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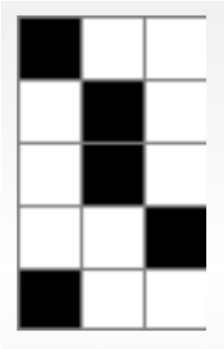
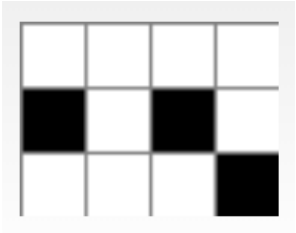
Worksheet

Image Processing through Linear Algebra 1

1. Write a matrix that represents each of the following images.

a. 

$$= \begin{bmatrix} 0 & 255 & 255 \\ 255 & 0 & 255 \\ 255 & 0 & 255 \\ 255 & 255 & 0 \\ 0 & 0 & 255 \end{bmatrix}$$

b. 

$$= \begin{bmatrix} 255 & 255 & 255 & 255 \\ 0 & 255 & 0 & 255 \\ 255 & 255 & 255 & 0 \end{bmatrix}$$

2. (I) Draw a visual representation of the following matrices.

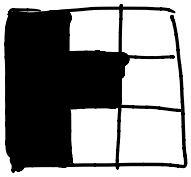
   (II) Write the matrix and draw the visual representation that corresponds to the reflection of each of the following about the

   a. x – axis

   b. y – axis

(I) Visual

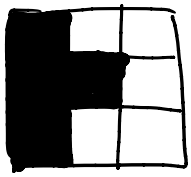
a. $\begin{bmatrix} 0 & 255 & 255 \\ 0 & 0 & 255 \\ 0 & 255 & 255 \end{bmatrix}$ =



(a) x-axis

matrix; a'_x = $\begin{bmatrix} 0 & 255 & 255 \\ 0 & 0 & 255 \\ 0 & 255 & 255 \end{bmatrix}$

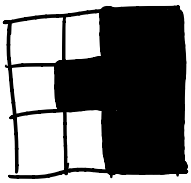
visual; a'_x =



(b) y-axis

matrix; a'_y = $\begin{bmatrix} 255 & 255 & 0 \\ 255 & 0 & 0 \\ 255 & 255 & 0 \end{bmatrix}$

visual; a'_y =



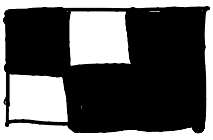
(I) Visual

b. $\begin{bmatrix} 255 & 0 & 0 \\ 0 & 255 & 0 \end{bmatrix}$

(a) x-axis

matrix; b'_x = $\begin{bmatrix} 0 & 255 & 0 \\ 255 & 0 & 0 \end{bmatrix}$

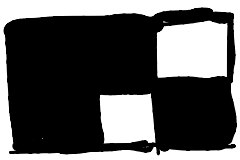
visual; b'_x =



(b) y-axis

matrix; b'_y = $\begin{bmatrix} 0 & 0 & 255 \\ 0 & 255 & 0 \end{bmatrix}$

visual; b'_y =



3. What size does the reverse identity matrix need to be used to reflect an image represented by a $4\ x\ 5$ matrix about the x-axis? Why?

4. What size does the reverse identity matrix need to be used to reflect an image represented by a $8\ x\ 6$ matrix about the y-axis? Why?

5. Use the reverse identity matrix of the appropriate size to reflect the images represented by the following matrices about the x-axis.

a. $\begin{bmatrix} 0 & 255 & 255 \\ 0 & 255 & 255 \\ 255 & 0 & 0 \\ 255 & 255 & 0 \end{bmatrix}$

b. $\begin{bmatrix} 255 & 0 & 0 \\ 0 & 255 & 0 \end{bmatrix}$

6. Use the reverse identity matrix of the appropriate size to reflect the images represented by the following matrices about the y-axis.

