An Introduction to Matlab

Version 3.0

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Contents

1 MATLAB

- Matlab is an interactive system for doing numerical computations.
- A numerical analyst called Cleve Moler wrote the first version of Matlab in the 1970s. It has since evolved into a successful commercial software package.
- Matlab relieves you of a lot of the mundane tasks associated with solving problems numerically. This allows you to spend more time thinking, and encourages you to experiment.
- Matlab makes use of highly respected algorithms and hence you can be confident about your results.
- Powerful operations can be performed using just one or two commands.
- You can build up your own set of functions for a particular application.
- Excellent graphics facilities are available, and the pictures can be inserted into LaTeX and Word documents.

These notes provide only a brief glimpse of the power and flexibility of the Matlab system. For a more comprehensive view we recommend the book

Matlab Guide 2nd ed.
D.J. Higham & N.J. Higham
SIAM Philadelphia, 2005, ISBN:
0-89871-578-4.

2 Starting Up

Matlab can be used in a number of different ways or modes; as an advanced calculator in the calculator mode, in a high level programming language mode and as a subroutine called from a C-program. More information on the first two of these modes is provided by these notes.
When used in calculator mode all Matlab commands are entered to the command line from the keyboard at the "command line prompt" indicated with '>>'.

Type `quit` at any time **to exit from Matlab**.
Extensive documentation is available, either via the command line by using the '`help topic`' command (see below) or via the internet. We recommend starting with the command

```
demo
```

(a link may also be provided on the top line of the command window). This brings up a separate window which gives access to a short video entitled "Getting Started" that describes the purpose of the various panes in the main Matlab window.
Help is available from the command line prompt. Type `help help` for "help" (which gives a brief synopsis of the help system), `help` for a list of topics. The first few lines of this read

```
HELP topics:      -
                  -
MatlabCode/matlab - (No table of contents file)
matlab/general    - General purpose commands.
matlab/ops        - Operators and special ...
matlab/lang       - Programming language ...
matlab/elmat      - Elementary matrices and ...
matlab/randfun    - Random matrices and ...
matlab/elfun      - Elementary math functions.
matlab/specfun    - Specialized math functions.
```

(truncated lines are shown with ...). Then to obtain help on "Elementary math functions", for instance, type

```
>> help elfun
```

Clicking on a key word, for example `sin` will provide further information together with a link to `doc sin` which provides the most extensive documentation on a keyword along with examples of its use.
Another useful facility is to use the

```
lookfor keyword
```

command, which searches the help files for the keyword. See Exercise 15.1 (page 22) for an example of its use.

3 Matlab as a Calculator

The basic arithmetic operators are `+ - * / ^` and these are used in conjunction with brackets: `( )`. The symbol `^` is used to get exponents (powers): `2^4=16`.
You should type in the commands shown at the prompt: `>>`.

```
>> 2 + 3/4*5
ans =
    5.7500
>>
```

Is this calculation `2 + 3/(4*5)` or `2 + (3/4)*5`? Matlab works according to the priorities:

1. quantities in brackets,
2. powers `2 + 3^2` $\Rightarrow$`2 + 9 = 11`,
3. `* /`, working left to right (`3*4/5=12/5`),
4. `+ -`, working left to right (`3+4-5=7-5`),

Thus, the earlier calculation was for `2 + (3/4)*5` by priority 3.

4 Numbers & Formats

Matlab recognizes several different kinds of numbers

Type	Examples
Integer	$1362, -217897$
Real	$1.234, -10.76$
Complex	$3.21 - 4.3i \quad (i = \sqrt{-1})$
Inf	Infinity (result of dividing by 0)
NaN	**N**ot **a** **N**umber, $0/0$

The "`e`" notation is used for very large or very small numbers:
`-1.3412e+03` $= -1.3412 \times 10^3 = -1341.2$
`-1.3412e-01` $= -1.3412 \times 10^{-1} = -0.13412$
All computations in MATLAB are done in double precision, which means about 15 significant figures. How Matlab prints numbers is controlled by the "format" command. Type `help format` for full list.
Should you wish to switch back to the default format then `format` will suffice.

Command	Example of Output
`>>format short`	`31.4162`(4–decimal places)
`>>format short e`	`3.1416e+01`
`>>format long e`	`3.141592653589793e+01`
`>>format short`	`31.4162`(4–decimal places)
`>>format bank`	`31.42`(2–decimal places)

The command

```
format compact
```

is also useful in that it suppresses blank lines in the output thus allowing more information to be displayed.

5 Variables

```
>> 3-2^4
ans =
   -13
>> ans*5
ans =
   -65
```

The result of the first calculation is labelled "`ans`" by Matlab and is used in the second calculation, where its value is changed.
We can use our own names to store numbers:

```
>> x = 3-2^4
x =
   -13
>> y = x*5
y =
   -65
```

so that `x` has the value -13 and $y = -65$. These can be used in subsequent calculations. These are examples of **assignment statements**: values are assigned to variables. Each variable must be assigned a value before it may be used on the right of an assignment statement.

5.1 Variable Names

Legal names consist of any combination of letters and digits, starting with a letter. These are allowable:

```
NetCost, Left2Pay, x3, X3, z25c5
```

These are **not** allowable:

```
Net-Cost, 2pay, %x, @sign
```

Use names that reflect the values they represent.
Special names: you should avoid using `eps` (which has the value `2.2204e-16`$= 2^{-54}$ which is the largest number such that `1 + eps` is indistinguishable from 1) and
`pi = 3.14159...` $= \pi$.
If you wish to do arithmetic with complex numbers,both `i` and `j` have the value $\sqrt{-1}$ unless you change them

```
>> i,j, i=3
ans = 0 + 1.0000i
ans = 0 + 1.0000i
i   = 3
```

See Section 8.4 for more on complex numbers.

6 Suppressing output

One often does not want to see the result of intermediate calculations—terminate the assignment statement or expression with semi–colon

```
>> x=-13; y = 5*x, z = x^2+y
y =
   -65
z =
   104
>>
```

the value of `x` is hidden. Note that we can place several statements on one line, separated by commas or semi–colons.

Exercise 6.1 *In each case find the value of the expression in Matlab and explain precisely the order in which the calculation was performed.*

i) `-2^3+9` ii) `2/3*3`
iii) `3*2/3` iv) `3*4-5^2*2-3`
v) `(2/3^2*5)*(3-4^3)^2` vi) `3*(3*4-2*5^2-3)`

7 Built–In Functions

7.1 Trigonometric Functions

Those known to Matlab are

`sin, cos, tan`
and their arguments should be in radians.
e.g. to work out the coordinates of a point on a circle of radius 5 centred at the origin and having an elevation $30^o = \pi/6$ radians:

```
>> x = 5*cos(pi/6), y = 5*sin(pi/6)
x =
    4.3301
y =
    2.5000
```

To work in degrees, use `sind`, `cosd` and `tand`. The inverse trig functions are called `asin, acos, atan` (as opposed to the usual arcsin or $\sin^{-1}$ etc.). The result is in radians.

```
>> acos(x/5), asin(y/5)
ans = 0.5236
ans = 0.5236
>> pi/6
ans = 0.5236
```

7.2 Other Elementary Functions

These include `sqrt, exp, log, log10`

```
>> x = 9;
>> sqrt(x),exp(x),log(sqrt(x)),log10(x^2+6)
ans =
     3
ans =
   8.1031e+03
ans =
    1.0986
ans =
    1.9395
```

`exp(x)` denotes the exponential function $\exp(x) = e^x$ and the inverse function is `log`:

```
>> format long e, exp(log(9)), log(exp(9))
ans = 9.000000000000002e+00
ans = 9
>> format short
```

and we see a tiny rounding error in the first calculation. `log10` gives logs to the base 10. A more complete list of elementary functions is given in Table 2 on page 42.

8 Vectors

These come in two flavours and we shall first describe **row vectors**: they are lists of numbers separated by either commas or spaces. The number of entries is known as the "length" of the vector and the entries are often referred to as "elements" or "components" of the vector. The entries must be enclosed in square brackets.

```
>> v = [ 1 3, sqrt(5)]
v =
    1.0000    3.0000    2.2361
>> length(v)
ans =
     3
```

Spaces can be vitally important:

```
>> v2 = [3+ 4 5]
v2 =
     7     5
>> v3 = [3 +4 5]
v3 =
     3     4     5
```

We can do certain arithmetic operations with vectors of the same length, such as `v` and `v3` in the previous section.

```
>> v + v3
ans =
    4.0000    7.0000    7.2361
>> v4 = 3*v
v4 =
    3.0000    9.0000    6.7082
>> v5 = 2*v -3*v3
v5 =
   -7.0000   -6.0000  -10.5279
>> v + v2
??? Error using ==> +
Matrix dimensions must agree.
```

i.e. the error is due to `v` and `v2` having different lengths.
A vector may be multiplied by a scalar (a number, see `v4` above), or added/subtracted to another vector of the **same** length. The operations are carried out elementwise.
We can build row vectors from existing ones:

```
>> w  = [1 2 3],  z = [8 9]
>> cd = [2*z,-w], sort(cd)
w =
     1     2     3
z =
     8     9
cd =
    16    18    -1    -2    -3
ans =
    -3    -2    -1    16    18
```

Notice the last command `sort`'ed the elements of `cd` into ascending order.
We can also change or look at the value of particular entries

```
>> w(2) = -2, w(3)
w =
     1    -2     3
ans =
     3
```

8.1 The Colon Notation

This is a shortcut for producing row vectors:

```
>>  1:4
ans =
     1     2     3     4
>> 3:7
ans =
     3     4     5     6     7
>> 1:-1
ans =
     []
```

More generally $a : b : c$ produces a vector of entries starting with the value a, incrementing by the value b until it gets to c (it will not produce a value beyond c). This is why `1:-1` produced the empty vector `[]`.

```
>> 0.32:0.1:0.6
ans =
    0.3200    0.4200    0.5200
>> -1.4:-0.3:-2
ans =
   -1.4000   -1.7000   -2.0000
```

8.2 Extracting Parts of Vectors

```
>>  r5 = [1:2:6, -1:-2:-7]
r5 =
  1  3  5  -1  -3  -5  -7
```

To get the 3rd to 6th entries:

```
>> r5(3:6)
ans =
     5    -1    -3    -5
```

To get alternate entries:

```
>> r5(1:2:7)
ans =
     1     5    -3    -7
```

What does `r5(6:-2:1)` give?
See `help colon` for a fuller description.

