

2018-2019 Sem 1

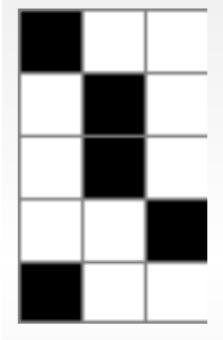
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Worksheet

Image Processing through Linear Algebra 1

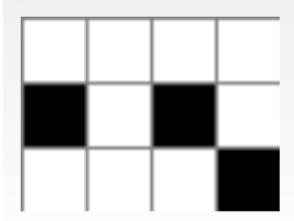
1. Write a matrix that represents each of the following images.

a.



$$= \begin{bmatrix} 0 & 255 & 255 & 255 & 255 \\ 255 & 0 & 255 & 255 & 255 \\ 255 & 255 & 0 & 255 & 255 \\ 255 & 255 & 255 & 0 & 255 \\ 0 & 0 & 255 & 255 & 0 \end{bmatrix}$$

b.



$$= \begin{bmatrix} 255 & 255 & 255 & 255 & 255 \\ 0 & 255 & 255 & 0 & 255 \\ 255 & 255 & 255 & 255 & 0 \end{bmatrix}$$

2. (I) Draw a visual representation of the following matrices.

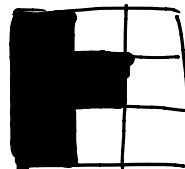
(II) Write the matrix and draw the visual representation that corresponds to the reflection of each of the following about the

a. x - axis

b. y - axis

(I) Visual

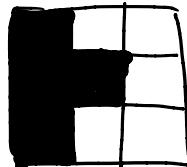
$$\text{a. } \begin{bmatrix} 0 & 255 & 255 \\ 0 & 0 & 255 \\ 0 & 255 & 255 \end{bmatrix} =$$



(a) x -axis

matrix; $a'_x = \begin{bmatrix} 0 & 255 & 255 \\ 0 & 0 & 255 \\ 0 & 255 & 255 \end{bmatrix}$

visual; $a'_x =$

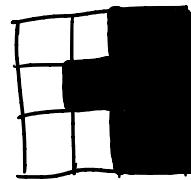


(b) y -axis

matrix; $a'_y =$

visual; $a'_y =$

$$\begin{bmatrix} 255 & 255 & 0 \\ 255 & 0 & 0 \\ 255 & 255 & 0 \end{bmatrix}$$



(I) Visual

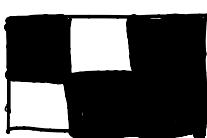
$$\text{b. } \begin{bmatrix} 255 & 0 & 0 \\ 0 & 255 & 0 \end{bmatrix}$$



(a) x -axis

matrix; $b'_x = \begin{bmatrix} 0 & 255 & 0 \\ 255 & 0 & 0 \end{bmatrix}$

visual; $b'_x =$



(b) y -axis

matrix; $b'_y = \begin{bmatrix} 0 & 0 & 255 \\ 0 & 255 & 0 \end{bmatrix}$

visual; $b'_y =$



ຫົວໜ້າ A : matrix ພິມເລກ

3

I' : reverse identity matrix

3. What size does the reverse identity matrix need to be used to reflect an image represented by a 4×5 matrix about the x -axis? Why?

ຂົວວັນ
reflection
 x -axis

$$; \quad I' \times \begin{matrix} A \\ 4 \times 5 \end{matrix}$$

[ອີກຕະລິກ ທີ່ I' ຈຳປັດຕົວ square matrix $(n \times n)$
ແມ່ນ = ດາວໂຫຼວງການຄົວຂອງລົງທະບຽນ
∴ ພາຍໃຕ້ I' ຈຳປັດຕົວ 4×4

4. What size does the reverse identity matrix need to be used to reflect an image represented by a 8×6 matrix about the y -axis? Why?

