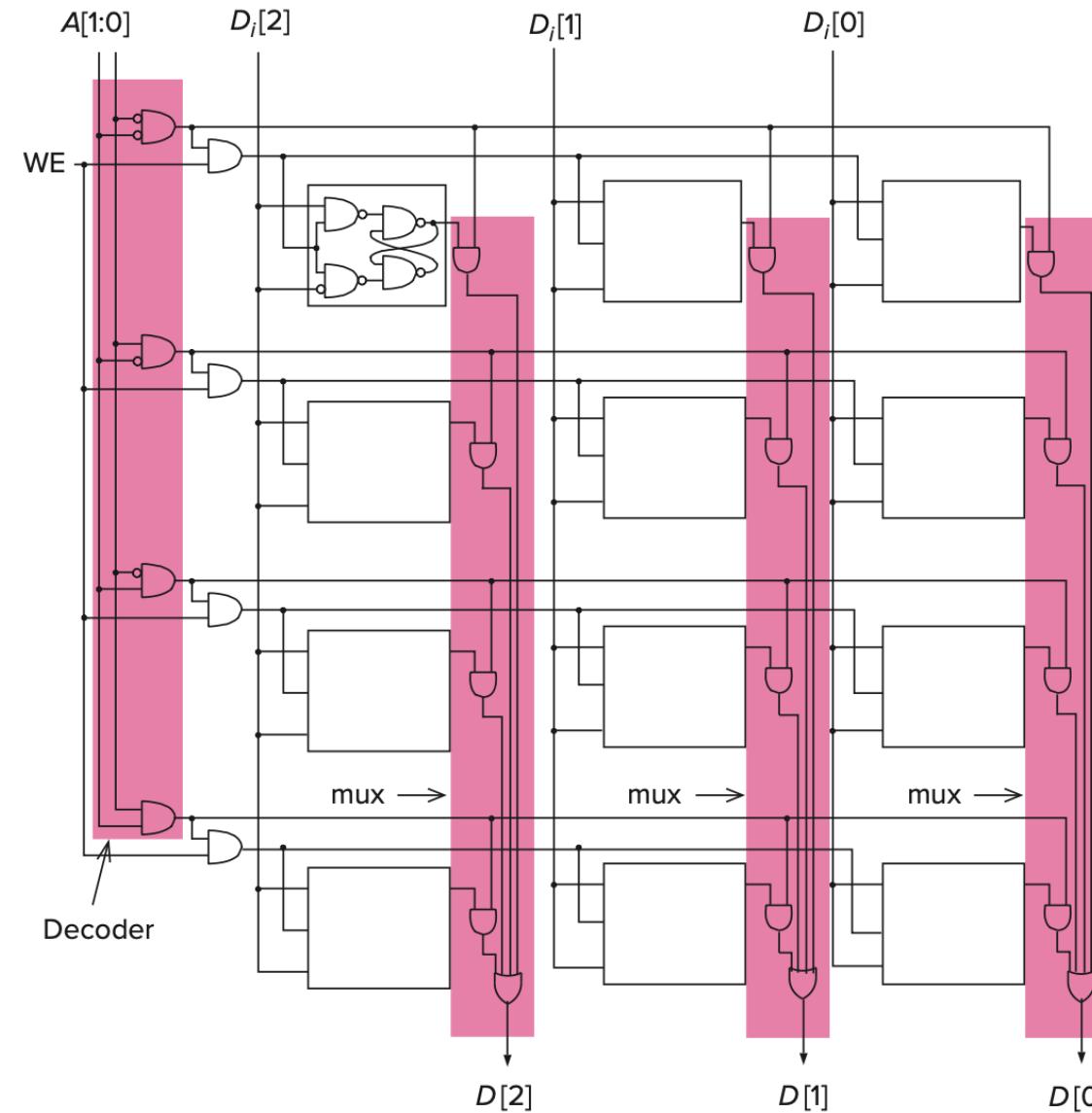
Address Space:

number of locations

(usually a power of 2)