8.3 Column Vectors

These have similar constructs to row vectors except that entries are separated by `;` or "newlines"

```
>> c = [ 1; 3; sqrt(5)]
c =
    1.0000
    3.0000
    2.2361
>> c2 = [3
4
5]
c2 =
     3
     4
     5
>> c3 = 2*c - 3*c2
c3 =
   -7.0000
   -6.0000
  -10.5279
```

so column vectors may be added or subtracted **provided that they have the same length**. The length of a vector (number of elements) can be determined by

```
>> length(c)
ans = 3
>> length(r5)
ans = 7
```

and does not distinguish between row and column vectors (compare with `size` described in §15.1). The `size` might be needed to determine the last element in a vector but this can be found by using the reserved word `end`:

```
>> c2(end), c2(end-1:end)
ans =
   4
ans =
   4   5
```

8.4 Transposing

We can convert a row vector into a column vector (and vice versa) by a process called *transposing* which is denoted by `'`.

```
>> w, w', [1 2 3], [1 2 3]'
w =
     1    -2     3
ans =
     1
    -2
     3
ans =
    1.0000
    3.0000
    2.2361
ans =
    1.0000    3.0000    2.2361
>> t = w + 2*[1 2 3]'
t =
    3.0000    4.0000    7.4721
>> T = 5*w'-2*[1 2 3]
T =
    3.0000
  -16.0000
   10.5279
```

If `x` is a *complex vector*, then `x'` gives the *complex conjugate transpose* of `x`:

```
>> x = [1+3i, 2-2i]
ans =
   1.0000 + 3.0000i   2.0000 - 2.0000i
>> x'
ans =
   1.0000 - 3.0000i
   2.0000 + 2.0000i
```

Note that the components of `x` were defined without a `*` operator; this means of defining complex numbers works even when the variable `i` already has a numeric value. To obtain the plain transpose of a complex number use `.'` as in

```
>> x.'
ans =
   1.0000 + 3.0000i
   2.0000 - 2.0000i
```

One must be aware at all times, as the next example shows:

```
>> i=3; [1+2i, 3-i, 3-1i]
ans =
 1.0000 + 2.0000i   0  3.0000 - 1.0000i
```

in which only the 2nd element has been influenced by the value of the variable `i`.

9 Keeping a record

Issuing the command

```
>> diary mysession
```

will cause all subsequent text that appears on the screen to be saved to the file `mysession` located in the directory in which Matlab was invoked. You may use any legal filename *except* the names `on` and `off`. The record may be terminated by

```
>> diary off
```

The file `mysession` may be edited with your favourite editor (the Matlab editor, emacs, or even Word) to remove any mistakes.
If you wish to quit Matlab midway through a calculation so as to continue at a later stage:

```
>> save thissession
```

will save the current values of all variables to a file called `thissession.mat`. **This file cannot be edited**. When you next startup Matlab, type

```
>> load thissession
```

and the computation can be resumed where you left off. We do not advocate this procedure except is special circumstances, but suggest making use of script files (see Section 10).
A list of variables used in the current session may be seen with

```
>> whos
```

They can also be seen in the "Workspace" pane of the main window. See `help whos` and `help save`.
`>> whos`

```
Name  Size  Elements Bytes Density Complex
  ans 1 by 1    1     8    Full    No
v   1 by 3    3    24    Full    No
  v1  1 by 2    2    16    Full    No
  v2  1 by 2    2    16    Full    No

Grand total 16 elements using 128 bytes
```

10 Script Files

Script files are ordinary ASCII (text) files that contain Matlab commands. It is essential that such files have names having an extension `.m` (e.g., `myfile.m`) and, for this reason, they are commonly known as *m-files*. The commands in this file may then be executed using
`>> myfile`
Note: the command does not include the file name extension `.m`.
Script files are created with the built-in editor (it is possible to change to your favourite editor in the Preferences window). Any text that follows `%` on a line is ignored. This enables descriptive comments to be included. It is possible, via a mouse menu, to highlight commands that appear in the "Command History" pane to create a script file. "Cut and Paste" can be used to copy individual commands from the "Command History" pane into a script file.

Exercise 10.1 *1. Type in the commands from §8.4 into a file called* `exsub.m`*. Its contents might look like:*

```
% My first script file: exsub.m
w, w', [1 2 3], [1 2 3]'
t = w + 2*[1 2 3]'
              % Use w to compute T
T = 5*w'-2*[1 2 3]
```

2. *Check in the "Current Folder" pane of the Matlab window (or use the command* **what***, which lists the m-files in the current directory) to see that the file is in the correct area.*

3. *Use the command* **type exsub** *to see the contents of the file.*

4. *Execute the file with the command* **exsub**.

It is only the output from the commands (and not the commands themselves) that are displayed on the screen. To see the commands in the command window prior to their execution:
`>> echo on`
and `echo off` will turn echoing off. Compare the effect of
`>> echo on, exsub, echo off`
with the results obtained earlier.
See §22 for the related topic of function files.

11 Keyboard Accelerators

One can recall previous Matlab commands in the Command Window by using the ↑ and ↓ cursor keys. Repeatedly pressing ↑ will review the previous commands (most recent first) and, if you want to re-execute the command, simply press the return key.
To recall the most recent command starting with `p`, say, type `p` at the prompt followed by ↑. Similarly, typing `pr` followed by ↑ will recall the most recent command starting with `pr`.
Once a command has been recalled, it may be edited (changed). You can use ← and → to move backwards and forwards through the line, characters may be inserted by typing at the current cursor position or deleted using the `Del` key. This is most commonly used when long command lines have been mistyped or when you want to re–execute a command that is very similar to one used previously.

The following emacs–like commands may also be used:

`cntrl a`	move to start of line
`cntrl e`	move to end of line
`cntrl f`	move forwards one character
`cntrl b`	move backwards one character
`cntrl d`	delete character under the cursor

Once the command is in the required form, press return.

Exercise 11.1 *Type in the commands*

```
>> x = -1:0.1:1;
>> plot(x,sin(pi*x),'w-')
>> hold on
>> plot(x,cos(pi*x),'r-')
```

Now use the cursor keys with suitable editing to execute:

```
>> x = -1:0.05:1;
>> plot(x,sin(2*pi*x),'w-')
>> plot(x,cos(2*pi*x),'r-.'), hold off
```

12 Arithmetic with Vectors

12.1 Inner Product (*)

We shall describe two ways in which a meaning may be attributed to the product of two vectors. In both cases the vectors concerned must have the **same length**.
The first product is the standard inner product. Suppose that $\underline{u}$ and $\underline{v}$ are two vectors of length n, $\underline{u}$ being a **row** vector and $\underline{v}$ a **column** vector:

$$\underline{u} = [u_1, u_2, \ldots, u_n], \qquad \underline{v} = \begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_n \end{bmatrix}.$$

The inner product is defined by multiplying the corresponding elements together and adding the results to give a single number (inner).

$$\underline{u} * \underline{v} = \sum_{i=1}^{n} u_i v_i.$$

For example, if $\underline{u} = [10, -11, 12]$, $\underline{v} = \begin{bmatrix} 20 \\ -21 \\ -22 \end{bmatrix}$

then $n = 3$ and

$\underline{u} * \underline{v} = 10 \times 20 + (-11) \times (-21) + 12 \times (-22) = 167.$

We can perform this product in Matlab by

```
>> u = [ 10, -11, 12], v = [20; -21; -22]
>> prod = u*v % row times column vector
```

Suppose we also define a row vector `w` and a column vector `z` by

```
>> w = [2, 1, 3], z = [7; 6; 5]
w =
     2     1     3
z =
     7
     6
     5
```

and we wish to form the inner products of `u` with `w` and `v` with `z`.

```
>> u*w
??? Error using ==> *
Inner matrix dimensions must agree.
```

an error results because `w` is not a column vector. Recall from page 6 that transposing (with `'`) turns column vectors into row vectors and vice versa. So, to form the inner product of two row vectors or two column vectors,

```
>> u*w'      % u & w are row vectors
ans =
    45
>> u*u'      % u is a row vector
ans =
   365
>> v'*z      % v & z are column vectors
ans =
   -96
```

The Euclidean length of a vector is an example of the **norm** of a vector; it is denoted by the symbol $\|\underline{u}\|$ and defined by

$$\|\underline{u}\| = \sqrt{\sum_{i=1}^{n} |u_i|^2},$$

where n is its dimension. This can be computed in Matlab in one of two ways:

```
>> [ sqrt(u*u'), norm(u)]
ans =
   19.1050    19.1050
```

where `norm` is a built–in Matlab function that accepts a vector as input and delivers a scalar as output. It can also be used to compute other norms: `help norm`.

