การบ้านพิเศษ

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$$\int_0^2 \int_0^3 (12x^2y + 6xy^2 - 10)\, dy\, dx$$

$$\int_0^2 \left[12x^2 \frac{y^2}{2} + 6x \frac{y^3}{3} - 10y \Big|_0^3 \right] dx$$

$$\int_0^2 \left[6x^2y^2 + 2xy^3 - 10y \Big|_0^3 \right] dx$$

$$\int_0^2 \left[(6x^2(3^2) + 2x(3^3) - 10(3)) - (0 + 0 - 0) \right] dx$$

$$\int_0^2 \left[54x^2 + 54x - 30 \right] dx$$

$$= 54 \frac{x^3}{3} + 54 \frac{x^2}{2} - 30x \Big|_0^2$$

$$= 18x^3 + 27x^2 - 30x \Big|_0^2$$

$$= 18(2^3) + 27(2^2) - 30(2)$$

$$= 192 \text{ \#}$$

1. จงหาค่าของ $\displaystyle\int_{0}^{\frac{\pi}{2}}\int_{0}^{\sin y} e^{x} \cos y \, dx\, dy$

$$\cancel{\int_{0}^{\frac{\pi}{2}} \left[e^{x} \cos y \Big|_{0}^{\sin y} \right] dy}$$

$$\int_{0}^{\frac{\pi}{2}} \left[e^{x} \cos y - \sin(y) + \cos(y) \Big|_{0}^{\sin y} \right] dy$$

$$\int_{0}^{\frac{\pi}{2}} \left[e^{\sin y} \cos y - \sin(y) + \cos(y) \right] dy$$

$$= e^{\sin y} \sin(y) + \cos(y) + \sin(y) \Big|_{0}^{\frac{\pi}{2}}$$

$$= e^{\sin(\frac{\pi}{2})} \sin(\tfrac{\pi}{2}) + \cos(\tfrac{\pi}{2}) + \sin(\tfrac{\pi}{2})$$

$$= 1 + 0 + 1 + 1$$

$$= 3$$

$$2.\ \int_0^{\frac{\pi}{2}}\int_0^{\cos\theta}\int_0^{r}\frac{z}{r}\sin\theta\,dz\,dr\,d\theta$$

$$=\int_0^{\frac{\pi}{2}}\int_0^{\cos\theta}\left[\frac{z^2}{2r}\sin\theta\,\Big|_0^r\right]dr\,d\theta$$

$$=\int_0^{\frac{\pi}{2}}\int_0^{\cos\theta}\left[\frac{r^2}{2r}\sin\theta-0\right]dr\,d\theta$$

$$=\int_0^{\frac{\pi}{2}}\left[\frac{r^2}{4}\sin\theta\,\Big|_0^{\cos\theta}\right]d\theta$$

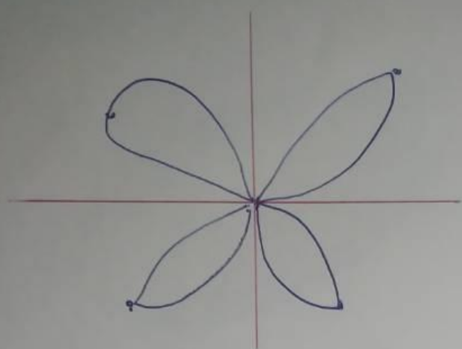
$$=\frac{1}{4}\int_0^{\frac{\pi}{2}}\cos^2\theta\cdot\sin\theta\,d\theta$$

$$=\frac{1}{4}\left[-\frac{\cos^3\theta}{3}\right]\Big|_0^{\frac{\pi}{2}}$$

$$=\frac{-1}{12}\cancel{\cos^3\left(\frac{\pi}{2}\right)}^{0}+\frac{1}{12}\cancel{\cos^3(0)}^{1}$$

$$=\frac{1}{12}\ \#$$

๓. จงหาพื้นที่ปิดล้อมด้วยเส้นโค้ง $r = 3\sin 2\theta$



$$A = 4\int_0^{\frac{\pi}{2}}\int^{3\sin 2\theta} r\,dr\,d\theta$$

$$= 4\int_0^{\frac{\pi}{2}} \left(\frac{r^2}{2}\Big|^{3\sin 2\theta}\right) d\theta$$

$$= 2\int_0^{\frac{\pi}{2}} (9\sin^2 2\theta - 0)\, d\theta$$

$$= 18\int_0^{\frac{\pi}{2}} \frac{1}{2}(1 - \cos 4\theta)\, d\theta$$

$$= 9\int_0^{\frac{\pi}{2}} (1 - \cos 4\theta)\, d\theta$$

$$= \left[\theta - 9\frac{\sin 4\theta}{4}\right|_0^{\frac{\pi}{2}}$$

$$= \left(9\frac{\pi}{2} - \frac{9}{4}\sin 2\pi\right) -$$

$$\left(9(0) - \frac{9}{4}\sin 0\right)$$

$$= \frac{9\pi}{2}$$ ตารางหน่วย

$r = 0$

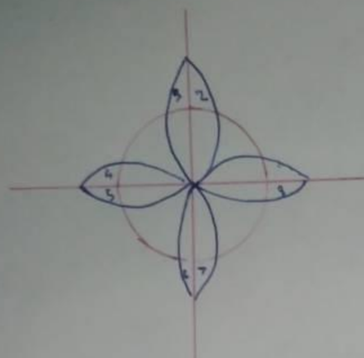
$0 = 3\sin 2\theta$

$0 = \sin 2\theta$

$\therefore 2\theta = 0^\circ, \pi, 2\pi,$

$\theta = 0, \frac{\pi}{2}$

4.



$r = 2 \quad -①$

$r = 4\cos 2\theta \quad -②$

$2 = 4\cos 2\theta$

$\frac{1}{2} = \cos 2\theta$

$\therefore\ 2\theta = \frac{\pi}{3}$

$\theta = \frac{\pi}{6}$

$$A = 8\int_0^{\frac{\pi}{2}}\int_2^{4\cos 2\theta} 1\, r\, dr\, d\theta$$

$$= \cancel{8}^{4}\int_0^{\frac{\pi}{6}} \left[\frac{r^2}{\cancel{2}}\Big|_2^{4\cos 2\theta}\right] d\theta$$

$$= 4\int_0^{\frac{\pi}{6}} \left[16\cos^2 2\theta - 4\right] d\theta$$

$$= 4\int_0^{\frac{\pi}{6}} \left[\frac{16}{2}(1+\cos 4\theta) - 4\right] d\theta$$

$$= 4\int_0^{\frac{\pi}{6}} \left[8 + 8\cos 4\theta - 4\right] d\theta$$

$$= 4\int_0^{\frac{\pi}{6}} \left[4 + 8\cos 4\theta\right] d\theta$$

$$= 4\left[4\theta + \frac{8}{4}\sin 4\theta\right]\Big|_0^{\frac{\pi}{6}}$$

$$= (16\theta + 8\sin 4\theta)\Big|_0^{\frac{\pi}{6}}$$

$$= \frac{16\pi}{6} + 8\sin 4\frac{\pi}{6} - (0 + 8\sin 0)$$

$$= \frac{8\pi}{3} + 8\sin\frac{2\pi}{3}$$

$$= \frac{8\pi}{3} + 8\left(\frac{\sqrt{3}}{2}\right) = \frac{8\pi}{3} + 4\sqrt{3}$$

ตารางหน่วย