

Tree & Tree Algorithms

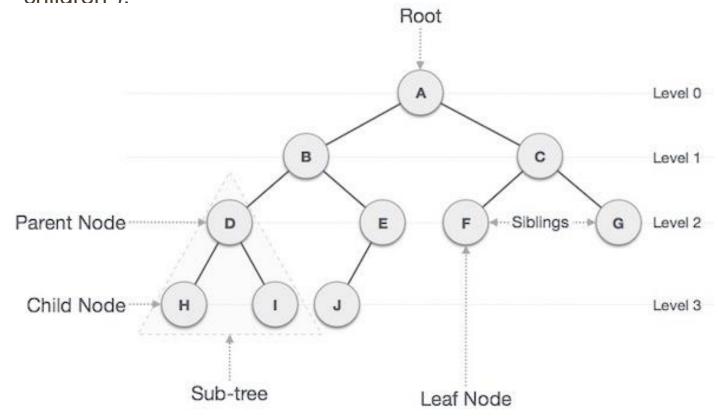
Data structure and Algorithms (310-2101)





Tree Structure

A tree is non-linear and a hierarchical data structure consisting of a collection of nodes such that each node of the tree stores a value, a list of references to nodes (the "children").



Tree Structure

Terminology

Path - Path refers to the sequence of nodes along the edges of a tree

Root – The node at the top of the tree is called root. There is only one root per tree and one path from the root node to any node.

Parent – Any node except the root node has one edge upward to a node called parent.

Child – The node below a given node connected by its edge downward is called its child node.

Leaf – The node which does not have any child node is called the leaf node.

Subtree – Subtree represents the descendants of a node.

Visiting – Visiting refers to checking the value of a node when control is on the node.

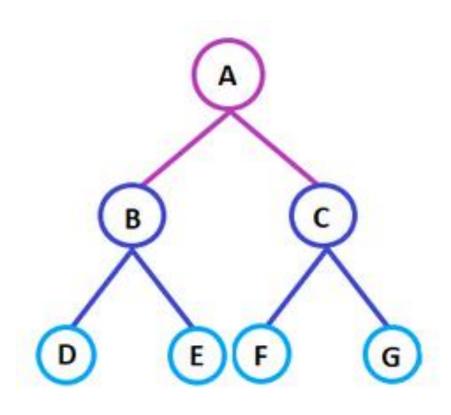
Traversing – Traversing means passing through nodes in a specific order.

Levels – Level of a node represents the generation of a node.

If the root node is at level 0, then its next child node is at level 1, its grandchild is at level 2, and so on.

keys – Key represents a value of a node based on which a search operation is to be carried out for a node.

Tree Structure

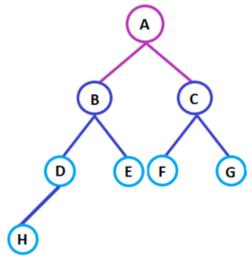


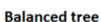
Example

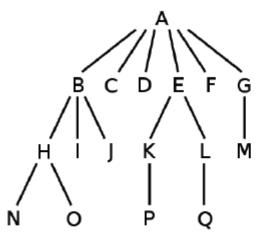
- □ Node A is the root node
- ☐ B is the parent of D and E
- D and E are the siblings
- □ D, E, and F are the leaf nodes
- □ A and B are the ancestors of E

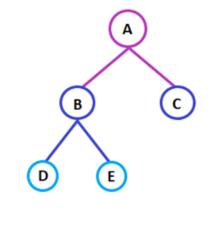
Type of Tree Structure

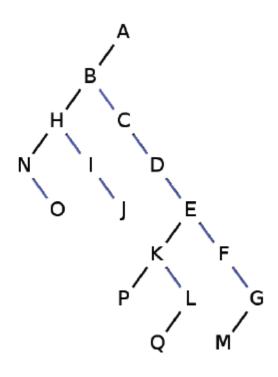
- 1. General Tree
- 2. Binary Tree
- 3. Balanced Tree
- 4. Binary Search Tree