Exercise 12.1 *The angle, θ, between two column vectors $\underline{x}$ and $\underline{y}$ is defined by*

$$\cos\theta = \frac{\underline{x}'\underline{y}}{\|\underline{x}\| \, \|\underline{y}\|}.$$

Use this formula to determine the cosine of the angle between

$$\underline{x} = [1, 2, 3]' \quad \textit{and} \quad \underline{y} = [3, 2, 1]'.$$

Hence show that the angle is 44.4153degrees.

[Hint: see `cosd` and `acosd`.]

12.2 Elementwise Product (`.*`)

The second way of forming the product of two vectors of the same length is known as the Hadamard product. It is rarely used in the course of normal mathematical calculations but is an invaluable Matlab feature. It involves vectors of the same type. If $\underline{u}$ and $\underline{v}$ are two vectors of the same type (both row vectors or both column vectors), the mathematical definition of this product, which we shall call the **Hadamard product**, is the **vector** having the components

$$\underline{u} \cdot * \underline{v} = [u_1 v_1, u_2 v_2, \ldots, u_n v_n].$$

The result is a vector of the same length and type as $\underline{u}$ and $\underline{v}$. Thus, we simply multiply the corresponding elements of two vectors. Summing the entries in the resulting vector would give their inner product.
For example, if $\underline{u} = [10, -11, 12]$, and $\underline{w} = [2, 1, 3]$ then $n = 3$ and

$$\begin{aligned} \underline{u}. * \underline{w} &= [10 \times 2, (-11) \times (1), 12 \times (3)] \\ &= [20, -11, 36] \end{aligned}$$

In Matlab, the product is computed with the operator `.*` and, using the vectors `u, v, w, z` defined on page 9,

```
>> u.*w
ans =
      20 -11 36
>> u.*v'
ans =
      200 231 -264
>> v.*z

ans =
      140
     -126
     -110
```

Perhaps the most common use of the Hadamard product is in the evaluation of mathematical expressions so that they may be plotted.

Example 12.1 *Tabulate the function* $y = x \sin \pi x$ *for* $x = 0, 0.25, \ldots, 1$.

The display is clearer with column vectors so we first define a vector of x-values: (see Transposing: §8.4)

```
>> x = (0:0.25:1)';
```

To evaluate y we have to multiply each element of the vector x by the corresponding element of the vector $\sin \pi x$:

$x \times$	$\sin \pi x =$	$x \sin \pi x$
$0 \times$	$0 =$	0
$0.2500 \times$	$0.7071 =$	0.1768
$0.5000 \times$	$1.0000 =$	0.5000
$0.7500 \times$	$0.7071 =$	0.5303
$1.0000 \times$	$0.0000 =$	0.0000

To carry this out in Matlab:

```
>>  y = x.*sin(pi*x)
y =
         0
    0.1768
    0.5000
    0.5303
    0.0000
```

Note: a) the use of `pi`, b) `x` and `sin(pi*x)` are both column vectors (the `sin` function is applied to each element of the vector). Thus, the Hadamard product of these is also a column vector.

Exercise 12.2 *Enter the vectors*

$$\underline{U} = [6, 2, 4], \quad \underline{V} = [3, -2, 3, 0],$$

$$\underline{W} = \begin{bmatrix} 3 \\ -4 \\ 2 \\ -6 \end{bmatrix}, \quad \underline{Z} = \begin{bmatrix} 3 \\ 2 \\ 2 \\ 7 \end{bmatrix}$$

into Matlab. Which of the products
`U*V, V*W, U*V', V*W', W*Z', U.*V`
`U'*V, V'*W, W'*Z, U.*W, W.*Z, V.*W`
is legal? State whether the legal products are row or column vectors and give the values of the legal results.

12.3 Elementwise Division (./)

In Matlab, the operator `./` is defined to give element by element division of one vector by another—it is therefore only defined for vectors of the same size and type.

```
>> a = 1:5, b = 6:10, a./b
a =
     1     2     3     4     5
b =
     6     7     8     9    10
ans =
 0.1667  0.2857  0.3750  0.4444  0.5000
```

If we change to `format rat` (short for rational)

```
>> format rat
>> (1:5)./(6:10)
ans =
   1/6    2/7    3/8    4/9    1/2
>> format compact
```

the output is displayed in fractions. Note that

```
>> a./a
ans =
     1     1     1     1     1
>> c = -2:2, a./c
c =
    -2    -1     0     1     2
Warning: Divide by zero
ans =
 -0.5000  -2.0000  Inf   4.0000   2.5000
```

The previous calculation required division by 0—notice the `Inf`, denoting infinity, in the answer.

```
>> a.*b -24, ans./c
ans =
   -18   -10     0    12    26

Warning: Divide by zero
ans =
     9    10   NaN    12    13
```

Here we are warned about 0/0—giving a `NaN` (**N**ot **a** **N**umber).

Example 12.2 *Estimate the limit*

$$\lim_{x\to 0} \frac{\sin \pi x}{x}.$$

The idea is to observe the behaviour of the ratio $\frac{\sin \pi x}{x}$ for a sequence of values of x that approach zero. Suppose that we choose the sequence defined by the column vector

```
>> x = [0.1; 0.01; 0.001; 0.0001]
```

then

```
>> sin(pi*x)./x
ans =
    3.0902
    3.1411
    3.1416
    3.1416
```

which suggests that the values approach π. To get a better impression, we subtract the value of π from each entry in the output and, to display more decimal places, we change the format

```
>> format long
>> ans -pi
ans =
  -0.05142270984032
  -0.00051674577696
  -0.00000516771023
  -0.00000005167713
```

Can you explain the pattern revealed in these numbers?
We also need to use `./` to compute a scalar divided by a vector:

```
>> 1/x
??? Error using ==> /
Matrix dimensions must agree.
>> 1./x
ans =
   10    100    1000    10000
```

so `1./x` works, but `1/x` does not.

12.4 Elementwise Powers (.^)

The square of each element of a vector could be computed with the command `u.*u`. However, a neater way is to use the `.^` operator:

```
>> u = [10, 11, 12]; u.^2
ans =
   100   121   144
>> u.*u
ans =
   100   121   144
>> ans.^(1/2)
ans =
    10    11    12
>> u.^4
ans =
       10000        14641        20736
>> v.^2
ans =
   400
   441
   484
>> u.*w.^(-2)
ans =
    2.5000  -11.0000    1.3333
```

Recall that powers (`.^` in this case) are done first, before any other arithmetic operation. Fractional and decimal powers are allowed.
When the base is a scalar and the power is a vector we get

```
>> n = 0:4
n =
     0     1     2     3     4
>> 2.^n
ans =
     1     2     4     8    16
```

and, when both are vectors of the same dimension,

```
>> x = 1:3:15
x =
    1    4    7    10    13
>> x.^n
ans =
    1     4    49  1000    28561
```

13 Plotting Functions

In order to plot the graph of a function, $y = \sin 3\pi x$ for $0 \leq x \leq 1$, say, it is sampled at a sufficiently large number of points and the points (x, y) joined by straight lines. Suppose we take $N + 1$ sampling points equally spaced a distance h apart:

```
>> N = 10; h = 1/N; x = 0:h:1;
```

defines the set of points $x = 0, h, 2h, \ldots, 1-h, 1$ with $h = 0.1$. Alternately, we may use the command `linspace`: The general form of the command is `linspace (a,b,n)` which generates $n + 1$ equispaced points between a and b, inclusive. So, in this case we would use the command

```
>> x = linspace (0,1,11);
```

The corresponding y values are computed by

```
>> y = sin(3*pi*x);
```

and finally, we can plot the points with

```
>> plot(x,y)
```

The result is shown in Fig. 1 below, where it is clear that the value of N is too small.

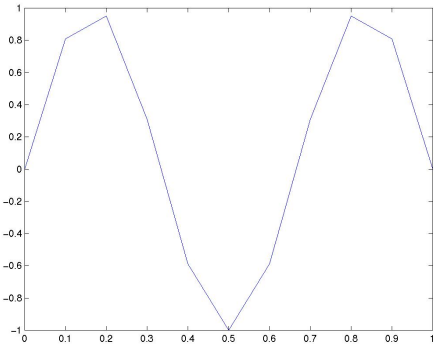


Fig. 1: Graph of $y = \sin 3\pi x$ for $0 \leq x \leq 1$ using $h = 0.1$.

On changing the value of N to 100:

```
>> N = 100; h = 1/N; x = 0:h:1;
>> y = sin(3*pi*x);  plot(x,y)
```

we get the picture shown in Fig. 2.

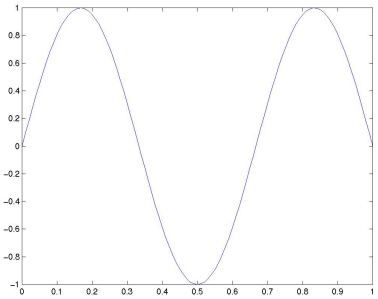


Fig. 2: Graph of $y = \sin 3\pi x$ for $0 \leq x \leq 1$ using $h = 0.01$.

13.1 Plotting—Titles & Labels

To put a title and label the axes, we use

```
>> title('Graph of y = sin(3pi x)')
>> xlabel('x axis')
>> ylabel('y-axis')
```

The *strings* enclosed in single quotes, can be anything of our choosing. Some simple LaTeX commands are available for formatting mathematical expressions and Greek characters—see Section 13.10.
See also `ezplot` the "Easy to use function plotter".

13.2 Grids

A dotted grid may be added by

```
>> grid on
```

and is removed with `grid off`.

