Departamento de Electrónica, Telecomunicações e Informática da Universidade de Aveiro

Algoritmos Avançados

2022/2023 — 1° Semestre

1st Project — Exhaustive Search

Deadline: November 6, 2022

Introduction

Design and test an **exhaustive search algorithm** to solve one of the following **graph problems**, as well as another **method using a greedy heuristics**.

Afterwards, analyze the computational complexity of the developed algorithms. To accomplish that:

- a) Perform a formal computational complexity analysis of the algorithms.
- b) Carry out a sequence of experiments, for successively larger problem instances, to register and analyze (1) the **number of basic operations** carried out, (2) the **execution time** and (3) the **number of solutions / configurations** tested.
- c) Compare the results of the experimental and the formal analysis.
- d) Determine the largest graph that you can process on your computer, without taking too much time.
- e) Estimate the execution time that would be required by much larger problem instances.
- f) Write a report (8 pages, max.).

Graphs for the Computational Experiments

The **graph instances** used in the **computational experiments** should represent the following scenario:

- graph **vertices** are 2D points on the XOY plane, with integer valued coordinates between 1 and 20.
- graph vertices should neither be coincident nor too close.
- the **number of edges** sharing a vertex is randomly determined.

Generate successively larger **random graphs**, with 4, 5, 6, ... vertices, using your **student number** as **seed**.

Use 12.5%, 25%, 50% and 75% of the maximum number of edges for the number of vertices.

Suggestion: store each graph in a file to be used for the computational experiments.

Depending on the problem, it might be helpful to represent each graph by its **adjacency matrix** or by its **incidence matrix**.

It might also be useful to graphically **visualize** the graph instances and the computed solutions.

Decision Problems – Set k to 12.5%, 25%, 50% and 75% of the number of graph vertices

- 1 For a given undirected graph <math>G(V, E), with n vertices and m edges, does G have a **clique with** k **vertices**? A **clique of** G is a subset of vertices, all adjacent to each other, i.e., defining a complete subgraph of G. Example: in a social network, a clique is a subset of people who all know each other.
- 2 For a given undirected graph G(V, E), with n vertices and m edges, does G have an **independent set with** k **vertices**? An **independent set of** G is a subset of vertices, no two of which are adjacent. Example: in a social network, an independent set is a subset of people who do not know each other.
- 3 For a given undirected graph G(V, E), with n vertices and m edges, does G have a **vertex cover** with k vertices? A **vertex cover of** G is a set C of vertices, such that each edge of G is incident to, at least, one vertex in C.
- 4 For a given undirected graph G(V, E), with n vertices and m edges, does G have a **dominating** set with k vertices? A **dominating** set of G is a subset D of vertices, such that every vertex not in D is adjacent to, at least, one vertex in D. Application example: dominating sets are used in wireless networking to find efficient routes.
- 5 For a given **directed graph** G(V, E), with n vertices and m edges, does G have a **closure with** k **vertices**? A **closure of** G is a set of vertices C, such that no edges leave C.

Decision Problems – Set k to 12.5%, 25%, 50% and 75% of the number of graph edges

- 6 For a given undirected graph G(V, E), with n vertices and m edges, does G have an **edge cover** with k edges? An edge cover is a set C of edges such that each vertex of G is incident to, at least, one edge in C.
- 7 For a given undirected graph G(V, E), with n vertices and m edges, does G have an **edge dominating set with** k **edges**? An **edge dominating set of** G is a subset D of edges, such that every edge not in D is adjacent to, at least, one edge in D.
- 8 For a given undirected graph G(V, E), with n vertices and m edges, does G have a **cut with** k **edges**? A **cut of** G **with** k **edges** is a partition of the graph's vertices into two complementary sets S and T, such that the number of edges between the set S and the set T is k.
- 9 For a given undirected graph G(V, E), with n vertices and m edges, does G have a **matching** with k edges? A **matching** in G is a set of pairwise non-adjacent edges, i.e., no two edges share a common vertex.