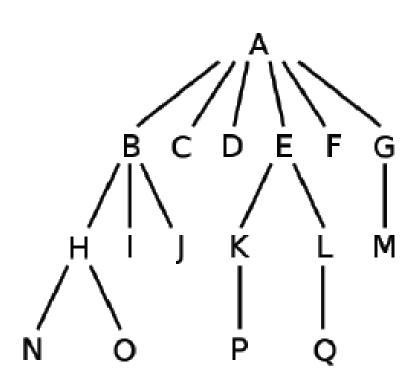


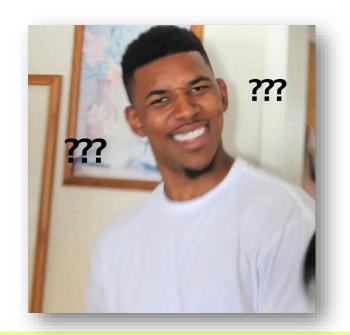
Unbalanced tree

5

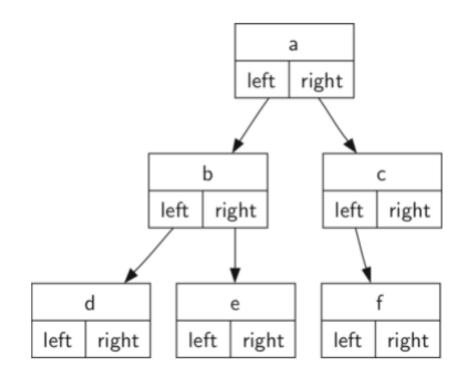
1.) General Tree

List of Lists Representation





2.) Binary Tree



```
class BinaryTree():
         def __init__(self, data):
             self.left = None
 4
             self.right = None
 5
             self.data = data
 6
     if __name__ == '__main__':
 9
         r = BinaryTree('A')
10
         r.left = BinaryTree('B')
11
         r.right = BinaryTree('C')
12
         r.left.left = BinaryTree('D')
13
         r.left.right = BinaryTree('E')
14
         r.right.left = BinaryTree('F')
15
16
```

2.) Binary Tree

```
class BinaryTree:
         def __init__(self, root):
             self.key = root
             self.left_child = None
 6
             self.right_child = None
         def insert_left(self, new_node):
             if self.left child == None:
                 self.left child = BinaryTree(new_node)
10
             else:
11
                 t = BinaryTree(new node)
12
                 t.left_child = self.left_child
13
                 self.left child = t
14
15
16
         def insert right(self, new node):
17
             if self.right child == None:
18
                 self.right child = BinaryTree(new node)
             else:
19
20
                 t = BinaryTree(new node)
                 t.right_child = self.right_child
21
22
                 self.right_child = t
23
```

```
def get_right_child(self):
24
25
             return self.right child
26
27
         def get left child(self):
             return self.left child
28
29
         def set root val(self, obj):
30
31
             self.key = obj
32
33
         def get_root_val(self):
             return self.key
34
35
```

2.) Binary Tree

```
35
36
     if __name__ == '__main__':
37
         r = BinaryTree('a')
38
         print(r.get_root_val())
39
         print(r.get_left_child())
40
41
42
         r.insert_left('b')
43
         print(r.get_left_child().get_root_val())
44
45
         r.insert_right('c')
46
         print(r.get_right_child().get_root_val())
47
48
         r.get_right_child().set_root_val('hello')
         print(r.get_right_child().get_root_val())
49
50
```

Can you sketch the result?

9

Heap : กองสะสม การทับถม

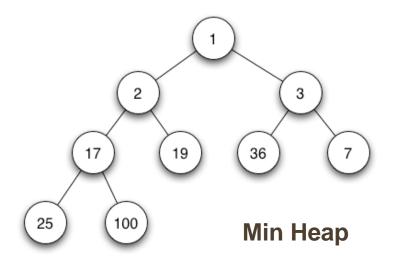
2.) Binary Tree: Binary Heap

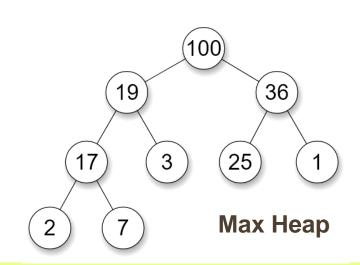
A binary heap is a heap data structure that takes the form of a binary tree.

Binary heaps are a common way of implementing priority queues.

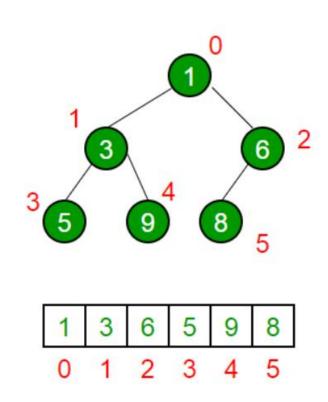
A binary heap is defined as a binary tree with two additional constraints:

- ☐ Shape property: a binary heap is a complete binary tree
- Heap property: the key stored in each node is either greater than or equal to (≥) or less than or equal to (≤) the keys in the node's children, according to some total order.