13.3 Line Styles & Colours

The default is to plot solid lines. A solid red line is produced by

```
>> plot(x,y,'r--x')
```

The third argument is a string comprising characters that specify the colour (red), the line style (dashed) and the symbol (`x`) to be drawn at each data point. The order in which they appear is unimportant and any, or all, may be omitted. The options for colours, styles and symbols include:

Colours		Line Styles/symbols	
`y`	yellow	`.`	point
`m`	magenta	`o`	circle
`c`	cyan	`x`	x-mark
`r`	red	`+`	plus
`g`	green	`-`	solid
`b`	blue	`*`	star
`w`	white	`:`	dotted
`k`	black	`-.`	dashdot
		`--`	dashed

The number of available plot symbols is wider than shown in this table. Use `help plot` to obtain a full list. See also `help shapes`.
The command `clf` clears the current figure while `close(1)` will close the graphics window labelled "Figure 1". To open a new figure window type `figure` or, to get a window labelled "Figure 9", for instance, type `figure (9)`. If "Figure 9" already exists, this command will bring this window to the foreground and the next plotting commands will be drawn on it.

13.4 Multi–plots

Several graphs may be drawn on the same figure as in

```
>> plot(x,y,'k-',x,cos(3*pi*x),'g--')
```

A descriptive legend may be included with

```
>> legend('Sin curve','Cos curve')
```

which will give a list of line–styles, as they appear in the plot command, followed by the brief description provided in the command.
For further information do `help plot` etc.
The result of the commands

```
>> plot(x,y,'k-',x,cos(3*pi*x),'g--')
>> legend('Sin curve','Cos curve')
>> title('Multi-plot')
>> xlabel('x axis'),  ylabel('y axis')
>> grid
```

is shown in Fig. 3. The legend may be moved either manually by dragging it with the mouse or as described in `help legend`.

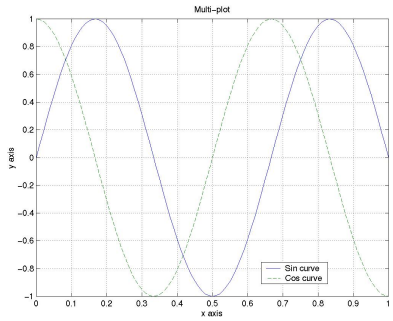


Fig. 3: Graph of $y = \sin 3\pi x$ and $y = \cos 3\pi x$ for $0 \leq x \leq 1$ using $h = 0.01$.

13.5 Hold

A call to `plot` clears the graphics window before plotting the current graph. This is not convenient if we wish to add further graphics to the figure at some later stage. To stop the window being cleared:

```
>> plot(x,y,'r-'), hold on
>> plot(x,y,'gx'), hold off
```

"`hold on`" holds the current picture; "`hold off`" releases it (but does not clear the window, which can be done with `clf`). "`hold`" on its own toggles the hold state.

13.6 Hard Copy

To obtain a printed copy select Print from the File menu on the Figure toolbar.
Alternatively one can save a figure to a file for later printing (or editing). A number of formats is available (use `help print` to obtain a list). To save the current figure in "Encapsulated Color PostScript" format, issue the Matlab command

```
print -depsc fig1
```

which will save a copy of the image in a file called `fig1.eps`.

```
print -f4 -djpeg90 figb
```

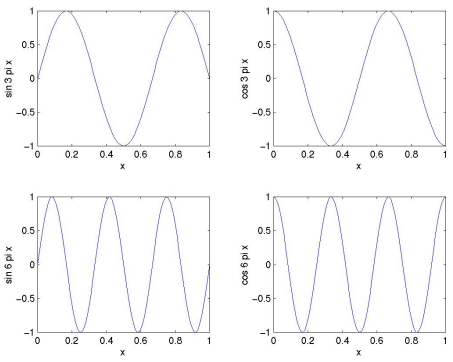
will save figure 4 as a jpeg file `figb.jpg` at a quality level of 90. It should be borne in mind that neither command (despite its name) sends the file to a printer.

13.7 Subplot

The graphics window may be split into an $m \times n$ array of smaller windows into each of which we may plot one or more graphs. The windows are counted 1 to mn row–wise, starting from the top left. Both `hold` and `grid` work on the current subplot.

```
>> subplot(221), plot(x,y)
>>   xlabel('x'),ylabel('sin 3 pi x')
>> subplot(222), plot(x,cos(3*pi*x))
>>   xlabel('x'),ylabel('cos 3 pi x')
>> subplot(223), plot(x,sin(6*pi*x))
>>   xlabel('x'),ylabel('sin 6 pi x')
>> subplot(224), plot(x,cos(6*pi*x))
>>   xlabel('x'),ylabel('cos 6 pi x')
```

`subplot(221)` (or `subplot(2,2,1)`) specifies that the window should be split into a 2×2 array and we select the first subwindow.



13.8 Zooming

We often need to "zoom in" on some portion of a plot in order to see more detail. Clicking on the "Zoom in" or "Zoom out" button on the Figure window is simplest but one can also use the command

```
>> zoom
```

Pointing the mouse to the relevant position on the plot and clicking the left mouse button will zoom in by a factor of two. This may be repeated to any desired level.

Clicking the right mouse button will zoom out by a factor of two.
Holding down the left mouse button and dragging the mouse will cause a rectangle to be outlined. Releasing the button causes the contents of the rectangle to fill the window.
`zoom off` turns off the zoom capability.

Exercise 13.1 *Draw graphs of the functions*

$$\begin{aligned} y &= \cos x \\ y &= x \end{aligned}$$

for $0 \leq x \leq 2$ *on the same window. Use the zoom facility to determine the point of intersection of the two curves (and, hence, the root of* $x = \cos x$*) to two significant figures.*

13.9 Figure Properties

All plot properties can be edited from the Figure window by selecting the Edit and Tools menus from the toolbar. For instance, to change the `linewidth` of a graph, click Edit and choose `Figure Properties...` from the menu. Clicking on the required curve will display its attributes which can be readily modified.
One of the shortcomings of editing the figure window in this way is the difficulty of reproducing the results at a later date. The recommended alternative involves using commands that directly control the graphics properties.
The current setting of any plot property can be determined by first obtaining its "handle number", which is simply a real number that we save to a named variable:

```
>> plt = plot (x,y.^3,'k--o')
plt  =
   188.0194
```

and then using the `get` command. This lists the settings for a number of properties that include

```
>>get(plt)
Color:            [0 0 0]
LineStyle:        '--'
LineWidth:        1
Marker:           'o'
MarkerSize:       6
XData:            [1 2 3]
```

```
YData:                [27 8 1]
ZData:                [1x0 double]
```

The colour is described by a rgb triple in which `[0 0 0]` denotes black and `[1 1 1]` denotes white. Properties can be changed with the `set` command, for example

```
>> set(plt,'markersize',12)
```

will change the size of the marker symbol 'o' while

```
>> set(plt,'linestyle',':','ydata',[1 8 27])
```

will change the lifestyle from dashed to dotted while also changing the y–coordinates of the data points. The commands

```
>> x = 0:.01:1; y=sin(3*pi*x);
>> plot(x,y,'k-',x,cos(3*pi*x),'g--')
>> legend('Sin curve','Cos curve')
>> title('Multi-plot ')
>> xlabel('x axis'),  ylabel('y axis')
>> set(gca,'fontsize',16,...
'ytick',-1:.5:1);
```

redraw Fig. 3 and the last line sets the font size to 16points and changes the tick-marks on the y-axis to $-1, -0.5, 0, 0.5, 1$—see Fig. 4. The `...` in the penultimate line tell Matlab that the line is split and continues on the next line.

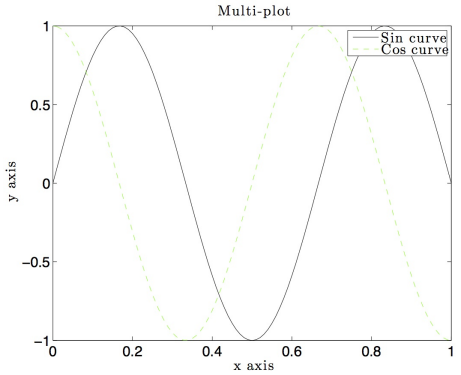


Fig. 4: Repeat of Fig. 3 with a font size of 16points and amended tick marks on the y-axis.

13.10 Formatted text on Plots

It is possible to typeset simple mathematical expressions (using LaTeX commands) in labels, legends, axes and text. We shall give two illustrations.

Example 13.1 *Plot the first 100 terms in the sequence* $\{y_n\}$ *given by* $y_n = \left(1 + \frac{1}{n}\right)^n$ *and illustrate how the sequence converges to the limit* $\mathrm{e} = \exp(1) = 2.7183....$ *as* $n \to \infty$.

Exercises such as this that require a certain amount of experimentation are best carried out by saving the commands in a script file. The contents of the file (which we call `latexplot.m`) are:

```
close all
figure(1);
set(0,'defaultaxesfontsize',12)
set(0,'defaulttextfontsize',16)
set(0,'defaulttextinterpreter','latex')
N = 100; n = 1:N;
y = (1+1./n).^n;
subplot(2,1,1)
plot(n,y,'.','markersize',8)
hold on
axis([0 N,2 3])
plot([0 N],[1, 1]*exp(1),'--')
text(40,2.4,'$y_n = (1+1/n)^n$')
text(10,2.8,'y = e')
xlabel('$n$'), ylabel('$y_n$')
```

The results are shown in the upper part of Fig. 5.

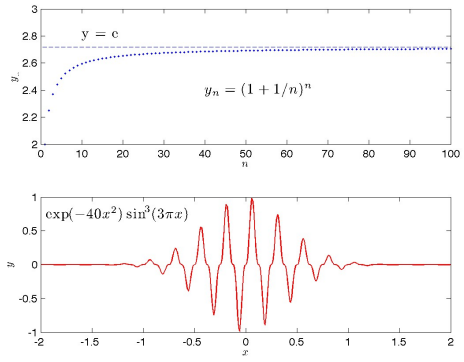


Fig. 5: The output from Example 13.1 (top) and Example 13.2 (bottom).