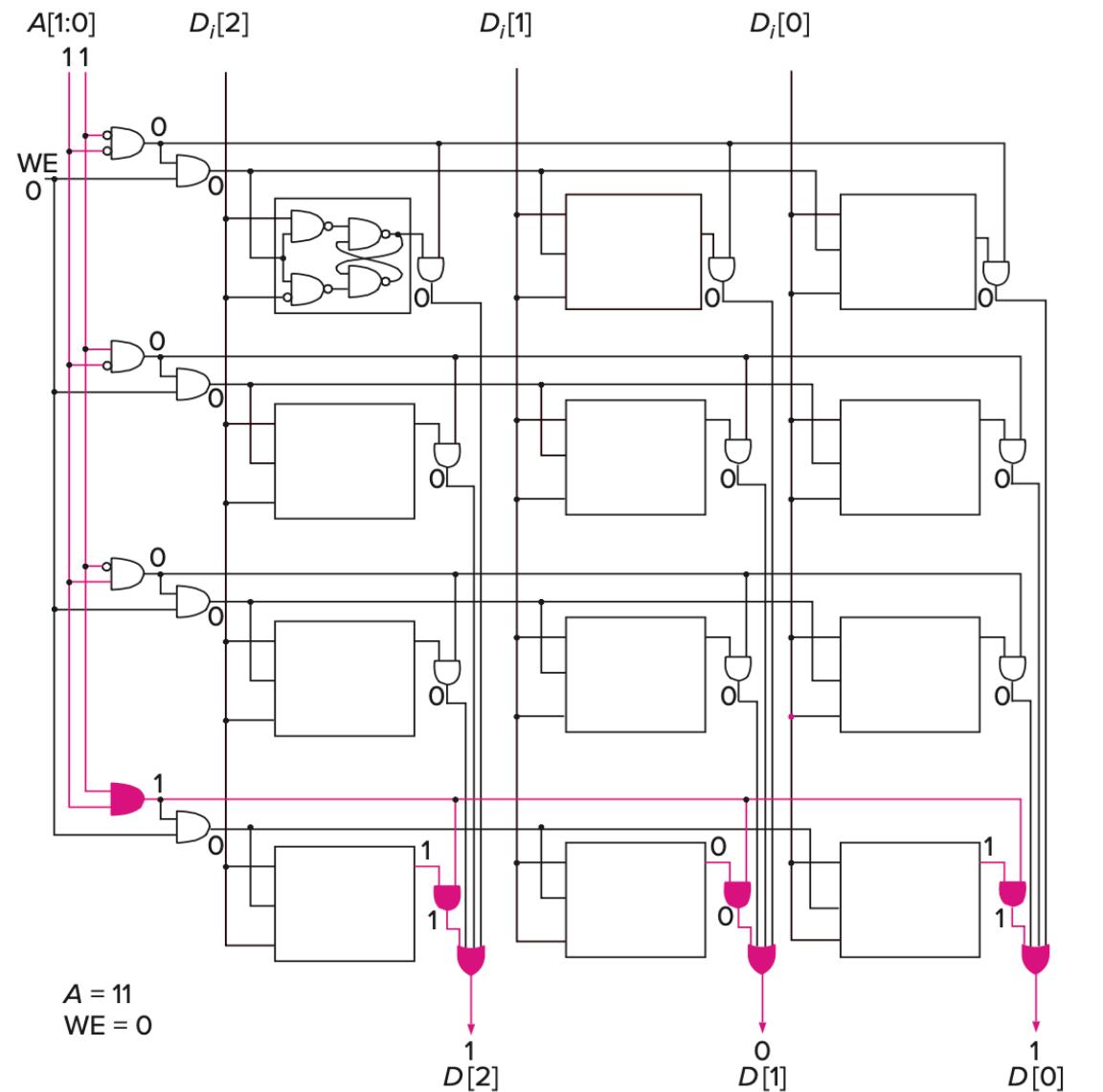


## $2^2$ -by-3-bit memory

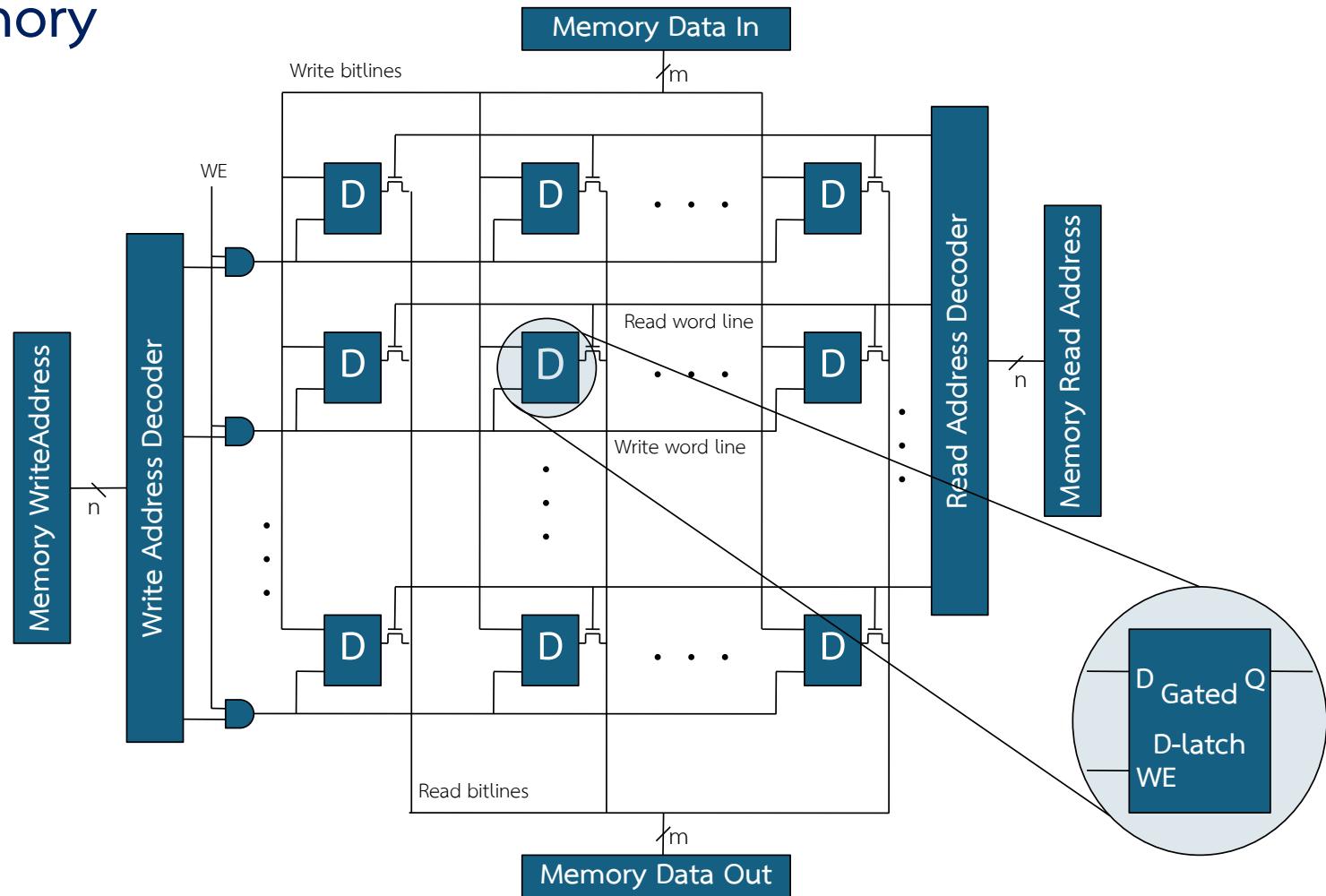


## $2^2$ -by-3-bit memory

Reading location 3<sup>rd</sup>



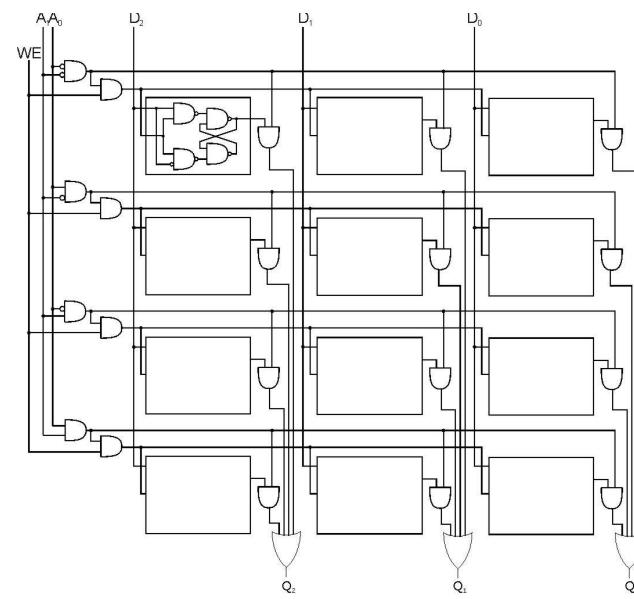
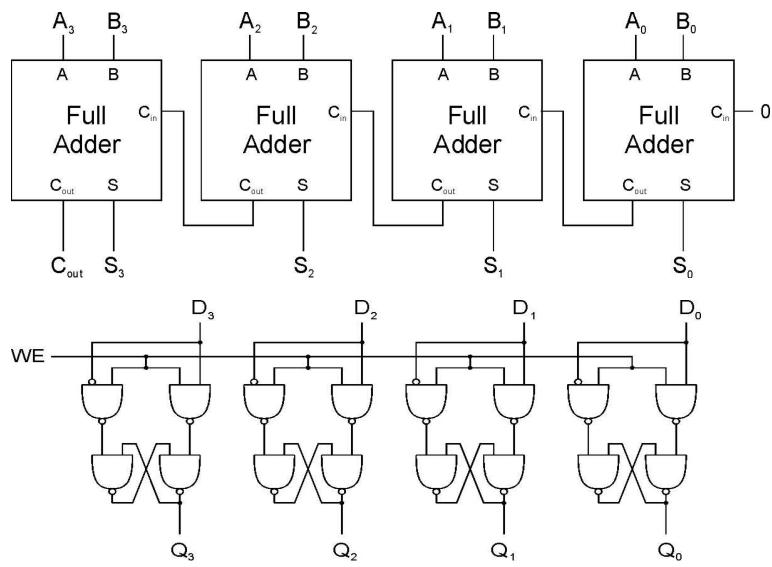
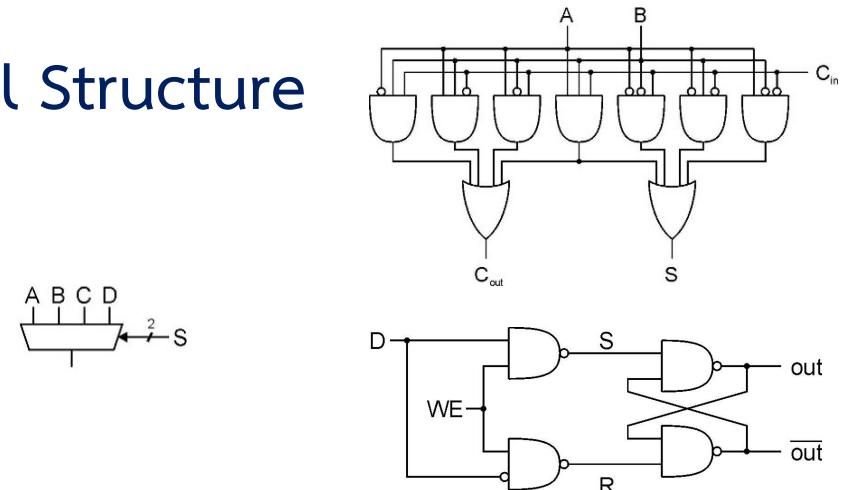
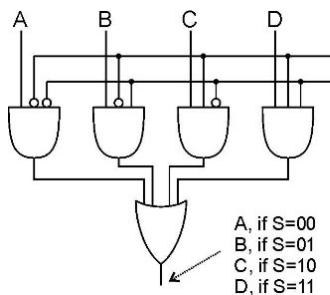
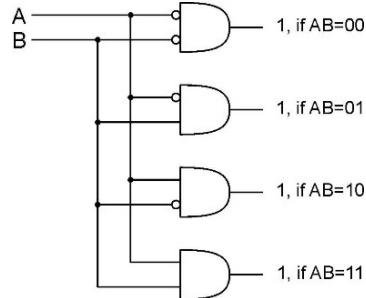
# SRAM Memory



## Wrap-up Memory

- This is a not the way actual memory is implemented.
  - fewer transistors, much more dense, relies on electrical properties
- But the logical structure is very similar.
  - address decoder
  - word select line
  - word write enable
- Two basic kinds of RAM (Random Access Memory)
  - Static RAM (SRAM): fast, not very dense (**bitcell is a latch**)
  - Dynamic RAM (DRAM): slower but denser, bit storage must be periodically refreshed
    - each bit-cell is a capacitor (like a leaky bucket) that decays

# Wrap-up Memory and Basic Logical Structure

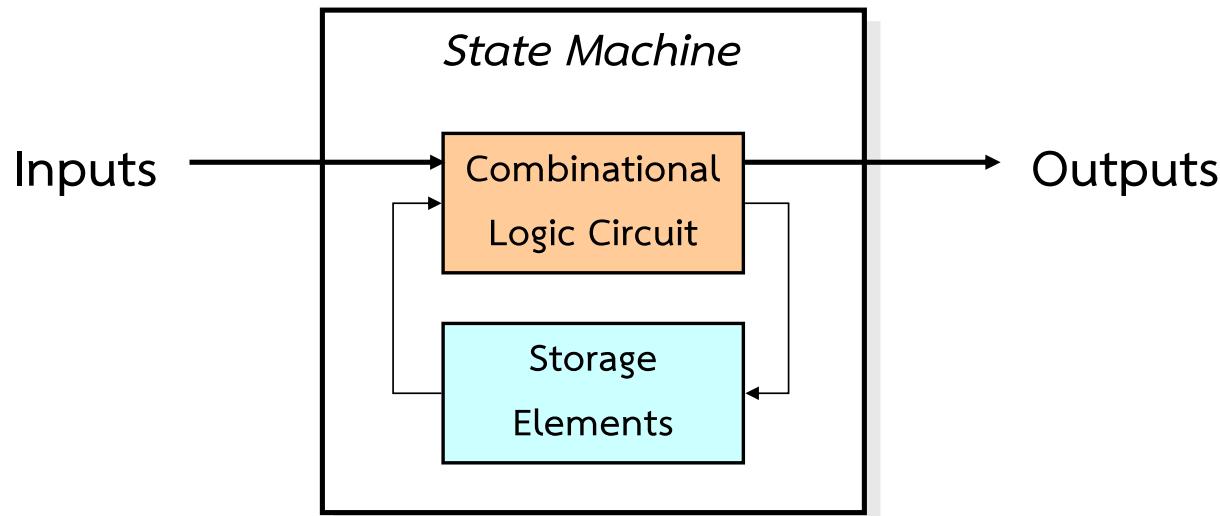


# Sequential Logic Circuits

# State Machine

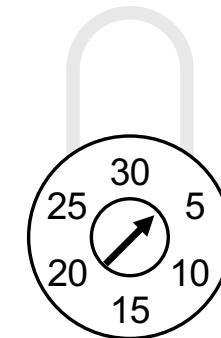
Another type of sequential circuit

- Combines combinational logic with storage
- “Remembers” state, and changes output (and state) based on **inputs** and **current state**



# Combinational vs. Sequential

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## Combinational

Success depends only on the  
values, not the order in which  
they are set

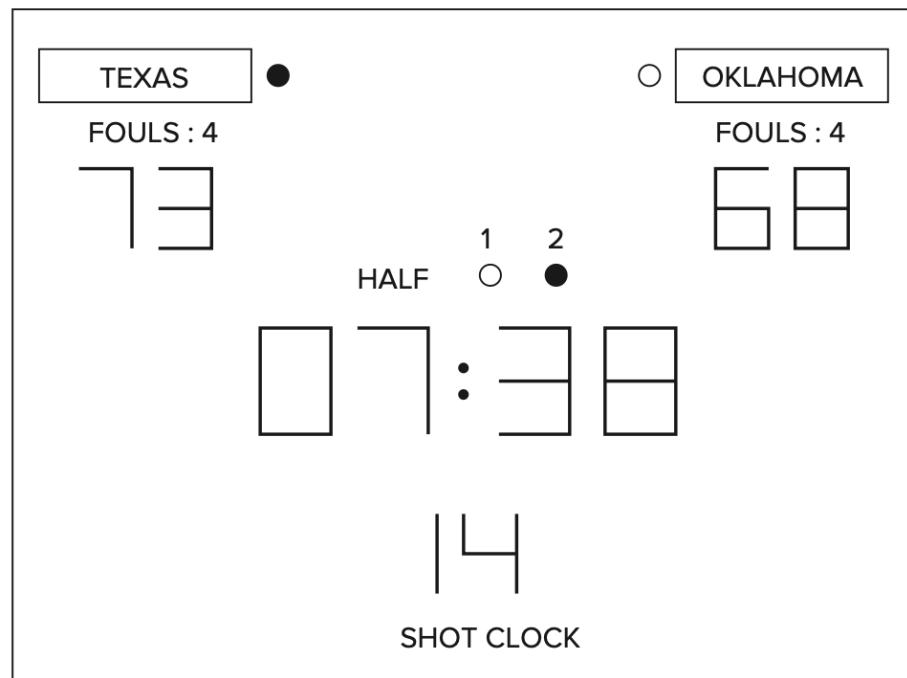
## Sequential

Success depends on  
the sequence of values  
(EX: R13-L22-R3)

## The Concept of State

- *The state of a system is a snapshot of all the relevant elements of the system at the moment the snapshot is taken.*

- Ex: State of a game of basketball

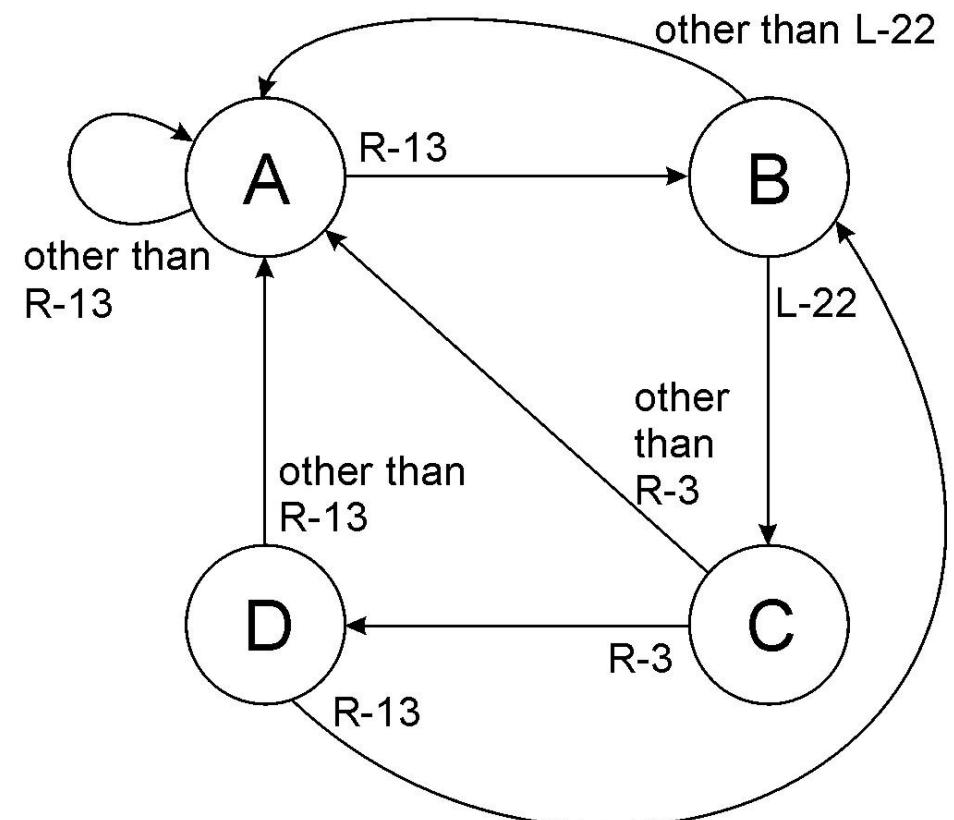


## State Diagram

\*\* R13-L22-R3 to Unlock

Back to Sequential Lock

- A: The lock is **not open**, and no relevant operations have been performed.
- B: The lock is **not open**, and the user has completed the **R-13** operation.
- C: The lock is **not open**, and the user has completed **R-13**, followed by **L-22**.
- D: The lock is **open**.