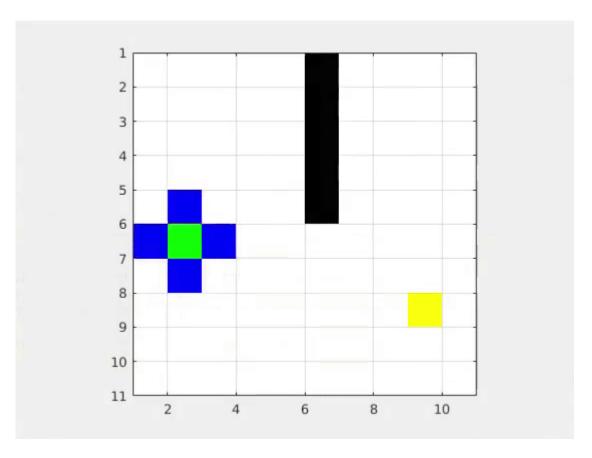




2.) Binary Tree: Binary Heap



Array Representation Of Binary Heap



Dijkstra's algorithm

Shortest Path Finding

2.) Binary Tree: Binary Heap

Do this...

```
[1] Create Binary Heap...
```

[2] Insert: 36, 5, 3, 10, 1 then print them and see the result...

```
if __name__ == '__main__':

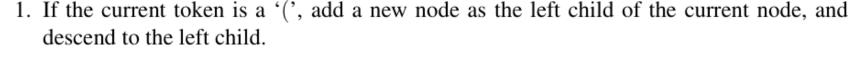
h = BinHeap()

h.build_heap([5, 9, 11, 14, 18, 19, 21, 33, 17, 27])

for idx in range(0, h.current_size+1):
    print(h.heap_list[idx])

print(h.heap_list[idx])
```

2.) Binary Tree: Parse Tree



- 2. If the current token is in the list ['+','-','/','*'], set the root value of the current node to the operator represented by the current token. Add a new node as the right child of the current node and descend to the right child.
- 3. If the current token is a number, set the root value of the current node to the number and return to the parent.
- 4. If the current token is a ')', go to the parent of the current node.

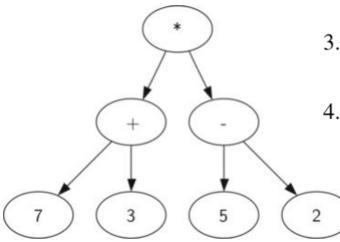


Figure 6.14: Parse Tree for ((7+3)*(5-2))

Tre



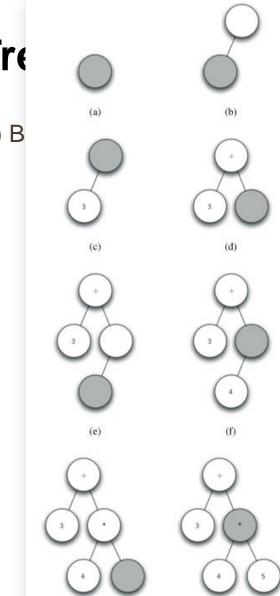


Figure 6.16: Tracing Parse Tree Construction

Application

$$(3+(4*5))$$

- 1. Create an empty tree.
- 2. Read (as the first token. By rule 1, create a new node as the left child of the root. Make the current node this new child.
- 3. Read 3 as the next token. By rule 3, set the root value of the current node to 3 and go back up the tree to the parent.
- 4. Read + as the next token. By rule 2, set the root value of the current node to + and add a new node as the right child. The new right child becomes the current node.
- 5. Read a (as the next token. By rule 1, create a new node as the left child of the current node. The new left child becomes the current node.
- 6. Read a 4 as the next token. By rule 3, set the value of the current node to 4. Make the parent of 4 the current node.
- 7. Read * as the next token. By rule 2, set the root value of the current node to * and create a new right child. The new right child becomes the current node.
- 8. Read 5 as the next token. By rule 3, set the root value of the current node to 5. Make the parent of 5 the current node.
- 9. Read) as the next token. By rule 4 we make the parent of * the current node.
- 10. Read) as the next token. By rule 4 we make the parent of + the current node. At this point there is no parent for + so we are done.

2.) Binary Tree: Parse Tree

Do this...