The salient features of these commands are

1. The `set` commands in lines 3–4 increase the size of the default font size used for

the axis labels, legends, titles and text. Line 4 tells Matlab to interpret any strings contained within $ symbols as LaTeX commands.

2. Defining a variable $N = 100$ makes it easier to experiment with a different number of sampling points.

3. The size of the plot symbol "." is changed from the default (6) to size 8 by the additional string followed in the `plot` command.

4. The `axis` command changes the dimensions of the plotting area to be $0 \le x \le N$ and $2 \le y \le 3$.

   The `axis` command has four parameters, the first two are the minimum and maximum values of x to use on the axis and the last two are the minimum and maximum values of y.

5. The command `text(40,2.4,'string')` prints string at the location with coordinates (40 2.4).

6. The string `y_n` gives subscripts: y_n, while `x^3` gives superscripts: x^3.

Example 13.2 *Draw a graph the function* $y = \mathrm{e}^{-3x^2} \sin^3(3\pi x)$ *on the interval* $-2 \le x \le 2$.

The appropriate commands are included in the script file for the previous example (so the default values continue to operate):

```
subplot(2,1,2)
x = -2:.01:2;
y = exp(-3*x.^2).*sin(8*pi*x).^3;
plot(x,y,'r-','linewidth',1)
xlabel('$x$'), ylabel('$y$')
text(-1.95,.75,'$ \exp(-40x^2)\sin^3(8\pi
print  -djpeg90 eplot1
```

The results are shown in the lower part of Fig. 5.

1. $\sin^3 8\pi x$ is typeset by the LaTeX string `$\sin^3 8\pi x$` and translates into the Matlab command `sin(8*pi*x).^3`—the position of the exponent is different.

2. Greek characters $\alpha, \beta, \ldots, \omega, \Omega$ are produced by the strings '`\alpha`', '`\beta`', ...,'`\omega`', '`\Omega`'. the integral symbol: $\int$ is produced by '`\int`'.

3. The thickness of the line used in the `plot` command is changed from its default value (0.5) to 2.

4. The graphics are saved in jpeg format to the file `eplot1`.

13.11 Controlling Axes

The look of a graph can be changed by using the `axis` command. We have already seen in Example 13.1 how the plotting area can be changed.
`axis equal` is required in order that a circle does not appear as an ellipse

```
>> clf, N = 100; t = (0:N)*2*pi/N;
>> x = cos(t); y = sin(t);
>> plot(x,y,'-',0,0,'.');
>> set(gca,'ytick',-1:.5:1)
>> axis equal
```

See Fig. 6. We recommend looking at `help axis` and experimenting with the commands `axis equal`, `axis off`, `axis square`, `axis normal`, `axis tight` in any order.

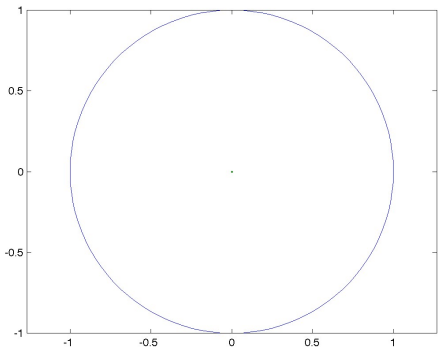


Fig. 6: Use of `axis equal` to get a circle to appear correctly.

14 Elementwise Examples

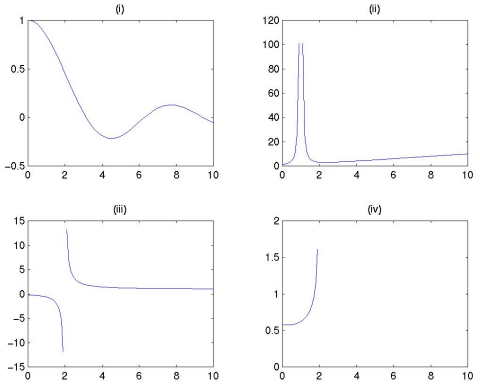
Example 14.1 *Draw graphs of the functions*

i) $y = \frac{\sin x}{x}$ *ii)* $u = \frac{1}{(x-1)^2} + x$

iii) $v = \frac{x^2+1}{x^2-4}$ *iv)* $w = \frac{(10-x)^{1/3}-2}{(4-x^2)^{1/2}}$

for $0 \leq x \leq 10$.

```
>> x = 0:0.1:10;
>> y = sin(x)./x;
>> subplot(221), plot(x,y), title('(i)')
Warning: Divide by zero
>> u = 1./(x-1).^2 + x;
>> subplot(222),plot(x,u), title('(ii)')
Warning: Divide by zero
>> v = (x.^2+1)./(x.^2-4);
>> subplot(223),plot(x,v),title('(iii)')
Warning: Divide by zero
>> w = ((10-x).^(1/3)-1)./sqrt(4-x.^2);
Warning: Divide by zero
>> subplot(224),plot(x,w),title('(iv)')
```



Note the repeated use of the "dot" (elementwise) operators.
Experiment by changing the axes (page 16), grids (page 12) and hold(page 13).

```
>>  subplot(222),axis([0 10 0 10])
>>  grid
>>  grid
>>  hold on
>>  plot(x,v,'--'), hold off,
>>  plot(x,y,':')
```

Exercise 14.1 *Tabulate the functions*

$$y = (x^2 + 3)\sin \pi x^2$$

and

$$z = \sin^2 \pi x/(x^{-2} + 3)$$

for $x = 0, 0.2, \ldots, 10$. *Hence, tabulate the function*

$$w = \frac{(x^2 + 3)\sin \pi x^2 \sin^2 \pi x}{(x^{-2} + 3)}.$$

Plot a graph of w *over the range* $0 \leq x \leq 10$.

15 Two–Dimensional Arrays

A rectangular array of numbers having m rows and n columns is referred to as an $m \times n$ matrix. It is usual in a mathematical setting to enclose such objects in either round or square brackets—Matlab insists on square ones. For example, when $m = 2, n = 3$ we have a 2×3 matrix such as

$$A = \begin{bmatrix} 5 & 7 & 9 \\ 1 & -3 & -7 \end{bmatrix}$$

To enter such an matrix into Matlab we type it in row by row using the same syntax as for vectors:

```
>> A = [5 7 9
        1 -3 -7]
A =
     5     7     9
     1    -3    -7
```

Rows may be separated by semi-colons rather than a new line:

```
>> B = [-1 2 5; 9 0 5]
B =
    -1     2     5
     9     0     5
>> C = [0, 1; 3, -2; 4, 2]
C =
     0     1
     3    -2
     4     2
>> D = [1:5; 6:10; 11:2:20]
D =
     1     2     3     4     5
     6     7     8     9    10
    11    13    15    17    19
```

So `A` and `B` are 2×3 matrices, `C` is 3×2 and `D` is 3×5.
In this context, a row vector is a $1 \times n$ matrix and a column vector a $m \times 1$ matrix.

15.1 Size of a matrix

We can get the size (dimensions) of a matrix with the command `size`

```
>> size(A), size(x)
ans =
     2     3
ans =
     3     1
>> size(ans)
ans =
     1     2
```

So `A` is 2×3 and x is 3×1 (a column vector). The last command `size(ans)` shows that the *value* returned by `size` is itself a 1×2 matrix (a row vector). We can save the results for use in subsequent calculations.

```
>> [r c] = size(A'), S = size(A')
r =
     3
c =
     2
S =
     3     2
```

Arrays can be reshaped. A simple example is:

```
>> A(:)
ans =
     5
     1
     7
    -3
     9
    -7
```

which converts `A` into a column vector by stacking its columns on top of each other. This could also be achieved using `reshape(A,6,1)`. The command

```
>> reshape(A,3,2)
ans =
     5    -3
     1     9
     7    -7
```

also redistributes the elements of `A` columnwise.

15.2 Transpose of a matrix

Transposing a vector changes it from a row to a column vector and vice versa (see §8.4)—recall that is also performs the conjugate of complex numbers. The extension of this idea to matrices is that transposing interchanges rows with the corresponding columns: the 1st row becomes the 1st column, and so on.

```
>> D, D'
D =
     1     2     3     4     5
     6     7     8     9    10
    11    13    15    17    19
ans =
     1     6    11
     2     7    13
     3     8    15
     4     9    17
     5    10    19
>> size(D), size(D')
ans =
     3     5
ans =
     5     3
```

15.3 Special Matrices

Matlab provides a number of useful built–in matrices of any desired size.
`ones(m,n)` gives an $m \times n$ matrix of 1's,

```
>> P = ones(2,3)
P =
     1     1     1
     1     1     1
```

`zeros(m,n)` gives an $m \times n$ matrix of 0's,

```
>> Z = zeros(2,3), zeros(size(P'))
Z =
     0     0     0
     0     0     0
ans =
     0     0
```

```
     0     0
     0     0
```

The second command illustrates how we can construct a matrix based on the size of an existing one. Try `ones(size(D))`.
An $n \times n$ matrix that has the same number of rows and columns and is called a **square** matrix.
A matrix is said to be **symmetric** if it is equal to its transpose (i.e. it is unchanged by transposition):

```
>> S = [2 -1 0; -1 2 -1; 0 -1 2],
S =
     2    -1     0
    -1     2    -1
     0    -1     2
>> St = S'
St =
     2    -1     0
    -1     2    -1
     0    -1     2
>> S-St
ans =
     0     0     0
     0     0     0
     0     0     0
```

15.4 The Identity Matrix

The $n \times n$ **identity** matrix is a matrix of zeros except for having ones along its leading diagonal (top left to bottom right). This is called `eye(n)` in Matlab (since mathematically it is usually denoted by I).

```
>> I = eye(3), x = [8; -4; 1], I*x
I =
     1     0     0
     0     1     0
     0     0     1
x =
     8
    -4
     1
ans =
     8
    -4
     1
```

Notice that multiplying the 3×1 vector `x` by the 3×3 identity `I` has no effect (it is like multiplying a number by 1).