## Finite State Machine

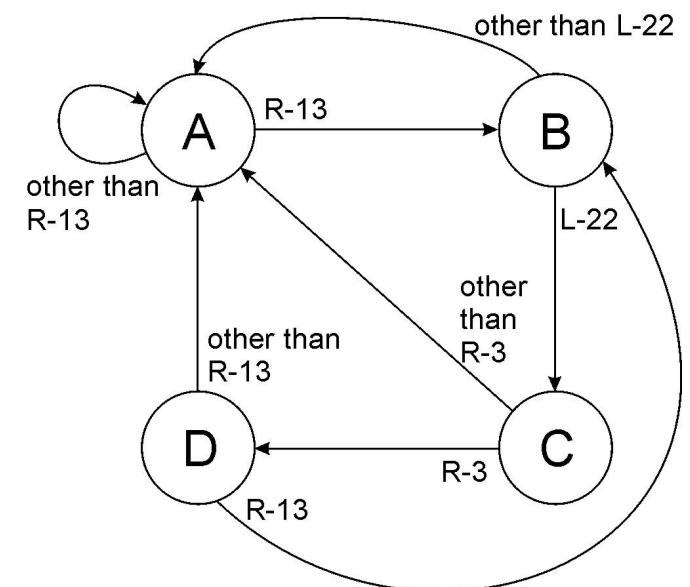
1. a finite number of states
2. a finite number of external inputs
3. a finite number of external outputs
4. an explicit specification of all state transitions
5. an explicit specification of what determines each external output value.

Often described by a state diagram

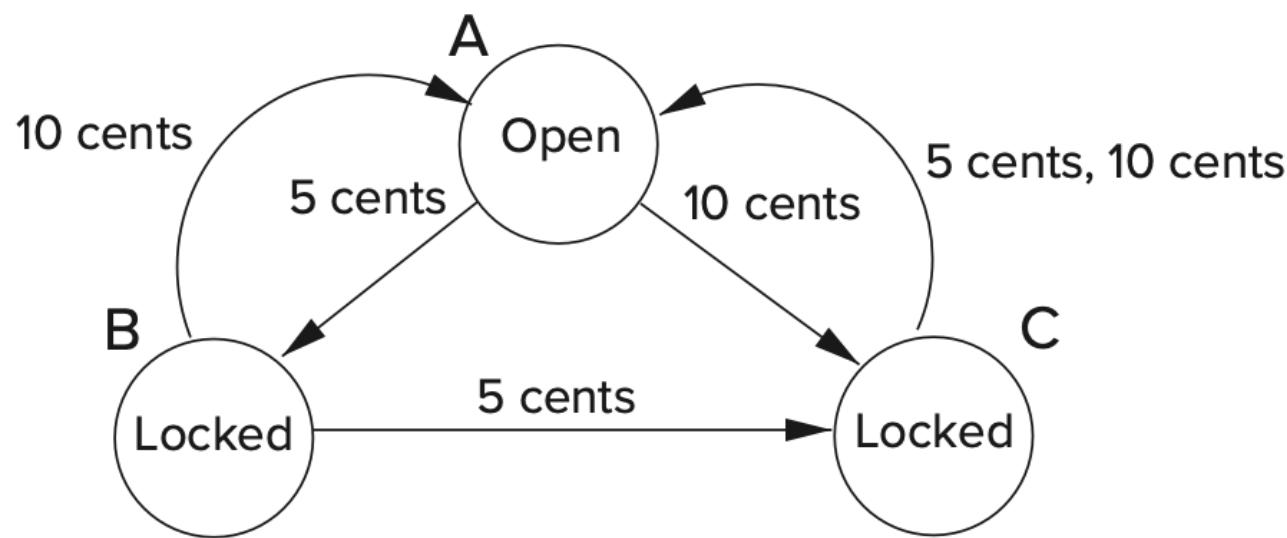
- Inputs may cause state transitions
- Outputs are associated with each state (or with each transition)

## Recall Combination Lock

- Four states A, B, C, and D
- The external inputs are R13, L22, R3, and R-other-than-13, L-other-than-22, and R-other-than-3
- The external output is either the lock is open, or the lock is not open
- The explicit specifications of all state transitions are shown by the arrows in the state diagram



Tao Bin



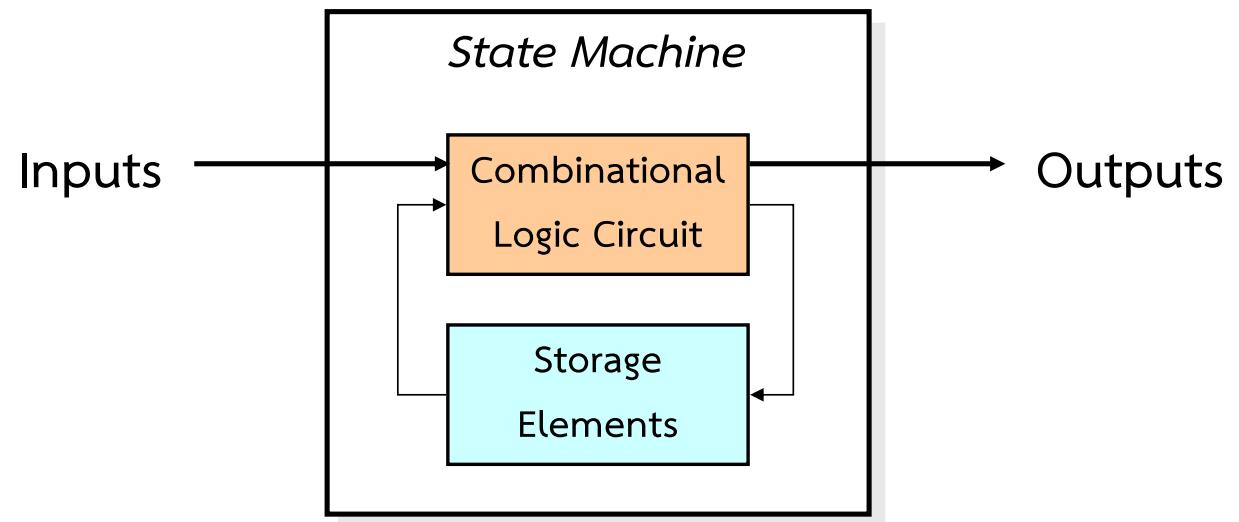
# Implementing a Finite State Machine

## Combinational logic

- Determine outputs and next state.

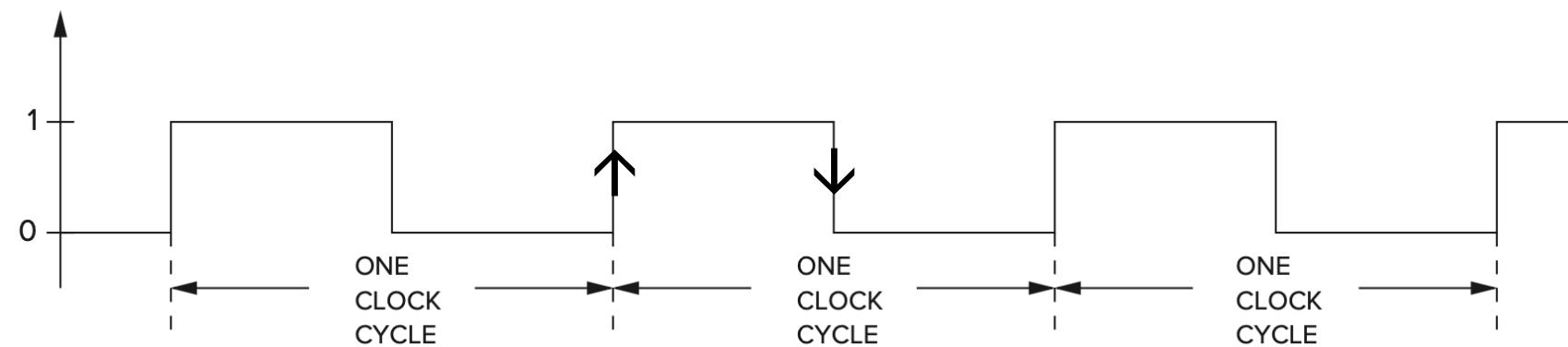
## Storage elements

- Maintain state representation.

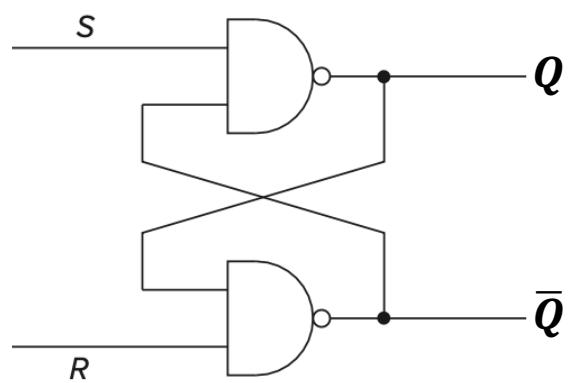


# The Clock

- Frequently, a clock circuit triggers transition from one state to the next.

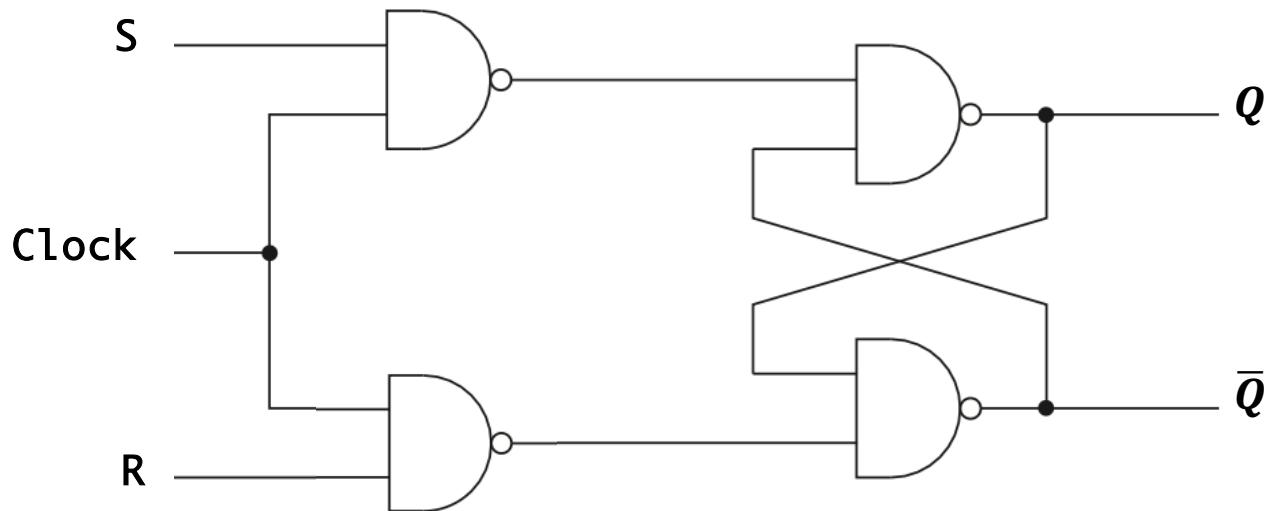


## Recall R-S Latch



S	R	$Q(t+1)$
0	0	-
0	1	1
1	0	0
1	1	$Q_t$

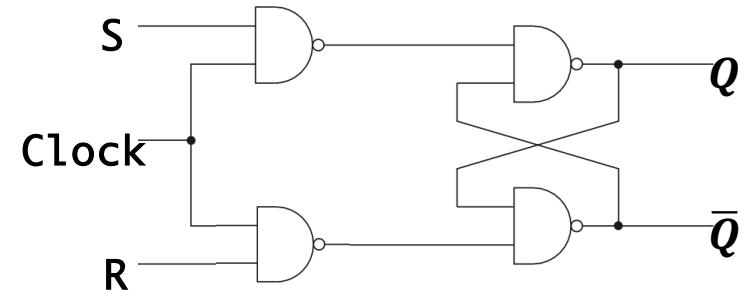
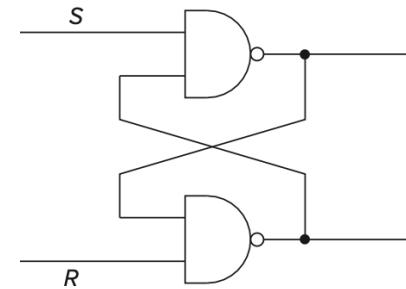
## Clocked RS Flip-Flop