- [1] Create Parse Tree
- [2] Print the result with postorder() and preorder() and see the difference

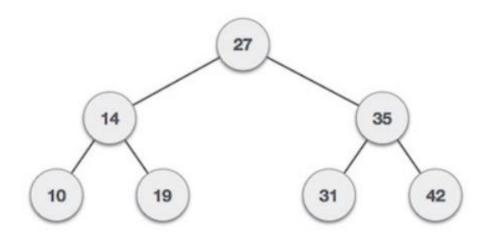
```
101
102    if __name__ == '__main__':
103         pt = build_parse_tree("( ( 10 + 5 ) * ( 3 - 2 ) )")
104         pt.postorder()
105
```

Tree: Binary Tree - Traversals

preorder In a preorder traversal, we visit the root node first, then recursively do a preorder traversal of the left subtree, followed by a recursive preorder traversal of the right subtree.

inorder In an inorder traversal, we recursively do an inorder traversal on the left subtree, visit the root node, and finally do a recursive inorder traversal of the right subtree.

postorder In a postorder traversal, we recursively do a postorder traversal of the left subtree and the right subtree followed by a visit to the root node.



Preorder: [27] [14] [10] [19] [35] [31] [42]

Inorder: [10] [14] [19] [27] [31] [35] [42]

Postorder: [10] [19] [14] [31] [42] [35] [27]

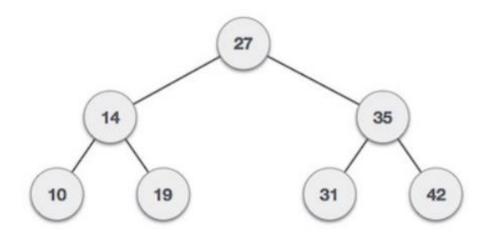
Tree: Binary Tree - Traversals

Preorder: [27] [14] [10] [19] [35] [31] [42]

Inorder: [10] [14] [19] [27] [31] [35] [42]

Postorder: [10] [19] [14] [31] [42] [35] [27]

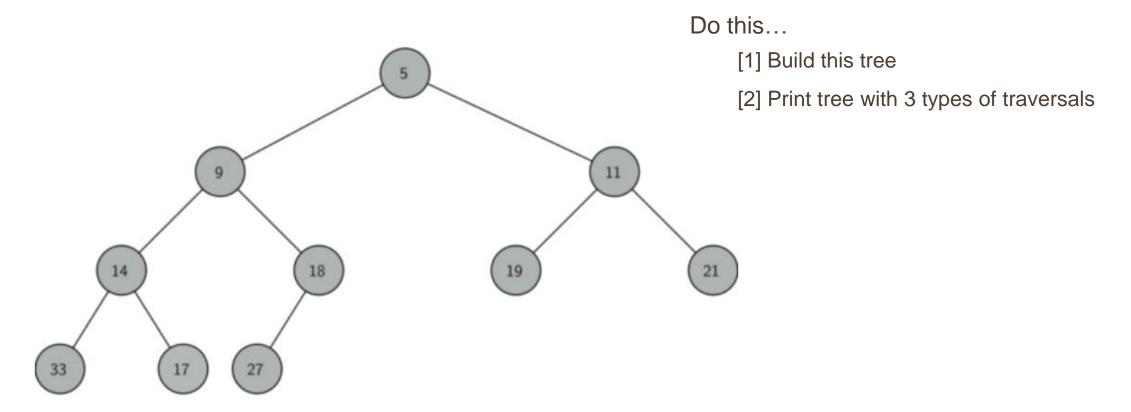
2.) Binary Tree: **Traversals**



```
# In-order Traversal
     def inorder(node):
         if node:
10
             inorder(node.left)
11
12
             print(node.data)
13
             inorder(node.right)
14
     # Pre-order Traversal (Breath-First Search:BFS)
15
     def preorder(node):
16
17
         if node:
18
             print(node.data)
             preorder(node.left)
19
             preorder(node.right)
20
21
     # Post-order Traversal (Depth-First Search:DFS)
22
23
     def postorder(node):
24
         if node:
25
             postorder(node.left)
             postorder(node.right)
26
             print(node.data)
27
28
```

Tree: Binary Tree - Traversals

2.) Binary Tree: **Traversals**

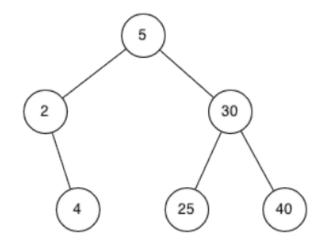


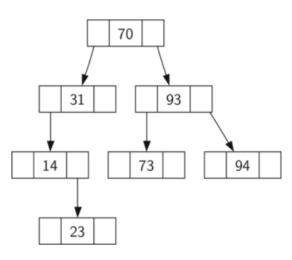
Tree: Binary Search Tree (BST)

3.) Binary Search Tree (BST)

A Binary Search Tree (BST) is a tree in which all the nodes follow the below-mentioned properties. The left sub-tree of a node has a key less than or equal to its parent node's key. The right sub-tree of a node has a key greater than to its parent node's key. Thus, BST divides all its sub-trees into two segments; the left sub-tree and the right sub-tree

```
left_subtree (keys) ≤ node (key) ≤ right_subtree (keys)
```





Tree: Binary Search Tree (BST)

3.) Binary Search Tree (BST): Implementation

Do this...