15.5 Diagonal Matrices

A diagonal matrix is similar to the identity matrix except that its diagonal entries are not necessarily equal to 1.

$$D = \begin{bmatrix} -3 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

is a 3×3 diagonal matrix. To construct this in Matlab, we could either type it in directly

```
>> D = [-3 0 0; 0 4 0; 0 0 2]
D =
    -3     0     0
     0     4     0
     0     0     2
```

but this becomes impractical when the dimension is large (e.g. a 100×100 diagonal matrix). We then use the `diag` function.We first define a vector `d`, say, containing the values of the diagonal entries (in order) then `diag(d)` gives the required matrix.

```
>>  d = [-3 4 2], D = diag(d)
d =
    -3     4     2
D =
    -3     0     0
     0     4     0
     0     0     2
```

On the other hand, if `A` is any matrix, the command `diag(A)` extracts its diagonal entries:

```
>> F = [0 1 8 7; 3 -2 -4 2; 4 2 1 1]
F =
     0     1     8     7
     3    -2    -4     2
     4     2     1     1
>> diag(F)
ans =
     0
    -2
     1
```

Notice that the matrix does not have to be square.

15.6 Building Matrices

It is often convenient to build large matrices from smaller ones:

```
>> C=[0 1; 3 -2; 4 2]; x=[8;-4;1];
>> G = [C x]
G =
     0     1     8
     3    -2    -4
     4     2     1
>> A, B, H = [A; B]
A =
     5     7     9
     1    -3    -7
B =
    -1     2     5
     9     0     5
H =
     5     7     9
     1    -3    -7
    -1     2     5
     9     0     5
```

so we have added an extra column (`x`) to `C` in order to form `G` and have stacked `A` and `B` on top of each other to form `H`.

```
>> J = [1:4; 5:8; 9:12; 20 0 5 4]
J =
     1     2     3     4
     5     6     7     8
     9    10    11    12
    20     0     5     4

>> K = [ diag(1:4) J; J' zeros(4,4)]
K =
   1   0   0   0   1   2   3   4
   0   2   0   0   5   6   7   8
   0   0   3   0   9  10  11  12
   0   0   0   4  20   0   5   4
   1   5   9  20   0   0   0   0
   2   6  10   0   0   0   0   0
   3   7  11   5   0   0   0   0
   4   8  12   4   0   0   0   0
```

The command `spy(K)` will produce a graphical display of the location of the nonzero entries in `K` (it will also give a value for `nz`—the number of nonzero entries):

```
>> spy(K), grid
```

The keyword `end` can also be used with multi-dimensional arrays

```
K(1:2,end-1:end)
ans =
    3   4
    7   8
```

15.7 Tabulating Functions

This has been addressed in earlier sections but we are now in a position to produce a more suitable table format.

Example 15.1
Tabulate the functions $y = 4\sin 3x$ *and* $u = 3\sin 4x$ *for* $x = 0, 0.1, 0.2, \ldots, 0.5$.

```
>> x = 0:0.1:0.5;
>> y = 4*sin(3*x); u = 3*sin(4*x);
>> [ x' y' u']
ans =
         0         0         0
    0.1000    1.1821    1.1683
    0.2000    2.2586    2.1521
    0.3000    3.1333    2.7961
    0.4000    3.7282    2.9987
    0.5000    3.9900    2.7279
```

Note the use of transpose (') to get column vectors. (we could replace the last command by `[x; y; u;]'`)
We could also have done this more directly:

```
>>  x = (0:0.1:0.5)';
>> [x 4*sin(3*x) 3*sin(4*x)]
```

15.8 Extracting Parts of Matrices

We may extract sections from a matrix in much the same way as for a vector (page 6).
Each element of a matrix is indexed according to which row and column it belongs to. The entry in the ith row and jth column is denoted mathematically by $A_{i,j}$ and, in Matlab, by `A(i,j)`. So

```
>> J
J =
     1     2     3     4
     5     6     7     8
```

```
     9    10    11    12
    20     0     5     4
>> J(1,1)
ans =
     1
>> J(2,3)
ans =
     7
>> J(4,3)
ans =
     5
>> J(4,5)
???  Index exceeds matrix dimensions.
>> J(4,1) = J(1,1) + 6
J =
     1     2     3     4
     5     6     7     8
     9    10    11    12
     7     0     5     4
>> J(1,1) = J(1,1) - 3*J(1,2)
J =
    -5     2     3     4
     5     6     7     8
     9    10    11    12
     7     0     5     4
```

In the following examples we extract i) the 3rd column, ii) the 2nd and 3rd columns, iii) the 4th row, and iv) the "central" 2×2 matrix. See §8.1.

```
>> J(:,3)          % 3rd column
ans =
     3
     7
    11
     5
>> J(:,2:3)        % columns 2 to 3
ans =
     2     3
     6     7
    10    11
     0     5
>> J(4,:)          % 4th row
ans =
     7     0     5     4
>> % To get rows 2 to 3 & cols 2 to 3:
>> J(2:3,2:3)
ans =
     6     7
    10    11
```

Thus, `:` on its own refers to the entire column or row depending on whether it is the first or the second index.

15.9 Elementwise Products (`.*`)

The elementwise product works as for vectors: corresponding elements are multiplied together—so the matrices involved must have the same size.

```
>> A, B
A =
     5     7     9
     1    -3    -7
B =
    -1     2     5
     9     0     5
>> A.*B
ans =
    -5    14    45
     9     0   -35
>> A.*C
??? Error using ==> .*
Matrix dimensions must agree.
>> A.*C'
ans =
     0    21    36
     1     6   -14
```

Elementwise powers `.^` and division `./` work in an analogous fashion.

15.10 Matrix–vector products

We turn next to the definition of the product of a matrix with a vector. This product is only defined for **column vectors** that have the same number of entries as the matrix has columns. So, if A is an $m \times n$ matrix and $\underline{x}$ is a column vector of length n, then the matrix–vector $A\underline{x}$ is legal.

An $m \times n$ matrix times an $n \times 1$ matrix $\Rightarrow$ a $m \times 1$ matrix.

We visualise A as being made up of m row vectors stacked on top of each other, then the product corresponds to taking the **inner** product

(See §12.1) of each row of A with the vector $\underline{x}$: The result is a column vector with m entries.

$$A\underline{x} = \begin{bmatrix} \boxed{5 & 7 & 9} \\ \boxed{1 & -3 & -7} \end{bmatrix} \begin{bmatrix} \boxed{8} \\ \boxed{-4} \\ \boxed{1} \end{bmatrix}$$

$$= \begin{bmatrix} 5\times 8 + 7\times(-4) + 9\times 1 \\ 1\times 8 + (-3)\times(-4) + (-7)\times 1 \end{bmatrix}$$

$$= \begin{bmatrix} 21 \\ 13 \end{bmatrix}$$

It is somewhat easier in Matlab:

```
>> A = [5 7 9; 1 -3 -7]
A =
     5     7     9
     1    -3    -7
>> x = [8; -4; 1]
x =
     8
    -4
     1
>> A*x
ans =
    21
    13
```

$$(m\times\boxed{n)\text{ times }(n}\times 1) \Rightarrow (m\times 1).$$

```
>> x*A
??? Error using ==> *
Inner matrix dimensions must agree.
```

Unlike multiplication in scalar arithmetic, `A*x` **is not the same as** `x*A`.

15.11 Matrix–Matrix Products

To form the product of an $m\times n$ matrix A and a $n\times p$ matrix B, written as AB, we visualise the first matrix (A) as being composed of m row vectors of length n stacked on top of each other while the second (B) is visualised as being made up of p column vectors of length n:

$$A = m \text{ rows} \left\{ \begin{bmatrix} \rule{2em}{0.4pt} \\ \rule{2em}{0.4pt} \\ \vdots \\ \rule{2em}{0.4pt} \end{bmatrix} \right. , \; B = \underbrace{\begin{bmatrix} \rule{0.4pt}{2em} & \rule{0.4pt}{2em} & \cdots & \rule{0.4pt}{2em} \end{bmatrix}}_{p \text{ columns}},$$

The entry in the ith row and jth column of the product is then the **inner**product of the ith row of A with the jth column of B. The product is an $m\times p$ matrix:

$$(m\times\boxed{n)\text{ times }(n}\times p) \Rightarrow (m\times p).$$

Check that you understand what is meant by working out the following examples by hand and comparing with the Matlab answers.

```
>> A = [5  7  9;  1 -3 -7]
A =
     5     7     9
     1    -3    -7
>> B = [0, 1; 3, -2; 4, 2]
B =
     0     1
     3    -2
     4     2
>> C = A*B
C =
    57     9
   -37    -7
>> D = B*A
D =
     1    -3    -7
    13    27    41
    22    22    22
>>  E = B'*A'
E =
    57   -37
     9    -7
```

We see that `E = C'` suggesting that

(A*B)' = B'*A'

Why is $B*A$ a 3×3 matrix while $A*B$ is 2×2?