Clock	R	S	$Q(t+1)$
$\downarrow$ or 0 or 1	X	X	$Q_t$ (no change)
$\uparrow$	0	0	$Q_t$ (no change)
$\uparrow$	0	1	1 (set)
$\uparrow$	1	0	0 (reset)
$\uparrow$	1	1	Not Allowed

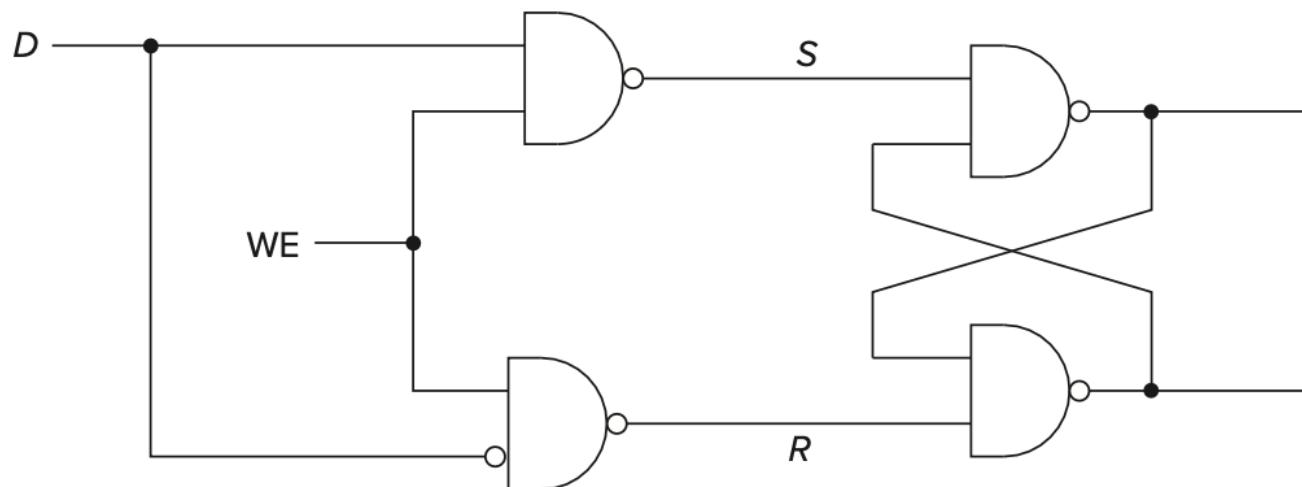
## Note

- The difference between **latch** and **flip flop**
  - Latch is level-triggered → outputs can change as soon as the inputs change)
  - Flip-Flop is edge-triggered → only changes state when a control signal goes from high to low or low to high

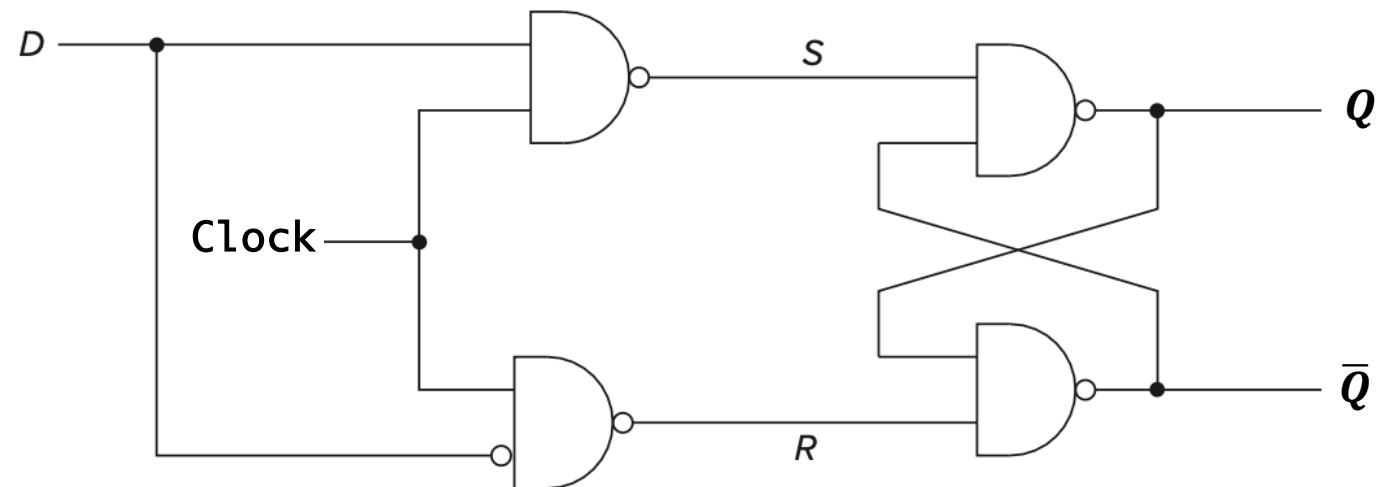
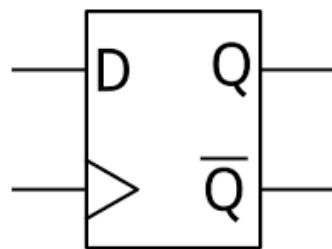


## D-Latch

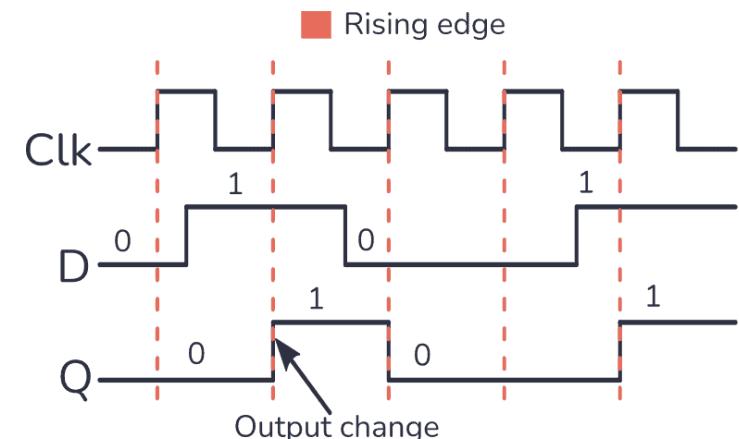
- Two inputs: D (Data) and WE (Write Enable)
  - when WE = 1, latch is **set to value of D** ; S = NOT(D), R = D
  - when WE = 0, latch **holds previous value** ; S = R = 1



## D Flip-Flop

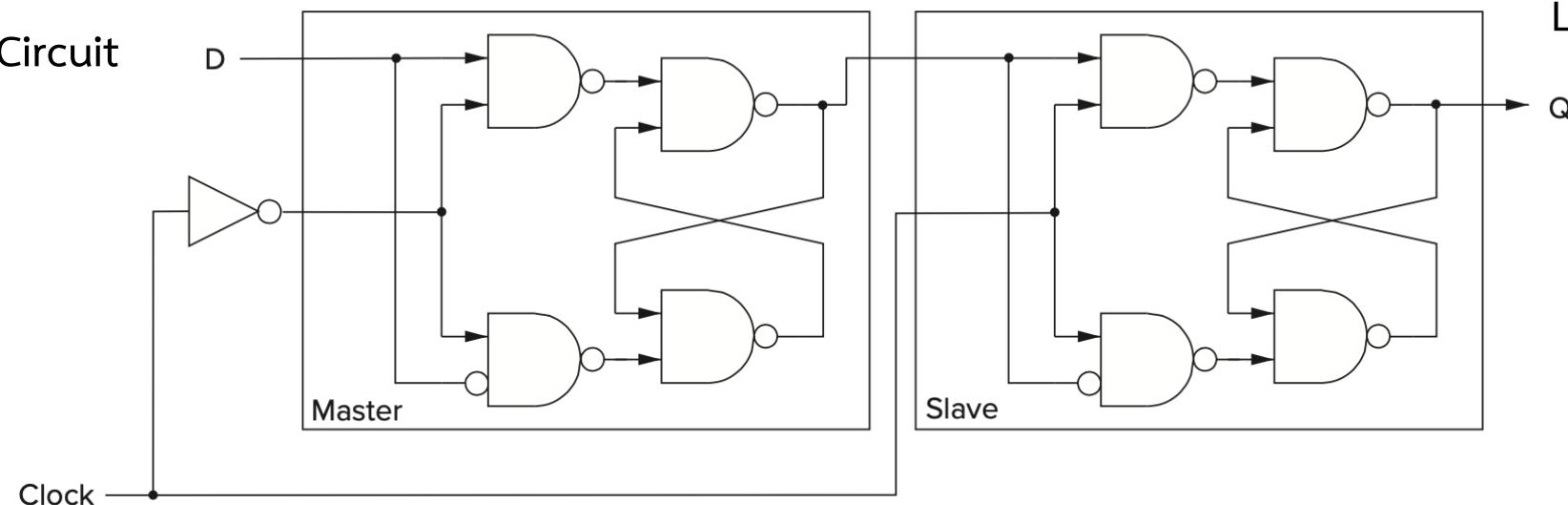


Clock	D	$Q(t+1)$
↓ or 0 or 1	X	$Q_t$ (no change)
↑	0	0 (reset)
↑	1	1 (set)



## Master / Slave flip-flop

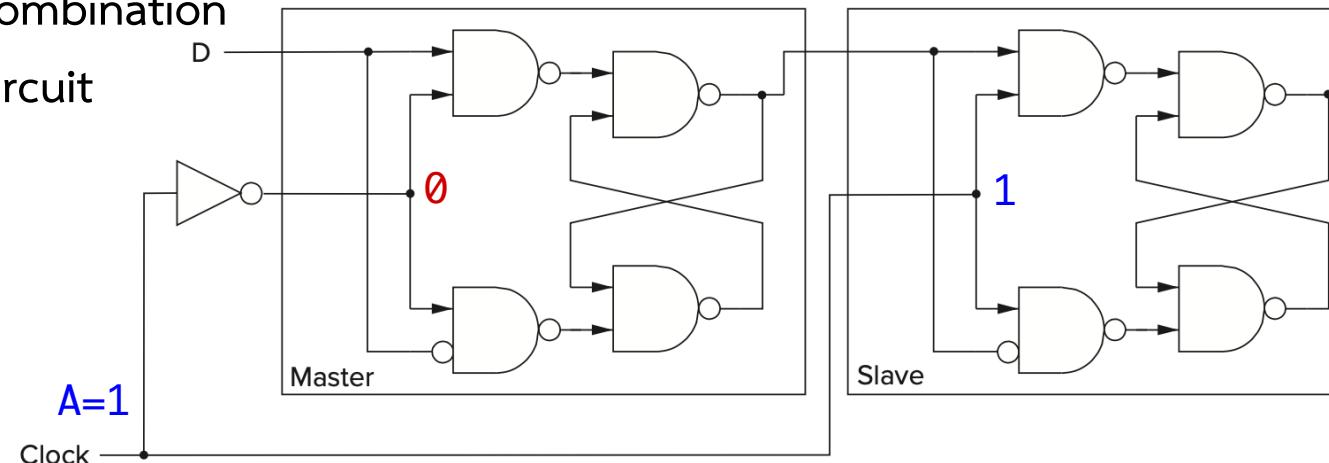
From Combination Logic Circuit



To Combination Logic Circuit

- A pair of gated D-latches, to isolate *next* state from *current* state
- Not write the next state values into the storage elements until the beginning of the next clock cycle