a. $\begin{bmatrix} 0 & 255 & 255 \\ 0 & 255 & 255 \\ 255 & 0 & 0 \\ 255 & 255 & 0 \end{bmatrix}$ $\times$ $\begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$

4x3 3x3

$= \begin{bmatrix} 255 & 255 & 255 \\ 255 & 255 & 255 \\ 0 & 0 & 255 \\ 0 & 255 & 255 \end{bmatrix}$ Ans

b. $\begin{bmatrix} 255 & 0 & 0 \\ 0 & 255 & 0 \end{bmatrix}$ $\times$ $\begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$

2x3 3x3

$= \begin{bmatrix} 0 & 0 & 255 \\ 0 & 255 & 0 \end{bmatrix}$ Ans

7. Convert each of the following matrices into vectors.

a. $A = \begin{bmatrix} 5 & 0 \\ 3 & 4 \end{bmatrix}$ $vec(A) = \begin{bmatrix} 5 \\ 3 \\ 0 \\ 4 \end{bmatrix}$.

b. $B = \begin{bmatrix} 2 & 3 \\ 1 & 6 \end{bmatrix}$ $\quad vec(B) = \begin{bmatrix} 2 \\ 1 \\ 3 \\ 6 \end{bmatrix}$.

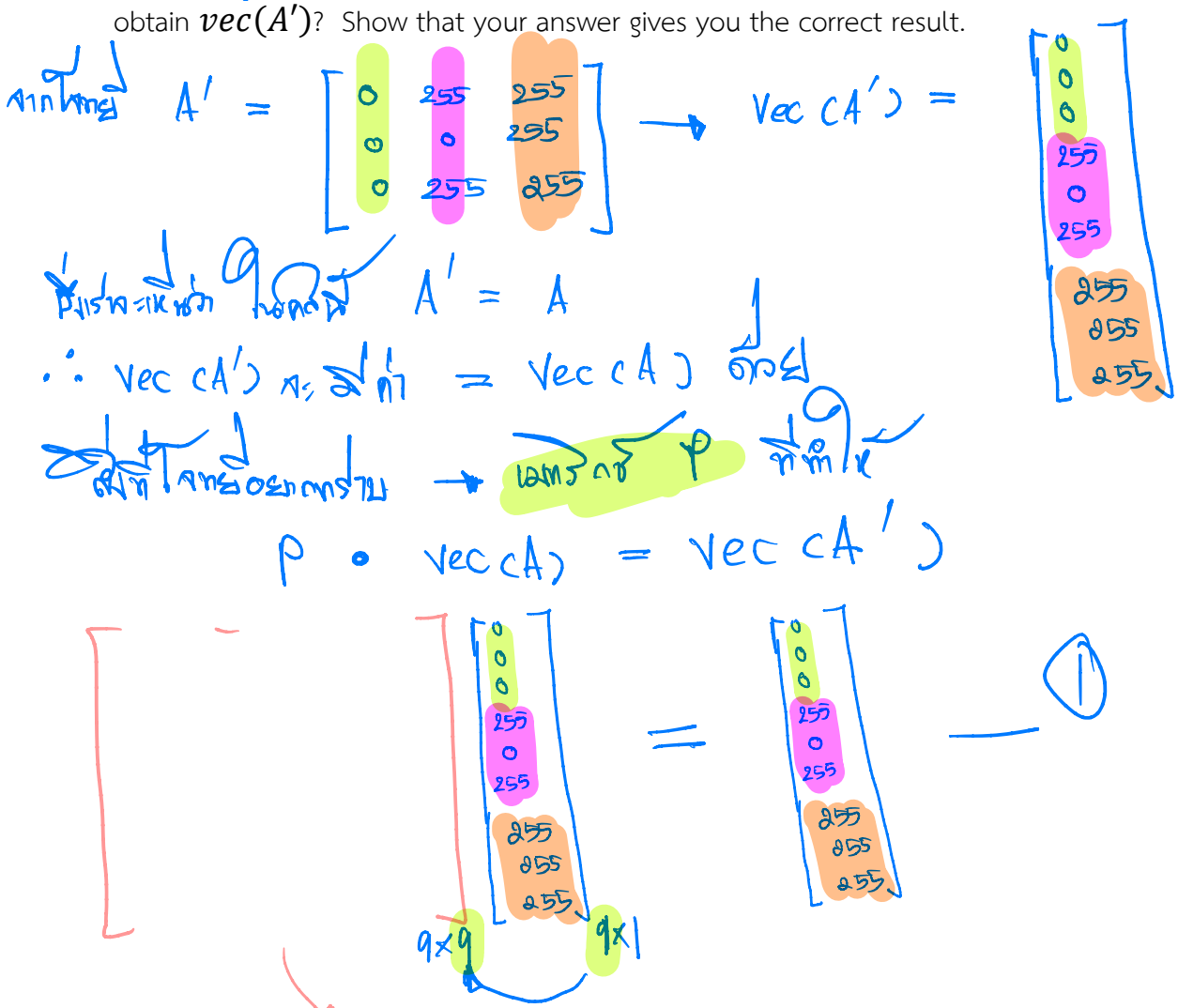
c. $C = \begin{bmatrix} 8 & 8 & 8 \\ 7 & 7 & 7 \\ 8 & 8 & 8 \end{bmatrix}$ $\quad vec(C) = \begin{bmatrix} 8 \\ 7 \\ 8 \\ 8 \\ 7 \\ 8 \\ 8 \\ 7 \\ 8 \end{bmatrix}$.