ຂົວວັນ
Bans reflection
 y -axis

$$A_{8 \times 6} \times I'_{n \times n}$$

[ອີກຕະລິກ ທີ່ I' ຈຳປັດຕົວ square matrix $(n \times n)$
ແມ່ນ = ດາວໂຫຼວງການຄົວຂອງລົງທະບຽນ
∴ ພາຍໃຕ້ I' ຈຳປັດຕົວ 6×6

5. Use the reverse identity matrix of the appropriate size to reflect the images represented by the following matrices about the x -axis.

x -axis;
ດ້ວຍ $I'_{n \times n} \times a$:

$$a. \begin{bmatrix} 0 & 255 & 255 \\ 0 & 255 & 255 \\ 255 & 0 & 0 \\ 255 & 255 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 255 & 255 \\ 0 & 255 & 255 \\ 255 & 0 & 0 \\ 255 & 255 & 0 \end{bmatrix} =$$

$$\begin{bmatrix} 255 & 0 & 0 \\ 255 & 0 & 0 \\ 0 & 255 & 0 \\ 0 & 0 & 255 \end{bmatrix}$$

b. $\begin{bmatrix} 255 & 0 & 0 \\ 0 & 255 & 0 \end{bmatrix}$

x -axis;
ດ້ວຍ $I'_{n \times n} \times b$:

$$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 255 & 0 & 0 \\ 0 & 255 & 0 \end{bmatrix} =$$

$$\begin{bmatrix} 0 & 255 & 0 \\ 255 & 0 & 0 \end{bmatrix}$$

6. Use the reverse identity matrix of the appropriate size to reflect the images represented by the following matrices about the y -axis.

a. $\begin{bmatrix} 0 & 255 & 255 \\ 0 & 255 & 255 \\ 255 & 0 & 0 \\ 255 & 255 & 0 \end{bmatrix}_{4 \times 3} \times \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}_{3 \times 3}$

$$= \begin{bmatrix} 255 & 255 & 255 \\ 255 & 255 & 255 \\ 0 & 0 & 255 \\ 0 & 255 & 255 \end{bmatrix}$$

b. $\begin{bmatrix} 255 & 0 & 0 \\ 0 & 255 & 0 \end{bmatrix}_{2 \times 3} \times \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}_{3 \times 3}$

$$= \begin{bmatrix} 0 & 0 & 255 \\ 0 & 255 & 0 \end{bmatrix}$$

7. Convert each of the following matrices into vectors.

a. $A = \begin{bmatrix} 5 & 0 \\ 3 & 4 \end{bmatrix}$

$$vec(A) = \begin{bmatrix} 5 \\ 3 \\ 0 \\ 4 \end{bmatrix}$$

b. $B = \begin{bmatrix} 2 & 3 \\ 1 & 6 \end{bmatrix}$

$$vec(B) = \begin{bmatrix} 2 \\ 1 \\ 3 \\ 6 \end{bmatrix}.$$

c. $C = \begin{bmatrix} 8 & 8 & 8 \\ 7 & 7 & 7 \\ 8 & 8 & 8 \end{bmatrix}$

$$vec(C) = \begin{bmatrix} 8 \\ 7 \\ 8 \\ 8 \\ 7 \\ 8 \\ 8 \\ 7 \\ 8 \end{bmatrix}.$$

8. Convert each of the following vectors into matrices.

a. $X = \begin{bmatrix} 3 \\ 5 \\ 6 \\ 1 \end{bmatrix}$

$$mat(X) = \begin{bmatrix} 3 & 6 \\ 5 & 1 \end{bmatrix}$$

b. $Y = \begin{bmatrix} 8 \\ 7 \\ 6 \\ 5 \\ 4 \\ 3 \\ 2 \\ 1 \\ 0 \end{bmatrix}$

$$mat(Y) = \begin{bmatrix} 8 & 5 & 2 \\ 7 & 4 & 1 \\ 6 & 3 & 0 \end{bmatrix}$$

9. Given the reflection of the image with matrix $A = \begin{bmatrix} 0 & 255 & 255 \\ 0 & 0 & 255 \\ 0 & 255 & 255 \end{bmatrix}$ about the x -axis would be A' . What matrix would we need to multiply $\text{vec}(A)$ by on the left in order to obtain $\text{vec}(A')$? Show that your answer gives you the correct result.