From Combination Logic Circuit



To Combination Logic Circuit

Clock	D	$Q(t+1)$
↓ or 0 or 1	X	$Q_t$ (no change)
↑	0	0
↑	1	1

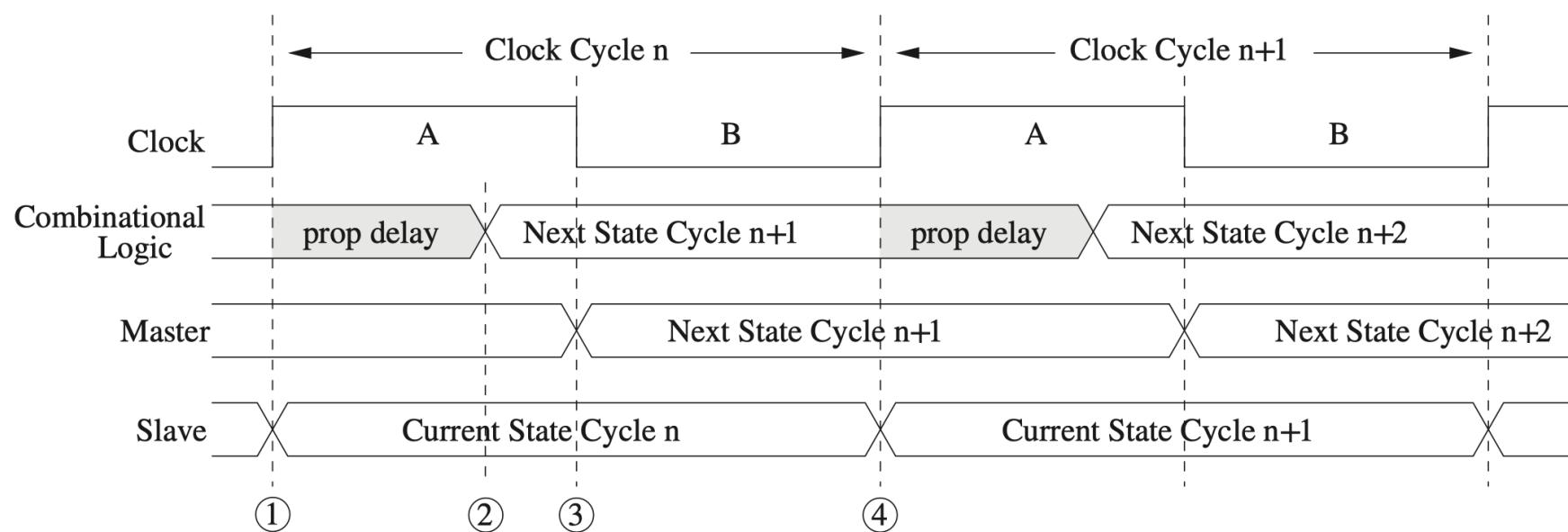
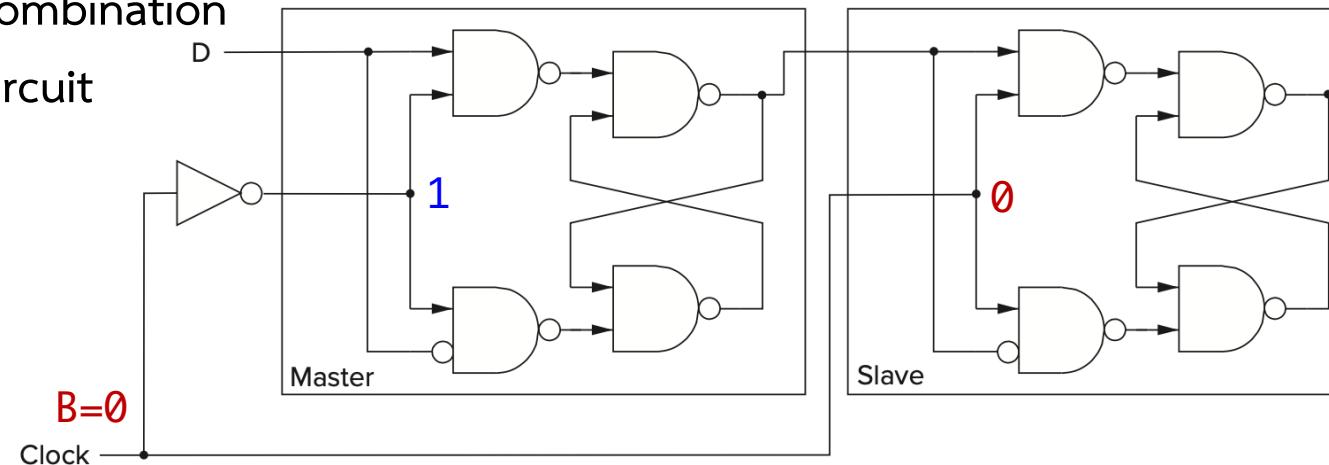


Figure 3.34 Timing diagram for a master/slave flip-flop.

From Combination Logic Circuit



To Combination Logic Circuit

Clock	D	$Q(t+1)$
↓ or 0 or 1	X	$Q_t$ (no change)
↑	0	0
↑	1	1

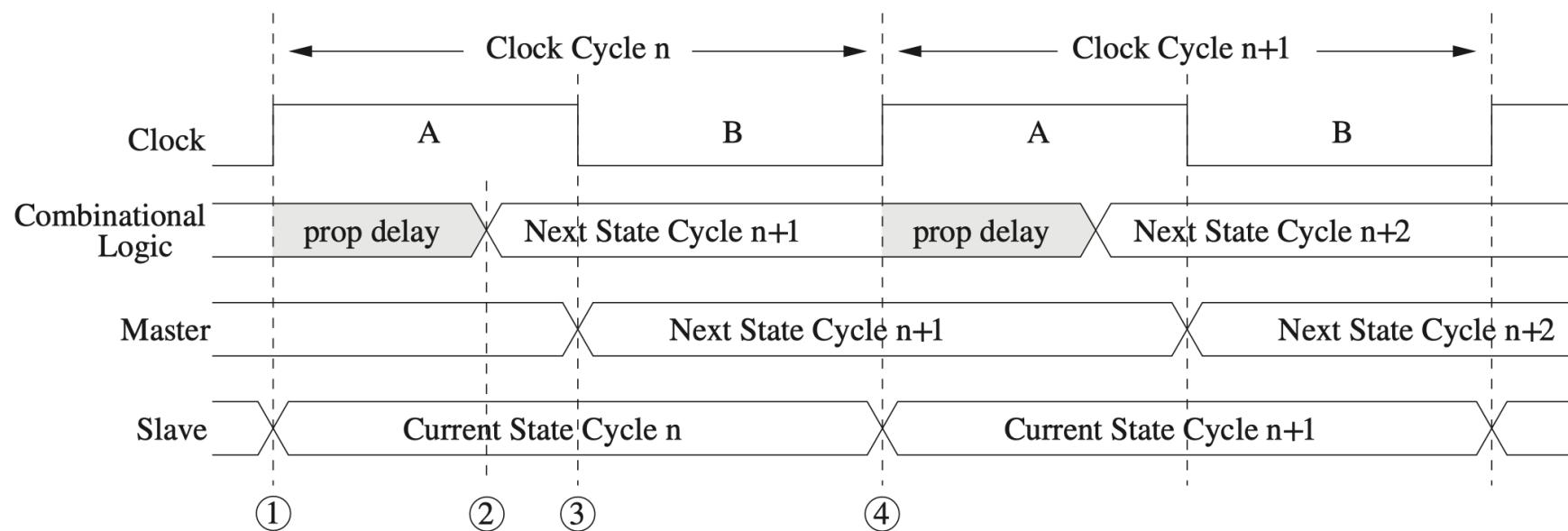
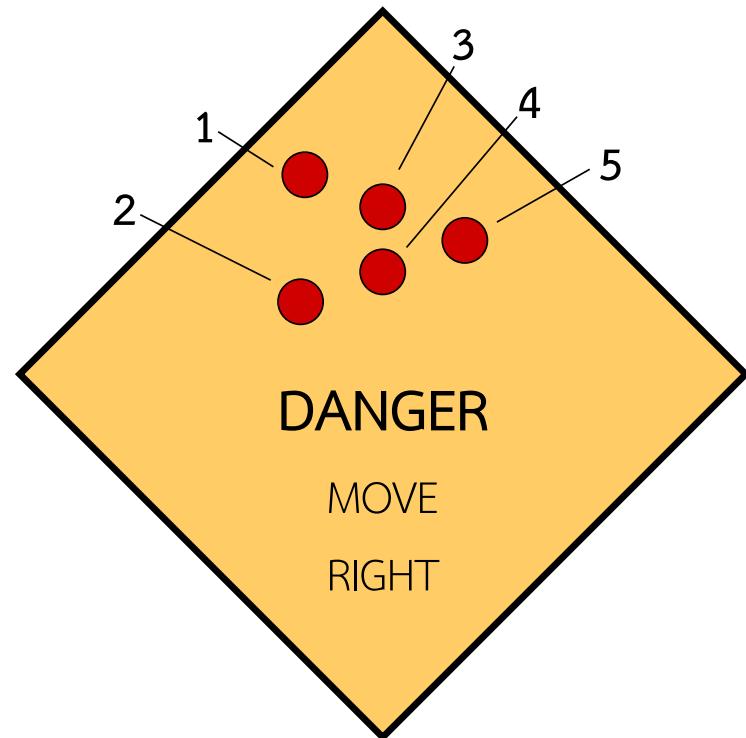


Figure 3.34 Timing diagram for a master/slave flip-flop.