Exercise 15.1 *It is often necessary to factorize a matrix, e.g., $A = BC$ or $A = S^TXS$ where the factors are required to have specific properties. Use the '`lookfor keyword`' command to make a list of factorizations commands in Matlab.*

15.12 Sparse Matrices

Matlab has powerful techniques for handling sparse matrices — these are generally large matrices (to make the extra work involved worthwhile) that have only a very small proportion of non–zero entries.

Example 15.2 *Create a sparse* 5×4 *matrix* S *having only 3 non–zero values:* $S_{1,2} = 10$, $S_{3,3} = 11$ *and* $S_{5,4} = 12$.

We first create 3 vectors containing the i–index, the j–index and the corresponding values of each term and we then use the `sparse` command.

```
>> i = [1, 3, 5]; j = [2,3,4];
>> v = [10 11 12];
>> S = sparse (i,j,v)
S =
   (1,2)        10
   (3,3)        11
   (5,4)        12
>> T = full(S)
T =
     0    10     0     0
     0     0     0     0
     0     0    11     0
     0     0     0     0
     0     0     0    12
```

The matrix `T` is a "full" version of the sparse matrix `S`.

Example 15.3 *Develop Matlab code to create, for any given value of* n*, the sparse (tridiagonal) matrix*

$$B = \begin{bmatrix} 1 & n-1 & & & & \\ -2 & 2 & n-2 & & & \\ & -3 & 3 & n-3 & & \\ & & \ddots & \ddots & \ddots & \\ & & & -n+1 & n-1 & 1 \\ & & & & -n & n \end{bmatrix}$$

We define three COLUMN vectors, one for each "diagonal" of non–zeros and then assemble the matrix using `spdiags` (short for sparse diagonals). The vectors are named `l` (lower diagonal), `d` (diagonal) and `u` (upper diagonal). They must all have the same length and only the **first** $n-1$ terms of `l` are used while the **last** $n-1$ terms of `u` are used. `spdiags` places these vectors in the diagonals labelled `-1`, `0` and `1` (`0` defers to the leading diagonal, negatively numbered diagonals lie below the leading diagonal, etc.)

```
>> n = 5;
>> d = (1:n)'; l = -(d+1)';
>> u = flipud(d')
>> B = spdiags([l d u],-1:1,n,n);
>> full(B)
ans =
     1     4     0     0     0
    -2     2     3     0     0
     0    -3     3     2     0
     0     0    -4     4     1
     0     0     0    -5     5
```

Notice the use of the command `flipud` that reverses the entries in a column vector. More generally `flipud`reverses the order of rows in a matrix (two dimensional array), while `fliplr` reverses the order of columns.

16 Systems of Linear Equations

Mathematical formulations of engineering problems often lead to sets of simultaneous linear equations.
A general system of linear equations can be expressed in terms of a coefficient matrix A, a right-hand-side (column) vector $\mathbf{b}$ and an unknown (column) vector $\mathbf{x}$ as

$$A\mathbf{x} = \mathbf{b}$$

or, componentwise, as

$$\begin{aligned} a_{1,1}x_1 + a_{1,2}x_2 + \cdots a_{1,n}x_n &= b_1 \\ a_{2,1}x_1 + a_{2,2}x_2 + \cdots a_{2,n}x_n &= b_2 \\ &\vdots \\ a_{n,1}x_1 + a_{n,2}x_2 + \cdots a_{n,n}x_n &= b_n \end{aligned}$$

When A is non-singular and square ($n \times n$), meaning that the number of *independent* equations is equal to the number of unknowns, the

system has a unique solution given by

$$\mathbf{x} = A^{-1}\mathbf{b}$$

where A^{-1} is the inverse of A. Thus, the solution vector $\mathbf{x}$ can, in principle, be calculated by taking the inverse of the coefficient matrix A and multiplying it on the right with the right-hand-side vector $\mathbf{b}$.

This approach based on the matrix inverse, though formally correct, is at best inefficient for practical applications (where the number of equations may be extremely large) but may also give rise to large numerical errors unless appropriate techniques are used. These issues are discussed in most courses and texts on numerical methods. Various stable and efficient solution techniques have been developed for solving linear equations and the most appropriate in any situation will depend on the properties of the coefficient matrix A. For instance, on whether or not it is symmetric, or positive definite or if it has a particular structure (sparse or full). Matlab is equipped with many of these special techniques in its routine library and many are invoked automatically.

The standard Matlab routine for solving systems of linear equations is invoked by calling the matrix left-division routine,

```
>> x = A \ b
```

where "\" is the matrix left-division operator known as "backslash" (see `help backslash`).

Exercise 16.1 *Enter the symmetric coefficient matrix and right–hand–side vector* $\mathbf{b}$ *given by*

$$A = \begin{bmatrix} 2 & -1 & 0 \\ 1 & -2 & 1 \\ 0 & -1 & 2 \end{bmatrix}, \qquad \mathbf{b} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$$

and solve the system of equations $A\mathbf{x} = \mathbf{b}$ *using the three alternative methods:*

i) $\mathbf{x} = A^{-1}\mathbf{b}$*, (the inverse* A^{-1} *may be computed in Matlab using* `inv(A)`*.)*

ii) `x = A \ b`*,*

iii) $\mathbf{x}^T A^T = \mathbf{b}^T$ *leading to* `xT = b' / A` *which makes use of the "slash" or "right division" operator "/". The required solution is then the transpose of the row vector* `xT`*.*

Exercise 16.2 *Use the backslash operator to solve the complex system of equations for which*

$$A = \begin{bmatrix} 2+2\mathrm{i} & -1 & 0 \\ -1 & 2-2\mathrm{i} & -1 \\ 0 & -1 & 2 \end{bmatrix}, \qquad \mathbf{b} = \begin{bmatrix} 1+\mathrm{i} \\ 0 \\ 1-\mathrm{i} \end{bmatrix}$$

Exercise 16.3 *Find information on the matrix inversion command '*`inv`*' using each of the methods listed in Section 2 for obtaining help. What kind of matrices are the '*`inv`*' command applicable to?*

Obviously problems may occur if the inverted matrix is nearly singular. Suggest a command that can be used to give an indication on whether the matrix is nearly singular or not. [Hint: see the topics referred to by '`help inv`*'.]*

16.1 Overdetermined systems

An overdetermined system of linear equations is a one with more equations (m) than unknowns (n), i.e., the coefficient matrix has more rows than columns ($m > n$). Overdetermined systems frequently appear in mathematical modelling when the parameters of a model are determined by fitting to experimental data. Formally the system looks the same as for square systems but the coefficient matrix is rectangular and so it is not possible to compute an inverse. In these cases a solution can be found by requiring that the magnitude of the residual vector $\mathbf{r}$, defined by

$$\mathbf{r} = A\mathbf{x} - \mathbf{b},$$

be minimized. The simplest and most frequently used measure of the magnitude of $\mathbf{r}$ is require the Euclidean length (or norm—see Section 12.1) which corresponds to the sum of squares of the components of the residual. This approach leads to the least squares solution of the overdetermined system. Hence the least squares solution is defined as the vector $\mathbf{x}$ that minimizes

$$\mathbf{r}^T\mathbf{r}.$$

It may be shown that the required solution satisfies the so–called "normal equations"

$$C\mathbf{x} = \mathbf{d}, \text{ where } C = A^T A \text{ and } \mathbf{d} = A^T\mathbf{b}.$$

It is well–known that the solution of this system can be overwhelmed by numerical rounding error in practice unless great care is taken in its solution (a large part of the difficulty is inherent in the loss of information in computing the matrix–matrix product $A^T A$). As in the solution of square systems of linear equations, special techniques have been developed to address these issues and they have been incorporated into the Matlab routine library.
This means that a direct solution to the problem of overdetermined equations is available in Matlab through its left division operator "\". When the matrix A is not square, the operation

```
x = A\b
```

automatically gives the least squares solution to $A\mathbf{x} = \mathbf{b}$. This is illustrated in the next example.

Example 16.1 *A spring is a mechanical element which, for the simplest model, is characterized by a linear force-deformation relationship*

$$F = kx,$$

F being the force loading the spring, k the spring constant or stiffness and x the spring deformation. In reality the linear force–deformation relationship is only an approximation, valid for small forces and deformations. A more accurate relationship, valid for larger deformations, is obtained if non–linear terms are taken into account. Suppose a spring model with a quadratic relationship

$$F = k_1 x + k_2 x^2$$

is to be used and that the model parameters, k_1 and k_2, are to be determined from experimental data. Five independent measurements of the force and the corresponding spring deformations are measured and these are presented in Table 1.