Optimization Problems

- 10 Find a **maximum clique** for a given undirected graph G(V, E), with n vertices and m edges. A **clique of** G is a subset of vertices, all adjacent to each other, i.e., defining a complete subgraph of G. A **maximum clique** is a clique with the **largest** possible number of vertices. Example: in a social network, a clique is a subset of people who all know each other.
- 11 Find a **maximum weight clique** for a given undirected graph G(V, E), whose **vertices carry positive weights**, with n vertices and m edges. A **clique of** G is a subset of vertices, all adjacent to each other, i.e., defining a complete subgraph of G. The **weight of a clique** is the sum of its vertices' weights. A **maximum weight clique** is a clique whose total weight is as large as possible.
- 12 Find a **maximum independent set** for a given undirected graph G(V, E), with n vertices and m edges. An **independent set of** G is a subset of vertices, no two of which are adjacent. A **maximum independent set** is an independent set of **largest** possible size. Example: in a social network, an independent set is a subset of people who do not know each other.
- 13 Find a **maximum weight independent set** for a given undirected graph G(V, E), whose **vertices carry positive weights**, with n vertices and m edges. An **independent set of** G is a subset of vertices, no two of which are adjacent. The **weight of an independent set** is the sum of its vertices' weights. A **maximum weight independent set** is an independent set whose total weight is as large as possible.
- 14 Find a **minimum vertex cover** for a given undirected graph G(V, E), with n vertices and m edges. A **vertex cover of** G is a set C of vertices, such that each edge of G is incident to, at least, one vertex in C. A **minimum vertex cover** is a vertex cover of **smallest** possible size.
- 15 Find a **minimum weight vertex cover** for a given undirected graph G(V, E), whose **vertices** carry **positive weights**, with n vertices and m edges. A **vertex cover of** G is a set C of vertices, such that each edge of G is incident to, at least, one vertex in C. The **weight of a vertex cover** is the sum of its vertices' weights. A **minimum weight vertex cover** is a vertex cover whose total weight is as small as possible.
- 16 Find a **minimum dominating set** for a given undirected graph G(V, E), with n vertices and m edges. A **dominating set of** G is a subset D of vertices, such that every vertex not in D is adjacent to, at least, one vertex in D. A **minimum dominating set** is a dominating set of **smallest** possible size. Application example: dominating sets are used in wireless networking to find efficient routes.
- 17 Find a **minimum weight dominating set** for a given undirected graph G(V, E), whose **vertices carry positive weights**, with n vertices and m edges. A **dominating set of** G is a subset D of vertices, such that every vertex not in D is adjacent to, at least, one vertex in D. The **weight of a dominating set** is the sum of its vertices' weights. A **minimum weight dominating set** is a dominating set whose total weight is as small as possible.
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- 20 Find a **minimum edge dominating set** for a given undirected graph G(V, E), with n vertices and m edges. An **edge dominating set** of G is a subset D of edges, such that every edge not in D is adjacent to, at least, one edge in D. A **minimum edge dominating set** is an edge dominating set of **smallest** possible size.
- 21 Find a **minimum weight edge dominating set** for a given undirected graph G(V, E), with n vertices and m edges. An **edge dominating set** of G is a subset D of edges, such that every edge not in D is adjacent to, at least, one edge in D. The **weight of an edge dominating set** is the sum of its edges' weights. A **minimum weight edge dominating set** is an edge dominating set whose total weight is as **small** as possible.
- 22 Find a **maximum cut** for a given undirected graph G(V, E), with n vertices and m edges. A **maximum cut of** G is a partition of the graph's vertices into two complementary sets S and T, such that the number of edges between the set S and the set T is as **large** as possible.
- 23 Find a **maximum weighted cut** for a given undirected graph G(V, E), with n vertices and m edges. A **maximum weighted cut** of G is a partition of the graph's vertices into two complementary sets S and T, such that the **sum of the weights of edges between the set S and the set T** is as large as possible.
- 24 Find a minimum cut for a given undirected graph G(V, E), with n vertices and m edges. A minimum cut of G is a partition of the graph's vertices into two complementary sets S and T, such that the number of edges between the set S and the set T is as **small** as possible.
- 25 Find a **minimum weighted cut** for a given undirected graph G(V, E), with n vertices and m edges. A **minimum weighted cut** of G is a partition of the graph's vertices into two complementary sets S and T, such that the **sum of the weights of edges between the set S and the set T** is as small as possible.
- 26 Find a **maximum weighted closure** for a given vertex-weighted directed graph G(V, E), with n vertices and m edges. A **closure of G** is a set of vertices C, such that no edges leave C. The **weight of a closure** is the sum of its vertices' weights. A **maximum weight closure** is a closure whose total weight is as large as possible.
- 27 Find a **minimum weighted closure** for a given vertex-weighted directed graph G(V, E), with n vertices and m edges. A **closure of G** is a set of vertices C, such that no edges leave C. The **weight of a closure** is the sum of its vertices' weights. A **minimum weight closure** is a closure whose total weight is as small as possible.
- 28 Find a **maximum matching** for a given undirected graph G(V, E), with n vertices and m edges. A **matching in** G is a set of pairwise non-adjacent edges, i.e., no two edges share a common vertex. A **maximum matching** is a matching of **largest** possible size. Application example: computational chemistry.
- 29 Find a **maximum weighted matching** for a given undirected graph G(V, E), with n vertices and m edges. A **matching in** G is a set of pairwise non-adjacent edges, i.e., no two edges share a common vertex. A **maximum weighted matching** is a matching for which the **sum of the weights** of its edges is as large as possible.
- 30 Find the **chromatic number** of a given undirected graph G(V, E), with n vertices and m edges. The **chromatic number of** G is the **smallest** number of colors needed to properly **color the vertices**

of graph G, i.e., to label the vertices with colors such that no two adjacent vertices have the same color. Application example: vertex coloring is used for register allocation in compilers.

31 – Find the **chromatic index** of a given undirected graph G(V, E), with n vertices and m edges. The **chromatic index of** G is the **smallest** number of colors needed to properly **color the edges** of graph G, i.e., to label the edges with colors such that no two edges sharing the same vertex have the same color. Application example: edge coloring is used for frequency assignment for fiber optics networks.

J. Madeira, October 11, 2022