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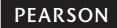
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Introduction to The Design & Analysis of Algorithms 3RD EDITION

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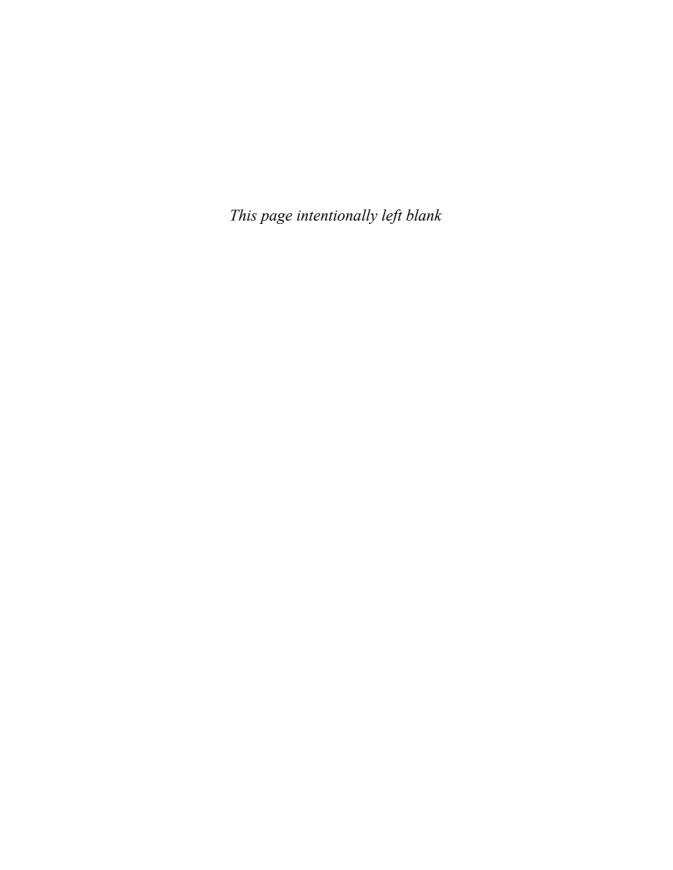
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Useful Formulas for the Analysis of Algorithms

This appendix contains a list of useful formulas and rules that are helpful in the mathematical analysis of algorithms. More advanced material can be found in [Gra94], [Gre07], [Pur04], and [Sed96].

Properties of Logarithms

All logarithm bases are assumed to be greater than 1 in the formulas below; $\lg x$ denotes the logarithm base 2, $\ln x$ denotes the logarithm base e = 2.71828...; x, y are arbitrary positive numbers.

1.
$$\log_a 1 = 0$$

2.
$$\log_a a = 1$$

$$3. \quad \log_a x^y = y \log_a x$$

$$4. \quad \log_a xy = \log_a x + \log_a y$$

$$5. \quad \log_a \frac{x}{y} = \log_a x - \log_a y$$

$$6. \quad a^{\log_b x} = x^{\log_b a}$$

7.
$$\log_a x = \frac{\log_b x}{\log_b a} = \log_a b \log_b x$$

Combinatorics

- 1. Number of permutations of an *n*-element set: P(n) = n!
- **2.** Number of k-combinations of an n-element set: $C(n, k) = \frac{n!}{k!(n-k)!}$
- 3. Number of subsets of an *n*-element set: 2^n

Important Summation Formulas

1.
$$\sum_{i=l}^{u} 1 = \underbrace{1 + 1 + \dots + 1}_{u-l+1 \text{ times}} = u - l + 1 \ (l, u \text{ are integer limits}, l \le u); \quad \sum_{i=1}^{n} 1 = n$$

2.
$$\sum_{i=1}^{n} i = 1 + 2 + \dots + n = \frac{n(n+1)}{2} \approx \frac{1}{2}n^2$$

3.
$$\sum_{i=1}^{n} i^2 = 1^2 + 2^2 + \dots + n^2 = \frac{n(n+1)(2n+1)}{6} \approx \frac{1}{3}n^3$$

4.
$$\sum_{i=1}^{n} i^{k} = 1^{k} + 2^{k} + \dots + n^{k} \approx \frac{1}{k+1} n^{k+1}$$

5.
$$\sum_{i=0}^{n} a^{i} = 1 + a + \dots + a^{n} = \frac{a^{n+1} - 1}{a - 1} \ (a \neq 1); \quad \sum_{i=0}^{n} 2^{i} = 2^{n+1} - 1$$

6.
$$\sum_{i=1}^{n} i 2^{i} = 1 \cdot 2 + 2 \cdot 2^{2} + \dots + n 2^{n} = (n-1)2^{n+1} + 2$$

7.
$$\sum_{i=1}^{n} \frac{1}{i} = 1 + \frac{1}{2} + \dots + \frac{1}{n} \approx \ln n + \gamma$$
, where $\gamma \approx 0.5772 \dots$ (Euler's constant)

$$8. \quad \sum_{i=1}^n \lg i \approx n \lg n$$

Sum Manipulation Rules

$$1. \quad \sum_{i=l}^{u} ca_i = c \sum_{i=l}^{u} a_i$$

2.
$$\sum_{i=1}^{u} (a_i \pm b_i) = \sum_{i=1}^{u} a_i \pm \sum_{i=1}^{u} b_i$$

3.
$$\sum_{i=l}^{u} a_i = \sum_{i=l}^{m} a_i + \sum_{i=m+1}^{u} a_i$$
, where $l \le m < u$

4.
$$\sum_{i=l}^{u} (a_i - a_{i-1}) = a_u - a_{l-1}$$

Approximation of a Sum by a Definite Integral

$$\int_{l-1}^{u} f(x)dx \le \sum_{i=l}^{u} f(i) \le \int_{l}^{u+1} f(x)dx \quad \text{for a nondecreasing } f(x)$$

$$\int_{l}^{u+1} f(x)dx \le \sum_{i=l}^{u} f(i) \le \int_{l-1}^{u} f(x)dx \quad \text{for a nonincreasing } f(x)$$

Floor and Ceiling Formulas

The *floor* of a real number x, denoted $\lfloor x \rfloor$, is defined as the greatest integer not larger than x (e.g., $\lfloor 3.8 \rfloor = 3$, $\lfloor -3.8 \rfloor = -4$, $\lfloor 3 \rfloor = 3$). The *ceiling* of a real number x, denoted $\lceil x \rceil$, is defined as the smallest integer not smaller than x (e.g., $\lceil 3.8 \rceil = 4$, $\lceil -3.8 \rceil = -3$, $\lceil 3 \rceil = 3$).

- **1.** $x 1 < \lfloor x \rfloor \le x \le \lceil x \rceil < x + 1$
- **2.** $\lfloor x+n\rfloor = \lfloor x\rfloor + n$ and $\lceil x+n\rceil = \lceil x\rceil + n$ for real x and integer n
- 3. $|n/2| + \lceil n/2 \rceil = n$
- **4.** $\lceil \lg(n+1) \rceil = \lfloor \lg n \rfloor + 1$

Miscellaneous

- **1.** $n! \approx \sqrt{2\pi n} \left(\frac{n}{e}\right)^n$ as $n \to \infty$ (Stirling's formula)
- 2. Modular arithmetic (n, m are integers, p is a positive integer)

$$(n+m) \bmod p = (n \bmod p + m \bmod p) \bmod p$$

 $(nm) \bmod p = ((n \bmod p)(m \bmod p)) \bmod p$