- [1] Download Binary Search Tree (BST) from Classroom
- [2] Enter the following code
- [3] Are there any error when we try to access [0], [1]?

```
if __name__ == '__main__':
199
200
          my_tree = BinarySearchTree()
201
          my\_tree[3] = 16
202
          my_tree[4] = 6
203
          my_tree[6] = 1
204
205
          my_tree[2] = 468
206
207
          print(my tree[0])
208
          print(my_tree[1])
          print(my_tree[2])
209
          print(my_tree[3])
210
211
          print(my_tree[4])
          print(my_tree[5])
212
          print(my_tree[6])
213
214
```

Take Home Message (Tree Structure)

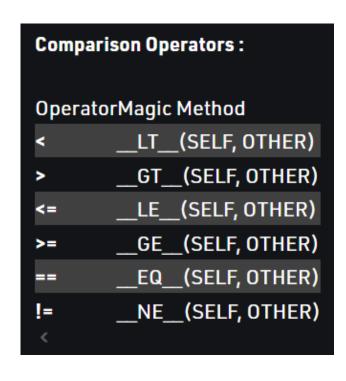
- [1] Tree Data Structure
 - Binary Heap
 - Binary Tree
 - Binary Search Tree (BST)
- [2] Binary Tree: Parse Tree
- [3] Binary Tree: Traversals

Overload Operator in Python

```
OperatorMagic Method
          _add__(self, other)
         __sub__(self, other)
         mul (self, other)
         __truediv__(self, other)
         __floordiv__(self, other)
         __mod__(self, other)
**
         pow (self, other)
>>
         __rshift__(self, other)
<<
          lshift (self, other)
         and (self, other)
         __or__(self, other)
         xor (self, other)
```

```
# Operator Overloading in Python
     # https://www.geeksforgeeks.org/operator-overloading-in-python/
     class complex:
         def _ init (self, a, b):
             self.a = a
10
11
             self.b = b
12
         # adding two objects
13
         def add (self, other):
14
15
             return self.a + other.a, self.b + other.b
16
17
    if __name__ == '__main__':
         Ob1 = complex(1, 2)
18
         Ob2 = complex(2, 3)
19
         0b3 = 0b1 + 0b2
20
         print(Ob3)
21
22
```

Overload Operator in Python



```
Assignment Operators:
OperatorMagic Method
         _ISUB__(SELF, OTHER)
         _IADD__(SELF, OTHER)
        __IMUL__(SELF, OTHER)
         IDIV (SELF, OTHER)
/=
         IFLOORDIV (SELF, OTHER)
//=
%=
         _IMOD__(SELF, OTHER)
**=
         IPOW_(SELF, OTHER)
         IRSHIFT_(SELF, OTHER)
>>=
        __ILSHIFT__(SELF, OTHER)
<<=
&=
         IAND__(SELF, OTHER)
         IOR_(SELF, OTHER)
1=
         _IXOR__(SELF, OTHER)
```

```
Unary Operators:
OperatorMagic Method
        _NEG__(SELF, OTHER)
         _POS__(SELF, OTHER)
        _INVERT__(SELF, OTHER)
```

Overload Function in Python

```
class Purchase:
         def __init__(self, basket, consumer):
             self.basket = list(basket)
 4
 5
             self.consumer = consumer
 6
         def __len__(self):
 8
             print('Call from overload function (>_<) this is fake length')</pre>
             return 10
 9
10
     if __name__ == '__main__':
         purchase = Purchase(['pencil ', 'book '], 'python ')
12
         print(len(purchase))
13
```

Overload Function in Python

```
11   if __name__ == '__main__':
12
13      print(dir(list))
14
```

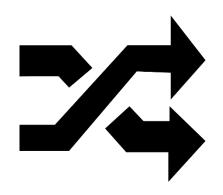
Python - Magic or Dunder Methods

Random Number Generator

คอมพิวเตอร์สามารถสร้างการสุ่มตัวเลขขึ้นมาได้อย่างไร

(How Do Computers Generate Random Numbers?)

