Answers to questions in

Lab 1: Filtering operations

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**Instructions**: Complete the lab according to the instructions in the notes and respond to the questions stated below. Keep the answers short and focus on what is essential. Illustrate with figures only when explicitly requested.

Good luck!

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**Question 1**: Repeat this exercise with the coordinates p and q set to (5, 9), (9, 5), (17, 9),

(17, 121), (5, 1) and (125, 1) respectively. What do you observe?

Answers:

Sinus waves

**Question 2**: Explain how a position (p, q) in the Fourier domain will be projected as a sine wave in the spatial domain. Illustrate with a Matlab figure.

Answers:

P is the frequency in the y direction and q in the x direction.

**Question 3**: How large is the amplitude? Write down the expression derived from Equation (4) in the notes. Complement the code (variable amplitude) accordingly.

Answers:

amplitude = abs(Fhat(u,v)/(sz^2))

6.1e-5

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**Question 4**: How does the direction and length of the sine wave depend on p and q? Write down the explicit expression that can be found in the lecture notes. Complement the code (variable wavelength) accordingly.

Answers:

wavelength = 1/sqrt(uc\*uc + vc\*vc)

**Question 5**: What happens when we pass the point in the center and either p or q exceeds half the image size? Explain and illustrate graphically with Matlab!

Answers:

Frequencies above half the length gets translated into negative frequencies instead.

**Question 6**: What is the purpose of the instructions following the question *What is done by these instructions?* in the code?

Answers:

The code shifts the frequencies that are above half the range to their corresponding negative frequencies and also shifts the frequency by one because of Matlab indexing. A (1,1) in Matlab is actually the (0,0) frequency.

**Question 7**: Why are these Fourier spectra concentrated to the borders of the images? Can you give a mathematical interpretation? Hint: think of the frequencies in the source image and consider the resulting image as a Fourier transform applied to a 2D function. It might be easier to analyze each dimension separately!

Answers:

Since the two images don’t change when looking at rows for one picture and columns for the other, we only get frequencies across the border. Since one of the frequencies are always zero we only get frequencies across the border.

**Question 8**: Why is the logarithm function applied?

Answers:

Since we have some very high value pixels and a lot of zeros, applying the log increases the contrast.

**Question 9**: What conclusions can be drawn regarding linearity? From your observations can you derive a mathematical expression in the general case?

Answers:

F(f + g) = F(f) + F(g)

**Question 10**: Are there any other ways to compute the last image? Remember what multiplication in Fourier domain equals to in the spatial domain! Perform these alternative computations in practice.

Answers:

Multiplication in spatial domain is Convolution in Fourier domain.

**Question 11**: What conclusions can be drawn from comparing the results with those in the previous exercise? See how the source images have changed and analyze the effects of scaling.

Answers:

We now have more lower frequencies in the x direction and higher frequencies in the y because of the new shape of the box.

**Question 12**: What can be said about possible similarities and differences? Hint: think of the frequencies and how they are affected by the rotation.

Answers:

Rotating images in the spatial domain also rotates in the Fourier domain. Rotation causes distances between the boxes to change in some directions, which changes the fourier domain a bit in the reconstruction, on some angles.

**Question 13**: What information is contained in the phase and in the magnitude of the Fourier transform?

Answers:

Magnitude is the intensity of the pixel, phase more controls the orientation and structure of the image.

**Question 14**: Show the impulse response and variance for the above-mentioned t-values. What are the variances of your discretized Gaussian kernel for t = 0.1, 0.3, 1.0, 10.0 and

100.0?

Answers:

t = 0.1 gives x,y = 0.01

t = 0.3 gives x,y = 0.28

t = 1.0 gives x,y = 1.0

t = 10.0 gives x,y = 10.0

t = 100 gives x,y = 100

**Question 15**: Are the results different from or similar to the estimated variance? How does the result correspond to the ideal continuous case? Lead: think of the relation between spatial and Fourier domains for different values of t.

Answers:

On low ts we can’t get a high enough resolution of the filter which causes the errors.

**Question 16**: Convolve a couple of images with Gaussian functions of different variances (like t = 1.0, 4.0, 16.0, 64.0 and 256.0) and present your results. What effects can you observe?

Answers:

The higher the blur, the less we observe. Higher ts give more blur.

**Question 17**: What are the positive and negative effects for each type of filter? Describe what you observe and name the effects that you recognize. How do the results depend on the filter parameters? Illustrate with Matlab figure(s).

Answers:

Gaussian: Gaussian filters smear out the pixels inside the filter. This looks good on gaussian noise, but not not sat noise. With sat noise, this filter doesn’t remove the noise, just smears it out on other pixels instead of removing it.

Mean: Median filter takes the median pixel value in a range. This works well on sat noise, since the noise will have no impact on the resulting pixel. It can get “off by one” errors on sharp edges. Also, the resulting image looks like it was painted.

Low pass: Behaves similarly to the gaussian. This filter removes high frequencies in the fourier domain which can be noise. Therefore, the resulting image can look a bit wavy because it could be missing some frequencies that “fix” the shape.

**Question 18**: What conclusions can you draw from comparing the results of the respective methods?

Answers:

Use different filters depending on the noise.

**Question 19**: What effects do you observe when subsampling the original image and the smoothed variants? Illustrate both filters with the best results found for iteration i = 4.

Answers:

Smoothing before subsampling can give a more readable image.

**Question 20**: What conclusions can you draw regarding the effects of smoothing when combined with subsampling? Hint: think in terms of frequencies and side effects.

Answers:

When we remove high frequencies, we get a wavy-pattern after subsampling.

If we don’t smooth before we subsample, we will get frequencies that we are not interested in aliased in.