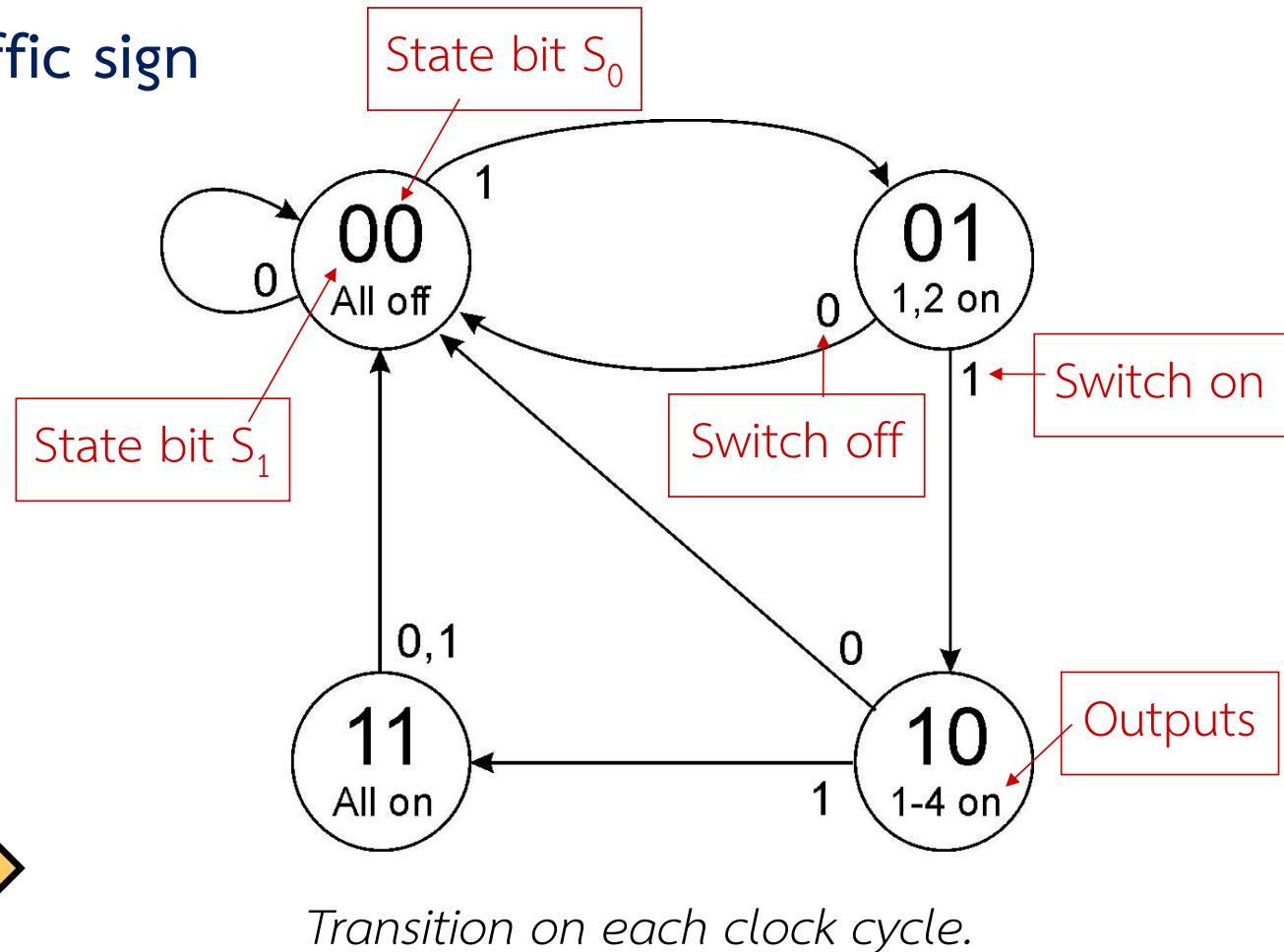
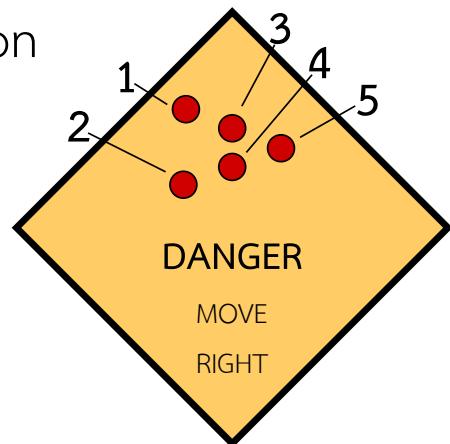
## Example: A blinking traffic sign

- No lights on
- 1 & 2 on
- 1, 2, 3, & 4 on
- 1, 2, 3, 4, & 5 on
- Repeat as long as **switch** is turned on

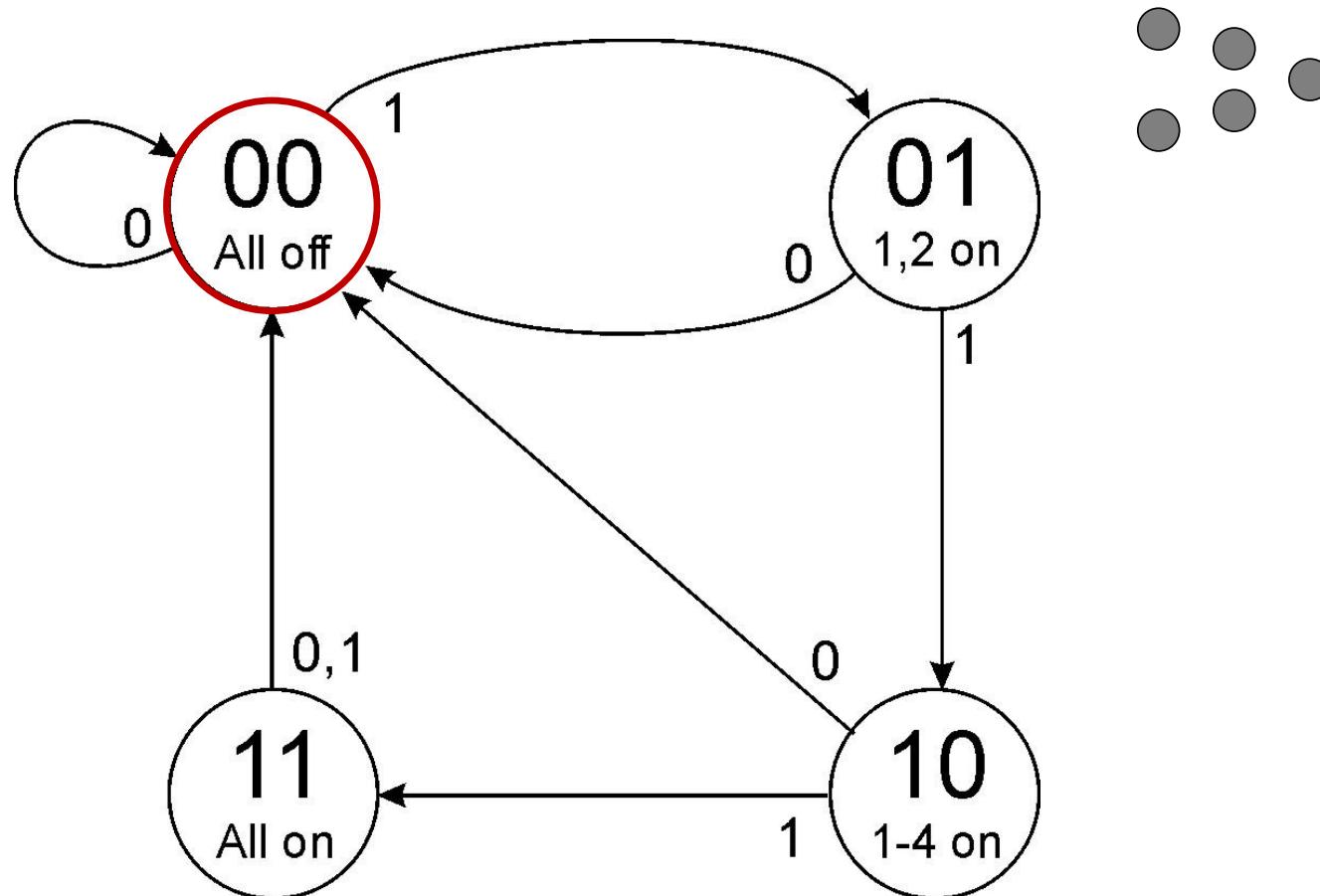


## Example: A blinking traffic sign

- No lights on
- 1 & 2 on
- 1, 2, 3, & 4 on
- 1, 2, 3, 4, & 5 on
- Repeat as long as switch is turned on