ตัวเลขสุ่ม (Random number) นั้นเป็นข้อมูลที่สามารถนำมาใช้ประโยชน์ได้หลากหลายงาน เช่น การเข้ารหัส การ ทำให้เกิดผลลัพธ์ที่มีความหลากหลายไม่ซ้ำซ้อน การสุ่มตัวอย่างเพื่อทดสอบความผิดพลาดของกระบวนการทำงาน





การสุ่มนั้นเป็นสิ่งที่ทำให้เกิดขึ้นในระบบคอมพิวเตอร์ได้ยาก เนื่องจากในระบบคอมพิวเตอร์ การสร้างผลลัพธ์ต่างๆ ให้เกิดขึ้น เป็นสิ่งที่มีสาเหตุและที่มาเสมอ ทำให้เราสามารถคำนวณหาผลลัพธ์ได้แน่นอน

Method to generate random number

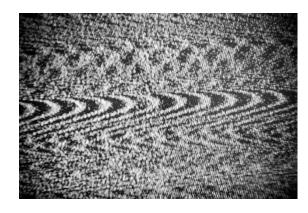
[1] True Random Numbers (TRNGs)

กระบวนการสุ่มซึ่งทำให้เกิดผลลัพธ์ทางกายภาพ (Physical Process) โดยการเก็บผลลัพธ์ที่ได้จากปรากฏการณ์ ทางฟิสิกส์ที่คาดหวังว่าผลลัพธ์มีลักษณะสุ่ม (physical phenomenon that expected to be random)

photoelectric effect,

cosmic background radiation,

atmospheric noise



Method to generate random number

[2] Pseudorandom Numbers (PRNGs)

ตัวเลขสุ่มถึงถูกสร้างขึ้นจาก Algorithm ที่ทำให้เกิดผลลัพธ์ในลักษณะสุ่ม ตัวเลขสุ่มที่ได้จากกรณีนี้เป็นตัวเลขที่ ไม่ได้เกิดการ Random จริง เพราะสุดท้ายแล้ว Algorithm ต่างๆ นั้นมี สมการ อยู่เบื้องหลังทำให้เราสามารถ คาดการณ์ผลลัพธ์ล่วงหน้าได้เสมอ

การสร้างตัวเลขสุ่มในรูปแบบนี้จะเกิดจากค่าตั้งต้น (Initial value)

ซึ่งมักจะถูกเรียกว่า Seed หรือ Key

ในทางปฏิบัติงานทางคอมพิวเตอร์ ตัวเลขสุ่ม หรือ ตัวแปรสุ่มที่ได้จาก PRNGs นั้นมีความสะดวกมากกว่า เพราะ สามารถทำการทดสอบตัวเองซ้ำ (Repeating itself) ในเหตุการณ์เดิมได้ (Reproduce same result later)

กระบวนการสุ่มในรูปแบบนี้จะประกอบไปด้วยกระบวนการพื้นฐาน 4 ขั้นตอน

- Accept some initial input number, that is a seed or key
- Apply that seed in a sequence of mathematical operations to generate the result.

That result is the random number

- Use that resulting random number as the seed for the next iteration
- Repeat the process to emulate randomness

Linear Congruential Generator

most common and oldest algorithm for generating pseudo randomized numbers

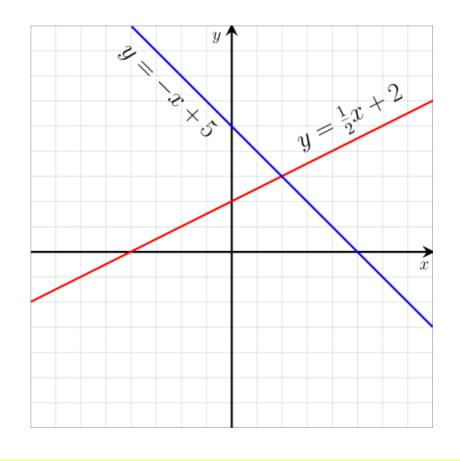
$$X_{n+1}=(aX_n+c) \, \bmod m$$

ตัวแปร X เป็น Sequence ของค่า Pseudorandom number

$$m, \ 0 < m$$
 — the "modulus" $a, \ 0 < a < m$ — the "multiplier"

$$c, \, 0 \leq c < m$$
 — the "increment"

$$X_0,\, 0 \leq X_0 < m$$
 — the "seed" or "start value"



Linear Congruential Generator

most common and oldest algorithm for generating pseudo randomized numbers

RNG

31

Linear Congruential Generator

most common and oldest algorithm for generating pseudo randomized numbers

Source	modulus m	multiplier a	increment c	output bits of seed in rand() or Random(L)
Borland C/C++	2 ³²	22695477	1	bits 3016 in <i>rand()</i> , 300 in <i>Irand()</i>
glibc (used by GCC) ^[17]	2 ³¹	1103515245	12345	bits 300
C++11's minstd_rand [22]	2 ³¹ – 1	48271	0	see MINSTD
MMIX by Donald Knuth	2 ⁶⁴	6364136223846793005	1442695040888963407	