$$\text{Thinking} \quad A' = \begin{bmatrix} 0 & 255 & 255 \\ 0 & 0 & 255 \\ 0 & 255 & 255 \end{bmatrix} \rightarrow \text{Vec}(A') = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 255 \\ 0 \\ 255 \\ 255 \\ 0 \\ 255 \end{bmatrix}$$

$$\text{P.S. } A' = A$$

$$\therefore \text{Vec}(A') = \text{Vec}(A)$$

$$\text{So } P \circ \text{Vec}(A) = \text{Vec}(A')$$

$$[\quad] \quad - \quad \begin{bmatrix} 0 \\ 0 \\ 0 \\ 255 \\ 0 \\ 255 \\ 255 \\ 0 \\ 255 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 255 \\ 0 \\ 255 \\ 255 \\ 0 \\ 255 \end{bmatrix} - \textcircled{1}$$

$$\text{So } \text{Vec}(A) = \text{Vec}(A')$$

Now we have to find the matrix P such that

$P \circ \text{Vec}(A) = \text{Vec}(A')$

$\therefore P$ is a 9×9 matrix

$P = I_{9 \times 9} \cdot \text{Ans}$

10. Consider an image whose matrix representation is $A = \begin{bmatrix} 0 & 255 & 255 \\ 0 & 0 & 255 \\ 0 & 255 & 255 \end{bmatrix}$. Suppose we

define a "neighbor" as an entry that is directly above, below, next to or diagonal from that entry. The blurred image A' is defined as the average each entry's "neighbors" (including the entry itself). What is the matrix A' ? And what is the 9×9 matrix P such that $A' = P * \text{vec}(A)$?

Sol
132 min

$$A' = \begin{bmatrix} \frac{0+255+0+0}{4} & \frac{0+255+255+0+0}{6} & \frac{255+255+0+255}{4} \\ \frac{0+255+0+0+0+255}{6} & \frac{0+255+255+0+0+255}{9} & \frac{255+255+0+0+255+255}{6} \\ \frac{0+0+0+255}{4} & \frac{0+0+255+0+255+255}{6} & \frac{0+255+255+255}{4} \end{bmatrix}$$

Matrix $\text{vec}(A)$ =

$$\begin{bmatrix} 0 \\ 0 \\ 0 \\ 255 \\ 0 \\ 255 \\ 255 \\ 255 \\ 255 \end{bmatrix}$$

$$= \begin{bmatrix} 255/4 & 255/2 & 255 \times \left(\frac{3}{4}\right) \\ 255/3 & 255 \times \left(\frac{5}{9}\right) & 255 \times \left(\frac{5}{6}\right) \\ 255/4 & 255/2 & 255 \times \left(\frac{3}{4}\right) \end{bmatrix}$$

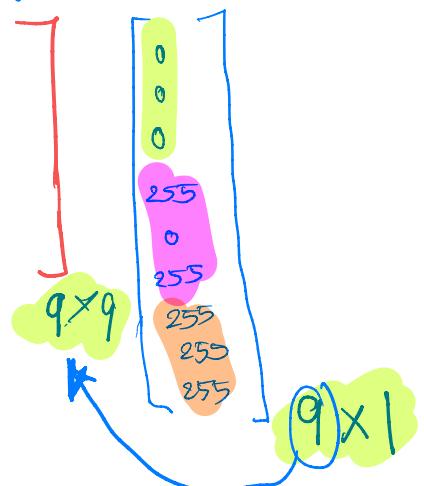
Ans

Transforming $P_{9 \times 9}$ to Φ

$$A' = P \times \text{vec}(A)$$

$$\begin{bmatrix} 255/4 & 255/2 & 255 \times \left(\frac{3}{4}\right) \\ 255/3 & 255 \times \left(\frac{5}{9}\right) & 255 \times \left(\frac{5}{6}\right) \\ 255/4 & 255/2 & 255 \times \left(\frac{3}{4}\right) \end{bmatrix}$$

$$= \boxed{\quad}$$



$$P = \begin{bmatrix} \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 & 0 \\ \frac{1}{6} & \frac{1}{6} & 0 & \frac{1}{6} & \frac{1}{6} & 0 & \frac{1}{6} & \frac{1}{6} & 0 \\ 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & 0 \\ \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & 0 & 0 & 0 \\ \frac{1}{9} & \frac{1}{9} \\ 0 & 0 & 0 & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} \\ 0 & \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 \\ 0 & \frac{1}{6} & \frac{1}{6} & 0 & \frac{1}{6} & \frac{1}{6} & 0 & \frac{1}{6} & \frac{1}{6} \\ 0 & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} \end{bmatrix}$$

~~Ans~~