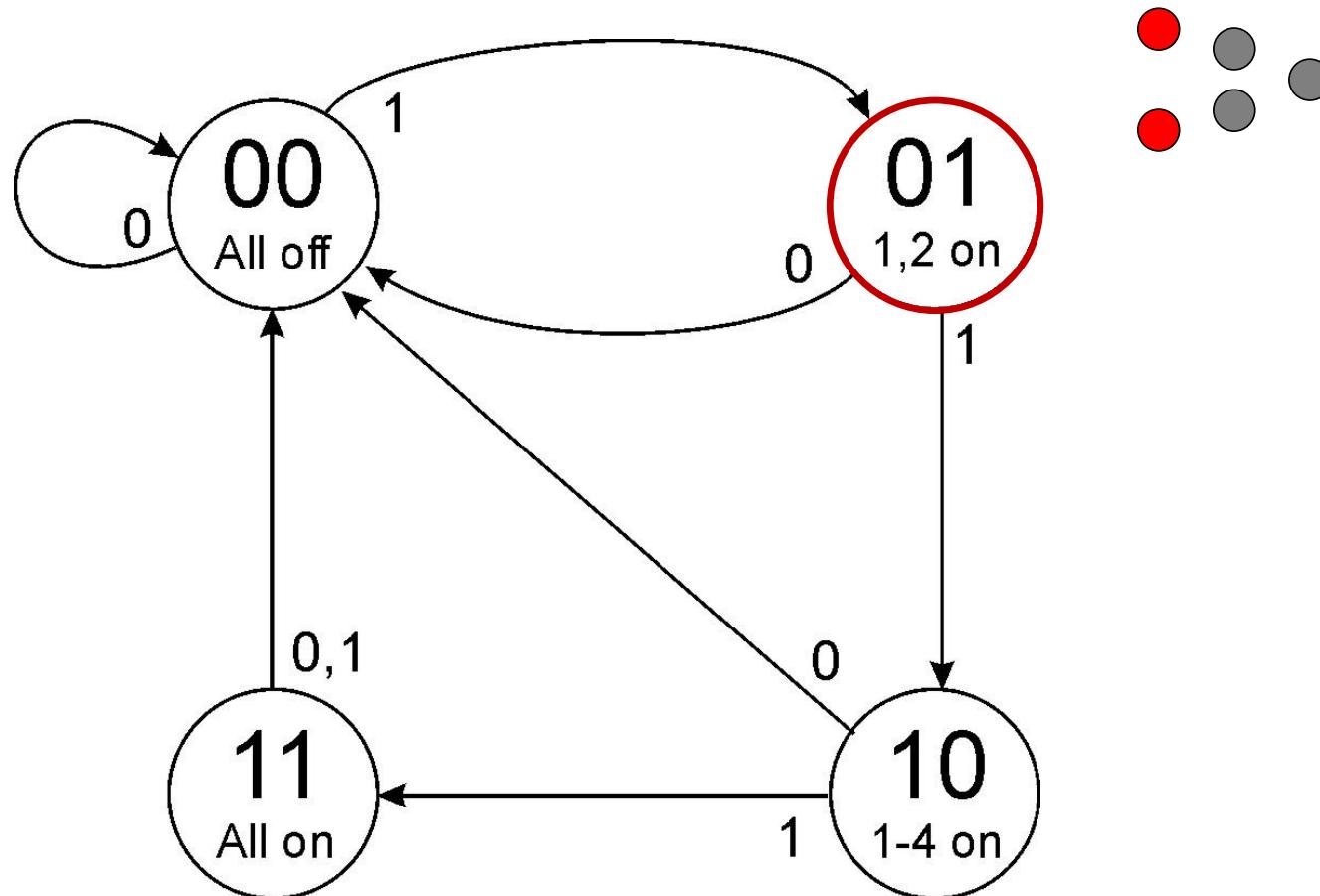


## Traffic Sign State Diagram: State 00



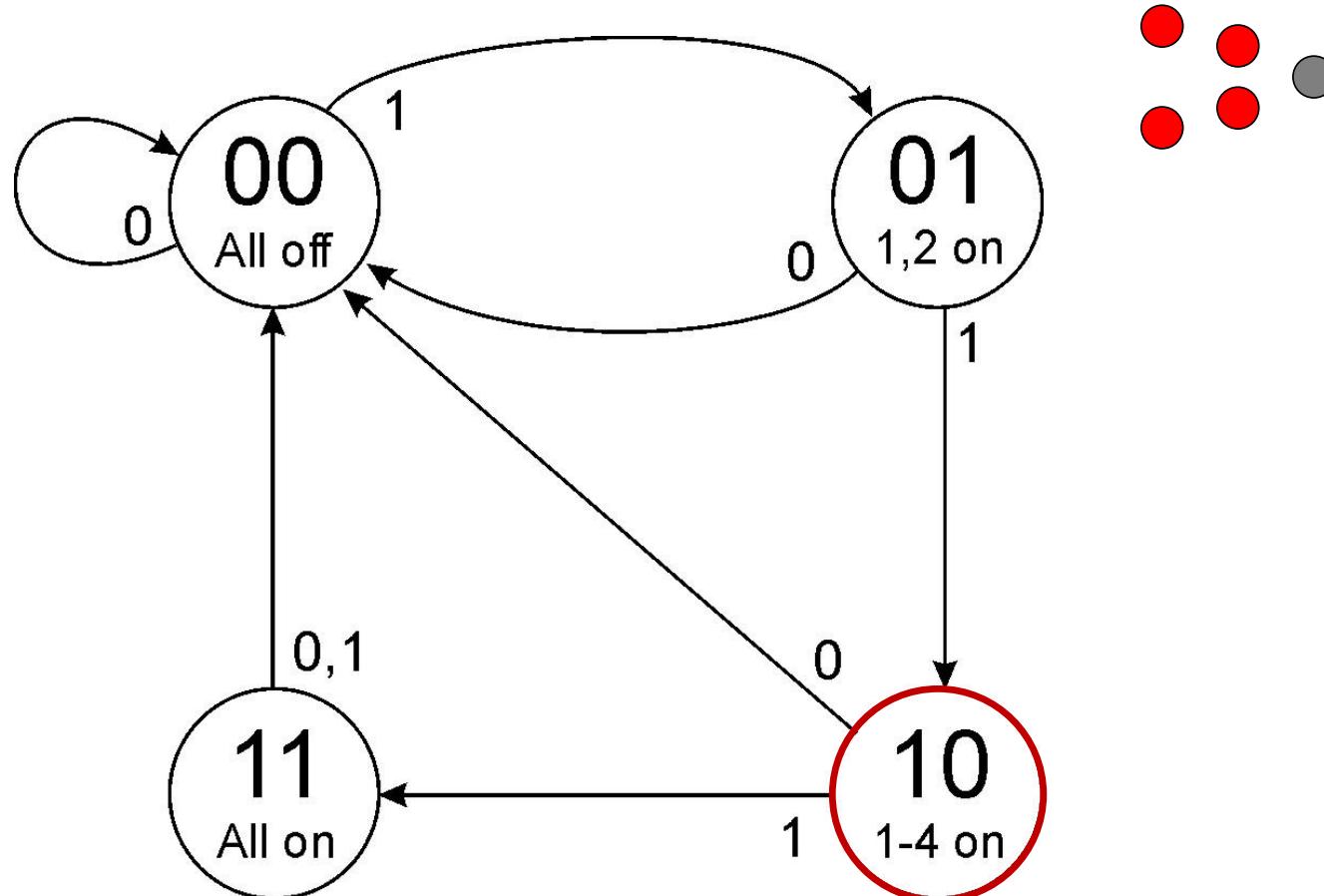
*Transition on each clock cycle.*

## Traffic Sign State Diagram: State 01



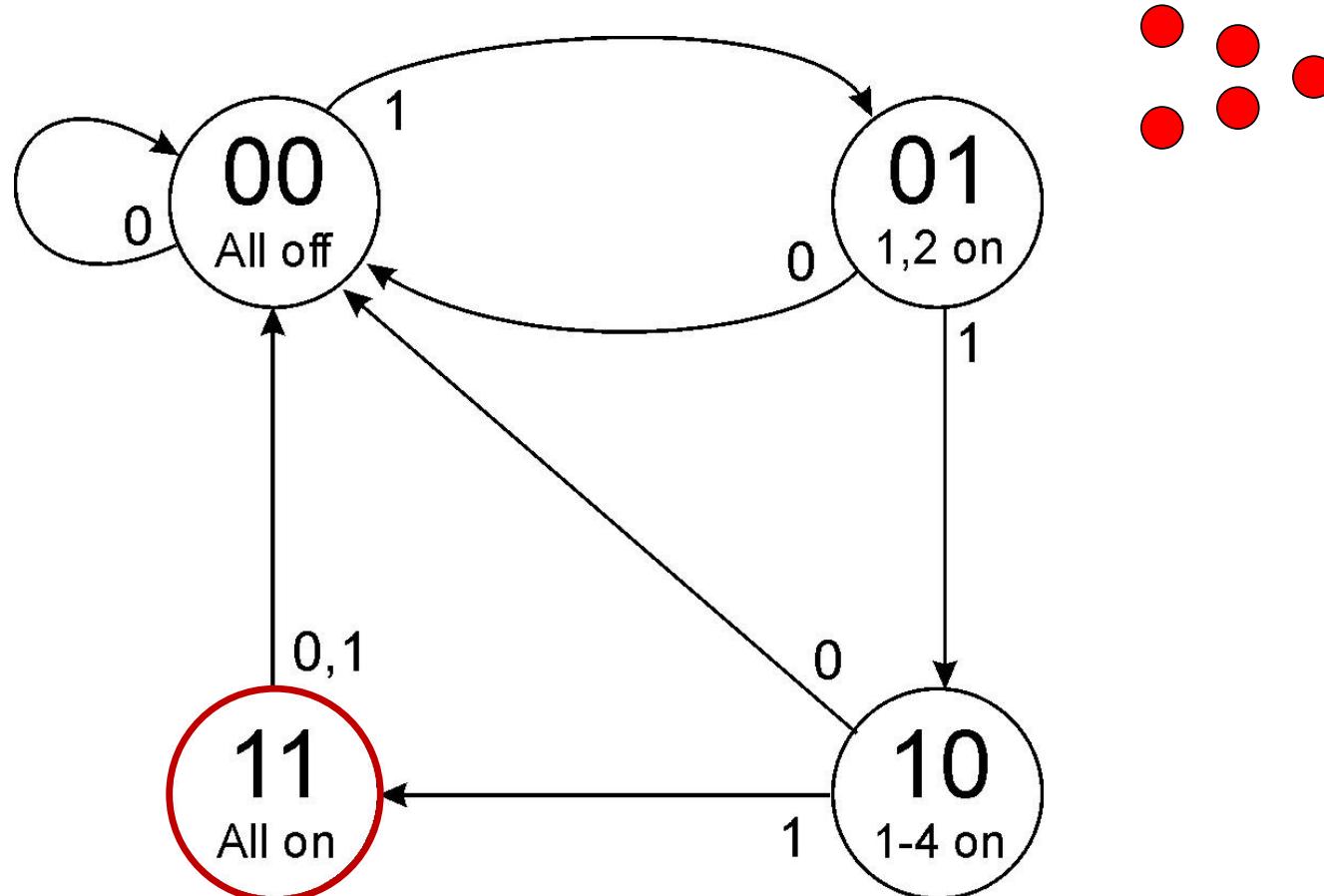
*Transition on each clock cycle.*

## Traffic Sign State Diagram: State 10



*Transition on each clock cycle.*

## Traffic Sign State Diagram: State 11



*Transition on each clock cycle.*

# Traffic Sign Truth Tables

Outputs  
(depend only on state:  $S_1 S_0$ )

$S_1$	$S_0$	W	X	V
0	0	0	0	0
0	1	1	0	0
1	0	1	1	0
1	1	1	1	1

Lights 1 and 2  
 Lights 3 and 4  
 Light 5

Next State:  $S_1' S_0'$   
(depend on state and input)