Linear Congruential Generator

```
from collections.abc import Generator
     def lcg(modulus: int, a: int, c: int, seed: int) -> Generator[int, None, None]:
 3
         """Linear congruential generator."""
 4
 5
        while True:
             seed = (a * seed + c) % modulus
 6
            yield seed
 8
     if __name__ == "__main__":
10
        m = 9
11
        a = 2
12
        c = 0
13
        seed = 1
                                          What does the "yield" keyword do?
14
15
         print("Random number")
16
         for x in lcg(m,a,c,seed):
             print(x, end=" ")
17
18
```

33

What does the "yield" keyword do?

yield นั้นมีความคล้ายกับ return สิ่งที่ทำให้ 2 keyword นี้มีความแตกต่างกัน คือ

return statement จะสิ้นสุด (terminate) เมื่อมีการเรียกฟังก์ชัน (execution of the function) yield statement จะหยุดชั่วคราว (pause) เมื่อมีการเรียกฟังก์ชัน (execution of the function)

```
def fun_generator():
         yield "Hello world!!"
         yield "Geeksforgeeks"
     if __name__ == "__main__":
         obj = fun_generator()
 6
         print(type(obj))
 8
10
         print(next(obj))
11
         print(next(obj))
12
```

```
<class 'generator'>
Hello world!!
Geeksforgeeks
```

ใช้ Yield เมื่อทำ Generator function จะให้ผลลัพธ์ที่รวดเร็วกว่า Return

RNG

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What does the "yield" keyword do?

- 🔷 **ข้อดีของ** yield
- ประหยัดหน่วยความจำ คืนค่าเฉพาะที่ต้องการ ไม่ต้องโหลดข้อมูลทั้งหมด
- 🗹 ทำให้โค้ดอ่านง่าย เหมาะกับการวนลูปข้อมูลขนาดใหญ่
- 🗹 รองรับการทำงานแบบขนาน สามารถสร้าง pipeline ของข้อมูลแบบ streaming ได้

♦ ใช้เมื่อไหร่?

- เมื่อทำงานกับ ข้อมูลขนาดใหญ่ เช่น อ่านไฟล์ทีละบรรทัด (yield จะช่วยให้ไม่ต้อง โหลดไฟล์ทั้งหมดลงใน RAM)
- เมื่อสร้าง stream ของข้อมูล เช่น คำนวณตัวเลขที่ต้องการเรียกใช้แบบต่อเนื่อง
- เมื่อสร้าง infinite sequence เช่น ตัวเลขฟีโบนักชีที่สามารถดึงค่าได้เรื่อย ๆ

```
def fibonacci():
    a, b = 0, 1
    while True:
        yield a
        a, b = b, a + b

fib = fibonacci()
for _ in range(5):
    print(next(fib)) # 0, 1, 1, 2, 3
```

```
# generator to print even numbers
     def print_even(test_list):
         for i in test list:
             if i % 2 == 0:
 4
 5
                 yield i
 6
    # initializing list
    test_list = [1, 4, 5, 6, 7]
10
    # printing initial list
     print("The original list is : " + str(test_list))
11
12
13
    # printing even numbers
     print("The even numbers in list are : ", end=" ")
14
    for j in print_even(test_list):
15
16
         print(j, end=" ")
```

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Characteristics of PRNG

- **Efficient:** PRNG can produce many numbers in a short time and is advantageous for applications that need many numbers
- **Deterministic:** A given sequence of numbers can be reproduced at a later date if the starting point in the sequence is known. Determinism is handy if you need to replay the same sequence of numbers again at a later stage.
- **Periodic:** PRNGs are periodic, which means that the **sequence will eventually repeat** itself. While periodicity is hardly ever a desirable characteristic, modern PRNGs have a period that is so long that it can be ignored for most practical purposes

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Python code to generate PRNG

```
? Example2.py > ...
     import random
     from datetime import datetime
      # Passing the current time as the seed value
      random.seed(datetime.now())
  6
      for i in range(5):
          print(random.randint(0, 10), end="\t")
  8
  9
```

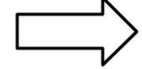
From statistics to probability

... we can generate a series of flips as our data

T, T, H, T, H, H...



From a simple coin...



And then calculate the results of what we observed

H: 43%, T: 57%

If we keep generating more data sets and calculations...