8. Convert each of the following vectors into matrices.

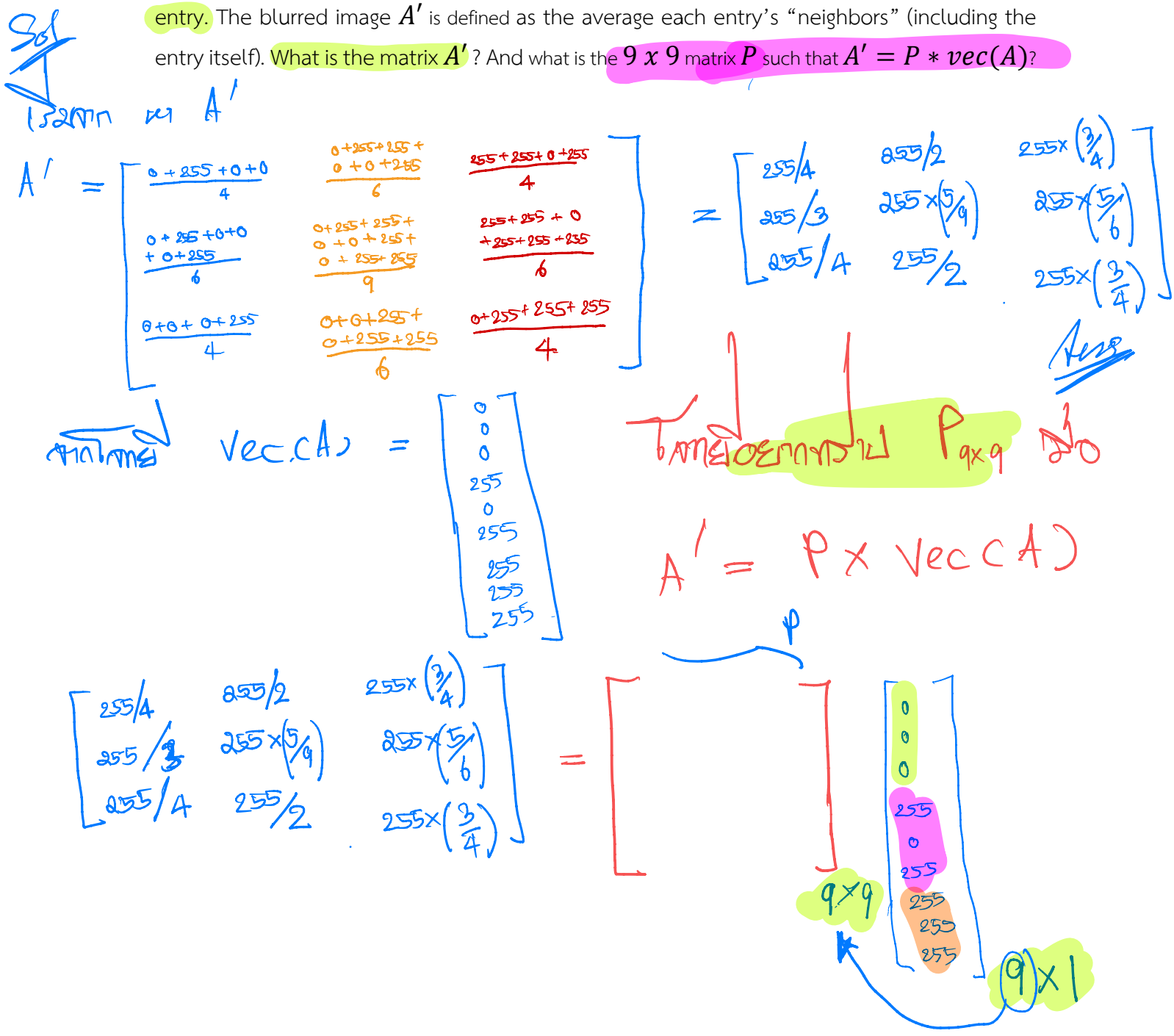
a. $X = \begin{bmatrix} 3 \\ 5 \\ 6 \\ 1 \end{bmatrix}$ $\quad mat(X) = \begin{bmatrix} 3 & 6 \\ 5 & 1 \end{bmatrix}$