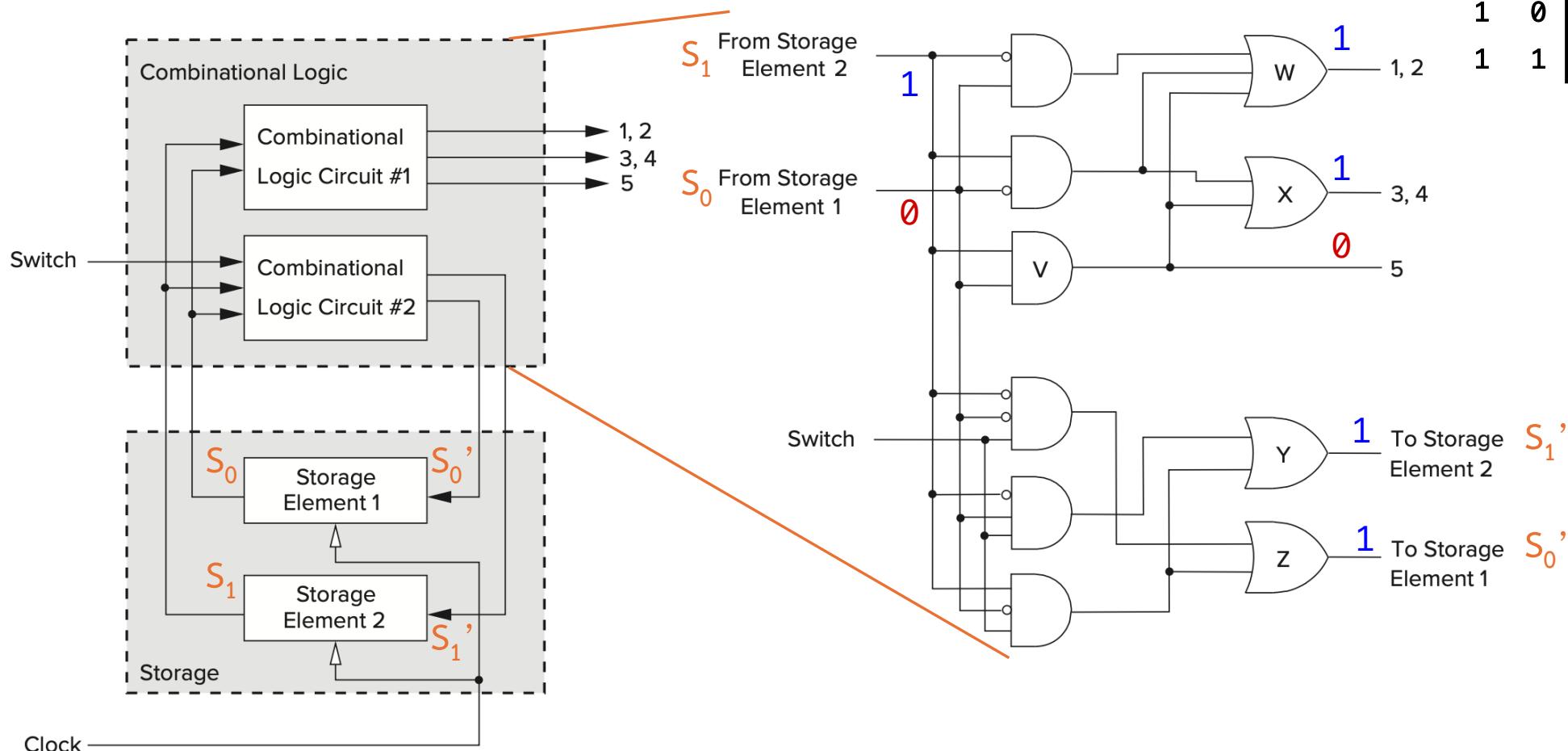
**Switch**

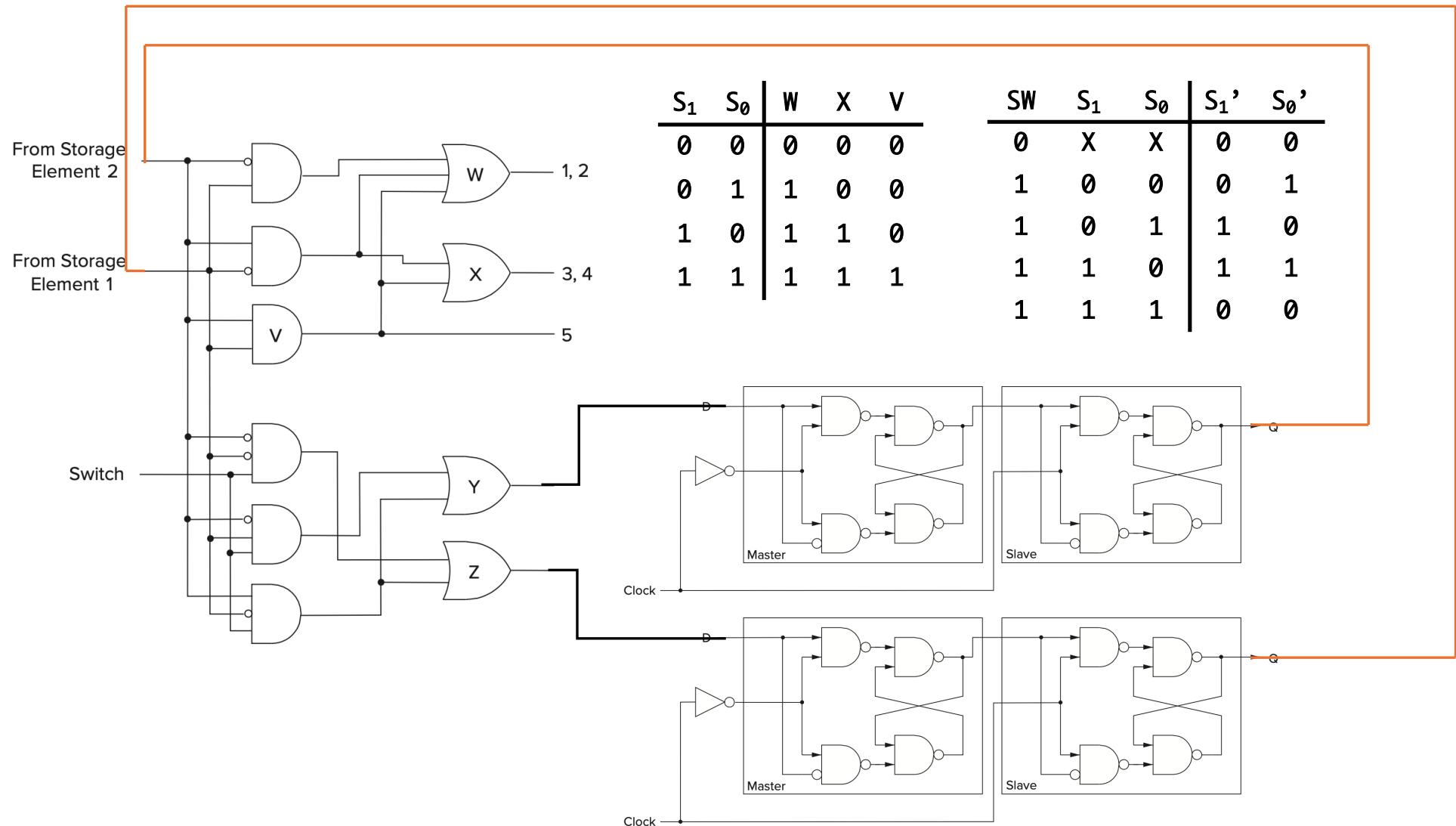
SW	$S_1$	$S_0$	$S_1'$	$S_0'$
0	X	X	0	0
1	0	0	0	1
1	0	1	1	0
1	1	0	1	1
1	1	1	0	0

 Whenever SW=0, next state is 00.

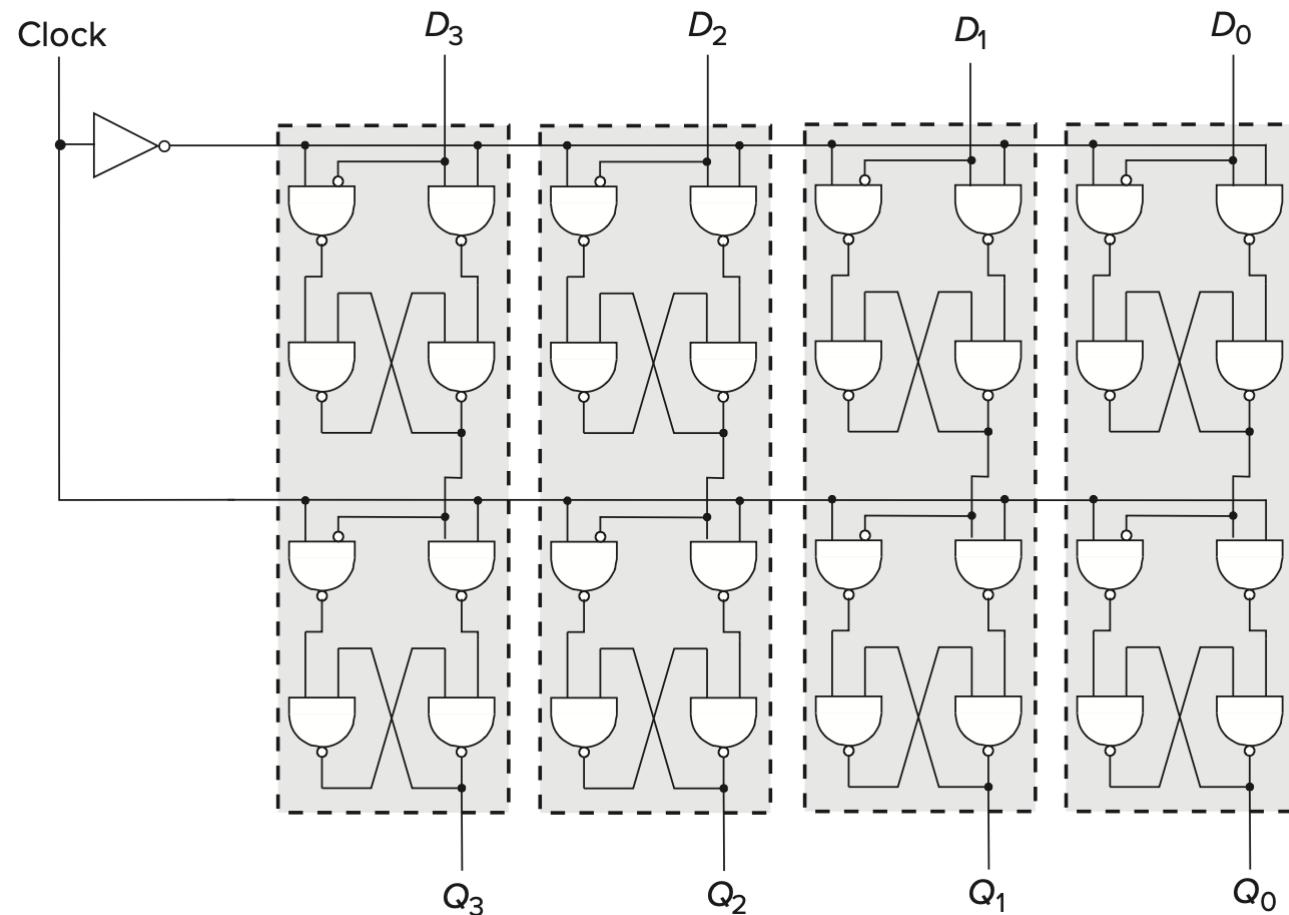
## Sequential logic circuit for the danger sign controller.

$S_1$	$S_0$	W	X	V
0	0	0	0	0
0	1	1	0	0
1	0	1	1	0
1	1	1	1	1





## 4-bit Register



# From Logic to Data Path

The data path of a computer is all the logic used to process information.

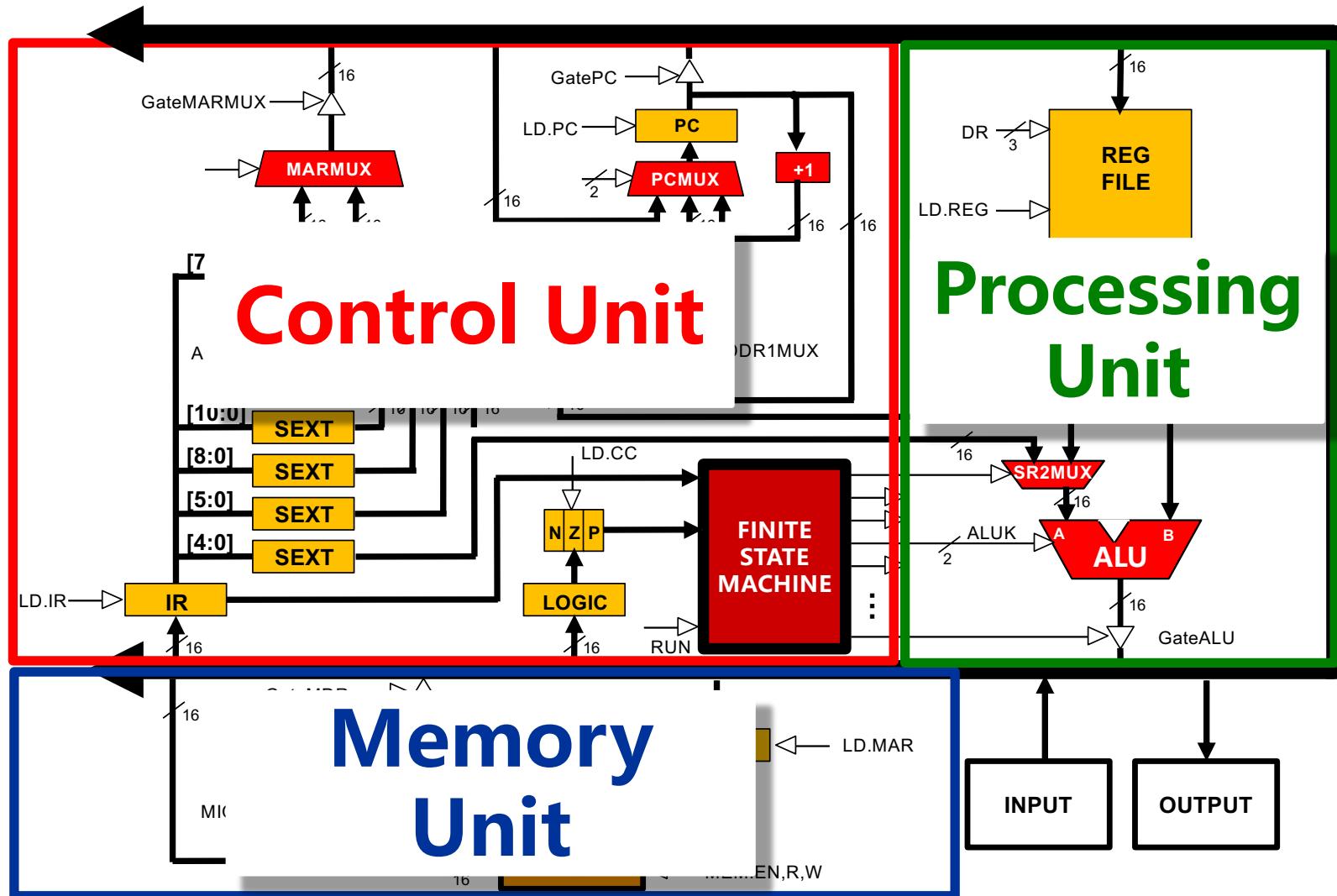
## Combinational Logic

- Decoders -- convert instructions into control signals
- Multiplexers -- select inputs and outputs
- ALU (Arithmetic and Logic Unit) -- operations on data

## Sequential Logic

- State machine -- coordinate control signals and data movement
- Registers and latches -- storage elements

## LC-3 Data Path Overview (Microarchitecture)



## The data path of the LC-3 computer

- Five MUXes
- An adder (shown as the ALU symbol with a + sign inside) and an ALU
- PC: Program Counter, IR: Instruction Register, MAR, and MDR are all 16-bit registers that store 16 bits of information each
- REG FILE consists of eight registers that each store 16 bits of information
- One bit of information can be stored in one flip-flop
- Three 1-bit registers, N, Z, and P (Negative Zero and Positive Flag)

## Wrap-Up

- MOS transistors are used as switches to implement logic functions
- Logic functions are usually expressed with AND, OR, and NOT
- Basic digital logic
  - Transistors / Gates
  - Storage (latches, flip-flops, memory)
  - State machines
- Some simple circuits
  - Adder, subtracter, adder/subtracter, Incrementor
  - Traffic sign state machine

# Homework

- Exercises:  
3.1, 3.2, 3.3, 3.5, 3.8, 3.10,  
3.25, 3.29, 3.30, 3.35, 3.40, 3.49