H, H, H, T, T, H... H: 53%, T: 47%

Repeat many many times....

We can take the average of these results.

H: ~50%, T: ~50%

RNG

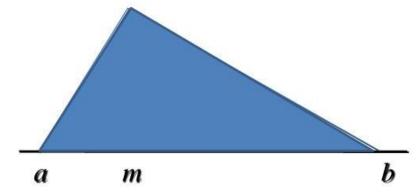
39

Transforming uniform random variables to other distributions

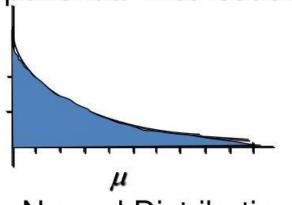
Uniform Distribution



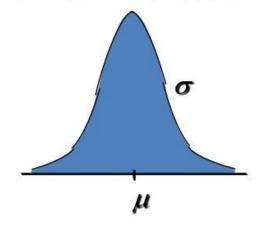
Triangular Distribution



Exponential Distribution



Normal Distribution



RNG

Just some idea to control rate of random value (result)





How to generate this possibility from typical rand function?



RNG

This is just for fun, don't need to take it serious

https://www.ascii-art-generator.org

โปรแกรมจัดการ Card ใน Deck พื้นฐาน

พัฒนาโปรแกรม จัดการสำรับไพ่ (Deck) ให้มีความสามารถต่อไปนี้

- 1. Put() วาง Card เข้าไปในสำรับไพ่ (ด้านบน)
- 2. Draw() หยิบ Card ใบบนสุดออกมาจากสำรับ
- 3. Show() แสดง Card ทั้งหมดในสำรับจาก บน -> ล่าง
- 4. Sort() เรียงตามตัวเลข 3 < 4 < 5 < ... < 10 < 11 < 12 < 13 < 1 < 2
 - ค่าของหน้าไพ่ club < diamonds < heart < spade
 - ล้าไพ่สองใบมีตัวเลขเท่ากัน ให้เปรียบเทียบจากหน้าไพ่
- 5. Shuffle() สลับตำแหน่งของหน้าไพ่ใน Deck
- 6. Sum() รวมตัวเลขของไพ่ในแต่ละหน้าแล้วทำการแสดงผล

```
Card.py > ...
      class Card:
          def __init__(self, value, suit):
  3
              self.value = None
  4
              self.suit = None
  5
              self.set(value, suit)
  6
  7 >
          def set(self, value, suit): ...
 30
          def print(self): ...
 31 >
 33
      if __name__ == "__main__":
 34
 35
 36
          import os
          os.system('cls')
 37
 38
          a = Card("5", "s")
 39
 40
          a.print()
 41
```

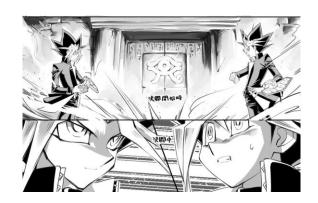
value suit set(value, suit) print()

นักศึกษาสามารถดัดแปลง แก้ไข Source Code ที่ให้ได้

```
Pick.py > ...
     from Card import *
  2
      class Deck:
  3
  4
          def __init__(self):
              cards = []
          def put():
  8
              pass
          def draw():
 10
 11
              pass
 12
 13
          def show():
 14
              pass
 15
 16
          def sort():
 17
              pass
 18
          def shuffle():
 19
 20
              pass
 21
 22
          def sum():
 23
              pass
 24
```

Homework???: Card Battle Game

- 1) แต่ละฝ่ายสุ่มหยิบไพ่ขึ้นมาใส่สำหรับของตัวเอง 5 ใบ
- 2) ใช้หน้าไพ่ (suit) เป็นตัวกำหนดการกระทำ [C] Draw เพิ่ม, [S] สุ่มทิ้งการ์ดอีกฝ่ายหนึ่งใบ,
 - [D] ลด Health Point ฝ่ายตรงข้าม, [H] ขโมยการ์ดฝ่ายตรงข้ามหนึ่งใบ
- 3) ใช้ตัวเลข (value) เป็นตัวกำหนด<u>โอกาส</u> ที่จะทำสำเร็จ
- 4) แต่ละรอบ (Turn) ให้แต่ละฝ่ายหยิบการ์ดเพิ่ม 1 ใบ
- 5) ฝ่ายที่ Health Point เป็น 0 ก่อนถือว่า "แพ้" => จบเกม
- 6) สำรับไพ่กองกลางมีจำนวนจำกัด (13 x 4) ใบ หมดสำหรับ => "เสมอ"



Health Point เริ่มต้นที่ 7