b. $Y = \begin{bmatrix} 8 \\ 7 \\ 6 \\ 5 \\ 4 \\ 3 \\ 2 \\ 1 \\ 0 \end{bmatrix}$ $\quad mat(Y) = \begin{bmatrix} 8 & 5 & 2 \\ 7 & 4 & 1 \\ 6 & 3 & 0 \end{bmatrix}$

9. Given the reflection of the image with matrix $A = \begin{bmatrix} 0 & 255 & 255 \\ 0 & 0 & 255 \\ 0 & 255 & 255 \end{bmatrix}$ about the x-axis would be A'. What matrix would we need to multiply $vec(A)$ by on the **left** in order to obtain $vec(A')$? Show that your answer gives you the correct result.



10. Consider an image whose matrix representation is $A = \begin{bmatrix} 0 & 255 & 255 \\ 0 & 0 & 255 \\ 0 & 255 & 255 \end{bmatrix}$. Suppose we define a "neighbor" as an entry that is directly above, below, next to or diagonal from that entry. The blurred image A' is defined as the average each entry's "neighbors" (including the entry itself). What is the matrix A' ? And what is the $9\ x\ 9$ matrix P such that $A' = P * vec(A)$?



$$P = \begin{bmatrix}
\frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 & 0 \\
\frac{1}{6} & \frac{1}{6} & 0 & \frac{1}{6} & \frac{1}{6} & 0 & \frac{1}{6} & \frac{1}{6} & 0 \\
0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & 0 \\
\frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & 0 & 0 & 0 \\
\frac{1}{9} & \frac{1}{9} & \frac{1}{9} & \frac{1}{9} & \frac{1}{9} & \frac{1}{9} & \frac{1}{9} & \frac{1}{9} & \frac{1}{9} \\
0 & 0 & 0 & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} \\
0 & \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 \\
0 & \frac{1}{6} & \frac{1}{6} & 0 & \frac{1}{6} & \frac{1}{6} & 0 & \frac{1}{6} & \frac{1}{6} \\
0 & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4}
\end{bmatrix}$$

Ans