Using the quadratic force-deformation relationship together with the experimental data yields an overdetermined system of linear equations and the components of the residual are given

Force F [N]	Deformation x [cm]
5	0.001
50	0.011
500	0.013
1000	0.30
2000	0.75

Table 1: Measured force-deformation data for spring.

by

$$\begin{aligned} r_1 &= x_1 k_1 + x_1^2 k_2 - F_1 \\ r_2 &= x_2 k_1 + x_2^2 k_2 - F_2 \\ r_3 &= x_3 k_1 + x_3^2 k_2 - F_3 \\ r_4 &= x_4 k_1 + x_4^2 k_2 - F_4 \\ r_5 &= x_5 k_1 + x_5^2 k_2 - F_5. \end{aligned}$$

These lead to the matrix and vector definitions

$$A = \begin{bmatrix} x_1 & x_1^2 \\ x_2 & x_2^2 \\ x_3 & x_3^2 \\ x_4 & x_4^2 \\ x_5 & x_5^2 \end{bmatrix} \text{ and } \mathbf{b} = \begin{bmatrix} F_1 \\ F_2 \\ F_3 \\ F_4 \\ F_5. \end{bmatrix}$$

The appropriate Matlab commands give (the components of `x` are all multiplied by `1e-2`, i.e., 10^{-2}, in order to change from cm to m)

```
>> x = [.001 .011 .13 .3 .75]*1e-2;
>> A = [x' (x').^2]
A =
    0.0000    0.0000
    0.0001    0.0000
    0.0013    0.0000
    0.0030    0.0000
    0.0075    0.0001
>> format short e
>> A
A =
   1.0000e-05   1.0000e-10
   1.1000e-04   1.2100e-08
   1.3000e-03   1.6900e-06
   3.0000e-03   9.0000e-06
   7.5000e-03   5.6250e-05
>> format, format compact
>> b = [5 50 500 1000 2000];
```

The second column of A is mainly zeros in standard format and so a switch to `format short`

e is used the least squares solution to this system is given by

```
>> k = A\b'
k =
  1.0e+07 *
    0.0386
   -1.5993
```

Thus, $\mathbf{k} \approx \begin{bmatrix} 0.39 \\ -16.0 \end{bmatrix} \times 10^6$ and the quadratic spring force-deformation relationship that optimally fits experimental data in the least squares sense is

$$F \approx 38.6 \times 10^4 x - 16.0 \times 10^6 x^2.$$

The data and solution may be plotted with the following commands

```
>>     % plot data points
>> plot(x,f,'o'), hold on
>> X = (0:.01:1)*max(x);
>>     % best fit curve
>> plot(X,[X' (X.^2)']*k,'-')
>> xlabel('x[m]'), ylabel('F[N]')
```

and the results are shown in Fig. 7.

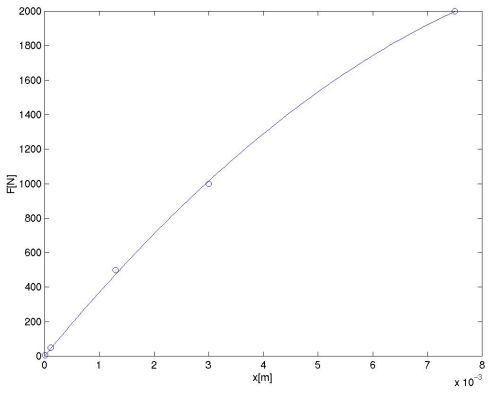


Fig. 7: Data for Example 16.1 (circles) and best least squares fit by a quadratic model (solid line).

Matlab has a routine `polyfit` for data fitting by polynomials: see "`help polyfit`". It is not applicable in this example because we require that the force – deformation law passes through the origin (so there is no constant term in the quadratic model that we used).

17 Characters, Strings and Text

The ability to process text in numerical processing is useful for the input and output of data to the screen or to files. In order to manage text, a new datatype of "character" is introduced. A piece of text is then simply a string (vector) or array of characters.

Example 17.1 *The assignment,*

```
>> t1 = 'A'
```

assigns the value `A` *to the* `1-by-1` *character array* `t1`*. The assignment,*

```
>> t2 = 'BCDE'
t2 =
BCDE
>> size(t2)
ans =
     1     4
```

assigns the value `BCDE` *to the* `1-by-4` *character array* `t2`*.*

Strings can be combined by using the operations for array manipulations.
The assignment,

```
>> t3 = [t1,t2]
```

assigns a value `ABCDE` to the `1-by-5` character array `t3`. The assignment,

```
>> t4 = [t3,' are the first 5      ';...
'characters in the alphabet.']
```

assigns the value
`'ABCDE are the first 5 '`
`'characters in the alphabet.'`
to the `2-by-27` character array `t4`. It is essential that the number of characters in both rows of the array `t4` is the same, otherwise an error will result. The three dots `...` signify that the command is continued on the following line
Sometimes it is necessary to convert a character to the corresponding number, or vice versa. These conversions are accomplished by the commands '`str2num`'—which converts a string to

the corresponding number, and two functions, '`int2str`' and '`num2str`', which convert, respectively, an integer and a real number to the corresponding character string. These commands are useful for producing titles and strings, such as '`The value of pi is 3.1416`'. This can be generated by the command
`['The value of pi is ',num2str(pi)]`.

```
>> N = 5; h = 1/N;
>> ['The value of N is ',int2str(N),...
', h = ',num2str(h)]
ans =
The value of N is 5, h = 0.2
```

18 Loops

There are occasions that we want to repeat a segment of code a number of different times (such occasions are less frequent than other programming languages because of the : notation).
A standard `for` loop has the form

```
>> for counter = 1:20
      ........
   end
```

which repeats the code as far as the `end` with the variable `counter=1` the first time, `counter=2` the second time, and so forth. Rather more generally

```
>> for counter = [23  11  19  5.4  6]
      ........
   end
```

repeats the code with `counter=23` the first time, `counter=11` the second time, and so forth.

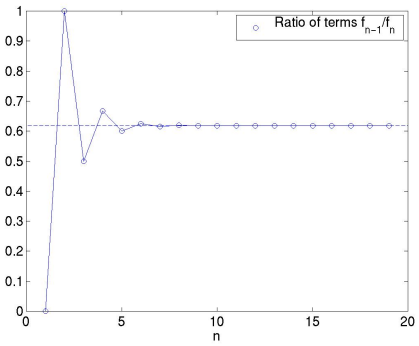
Example 18.1 *The Fibonnaci sequence starts off with the numbers* 0 *and* 1*, then succeeding terms are the sum of its two immediate predecessors. Mathematically,* $f_1 = 0$*,* $f_2 = 1$ *and*

$$f_n = f_{n-1} + f_{n-2}, \qquad n = 3, 4, 5, \ldots.$$

Test the assertion that the ratio f_{n-1}/f_n *of two successive values approaches the golden ratio* $(\sqrt{5} - 1)/2 = 0.6180\ldots$.

```
>> F(1) = 0; F(2) = 1;
>> for i = 3:20
     F(i) = F(i-1) + F(i-2);
   end
>> plot(1:19, F(1:19)./F(2:20),'o' )
>> hold on,  xlabel('n')
>> plot(1:19, F(1:19)./F(2:20),'-' )
>> legend('Ratio of terms f_{n-1}/f_n')
>> plot([0 20], (sqrt(5)-1)/2*[1,1],'--')
```

The last of these commands produces the dashed horizontal line.



Example 18.2 *Produce a list of the values of the sums*

$$\begin{aligned} S_{20} &= 1 + \tfrac{1}{2^2} + \tfrac{1}{3^2} + \cdots + \tfrac{1}{20^2} \\ S_{21} &= 1 + \tfrac{1}{2^2} + \tfrac{1}{3^2} + \cdots + \tfrac{1}{20^2} + \tfrac{1}{21^2} \\ &\vdots \\ S_{100} &= 1 + \tfrac{1}{2^2} + \tfrac{1}{3^2} + \cdots + \tfrac{1}{20^2} + \tfrac{1}{21^2} + \cdots + \tfrac{1}{100^2} \end{aligned}$$

There are a total of 81 sums. The first can be computed using `sum(1./(1:20).^2)` (The function `sum` with a vector argument sums its components. See §21.2].) A suitable piece of Matlab code might be

```
>> S = zeros(100,1);
>> S(20) = sum(1./(1:20).^2);
>> for n = 21:100
>>   S(n) = S(n-1) + 1/n^2;
>> end
>> clf; plot(S,'.',[20 100],[1,1]*pi^2/6,'-')
>> axis([20 100 1.5 1.7])
>> [ (98:100)'  S(98:100)]
ans =
   98.0000    1.6364
```

```
   99.0000    1.6365
  100.0000    1.6366
```

where a column vector `S` was created to hold the answers. The first sum was computed directly using the `sum` command then each succeeding sum was found by adding $1/n^2$ to its predecessor. The little table at the end shows the values of the last three sums—it appears that they are approaching a limit (the value of the limit is $\pi^2/6 = 1.64493\ldots$).
A more elegant solution is given by

```
>> n = 1:100;
>> S = cumsum(1./n.^2);
```

where `cumsum` calculates the cumulative sum of entries in a vector.

Exercise 18.1 *Repeat Example 18.2 to include 181 sums (i.e., the final sum should include the term* $1/200^2$*.)*

19 Timing

Matlab allows the timing of sections of code by providing the functions `tic` and `toc`. `tic` switches on a stopwatch while `toc` stops it and returns the CPU time (Central Processor Unit) in seconds. The timings will vary depending on the model of computer being used and its current load.

```
>> tic, sum((1:10000).^2);toc
Elapsed time is 0.000124 seconds.
>> tic, sum((1:10000).^2);toc
Elapsed time is 0.000047 seconds.
>> tic, s = sum((1:10000).^2);T = toc
T =
   8.2059e-05
```

The first two instances illustrate that there can be considerable variation in successive calls to the same operations. The third instance shows that the elapsed time can be assigned to a variable.

20 Logicals

Matlab represents `true` and `false` by means of the integers 0 and 1.

```
          true = 1,    false = 0
```

If at some point in a calculation a scalar `x`, say, has been assigned a value, we may make certain logical tests on it:

`x == 2`	is `x` equal to 2?
`x ~= 2`	is `x` **not** equal to 2?
`x > 2`	is `x` greater than 2?
`x < 2`	is `x` less than 2?
`x >= 2`	is `x` greater than or equal to 2?
`x <= 2`	is `x` less than or equal to 2?

Pay particular attention to the fact that the test for equality involves two equal signs `==`.

```
>> x = pi
x =
    3.1416
>> x ~= 3,  x ~= pi
ans =
     1
ans =
     0
```

When `x` is a vector or a matrix, these tests are performed elementwise:

```
x =
   -2.0000    3.1416    5.0000
   -1.0000         0    1.0000
>> x == 0
ans =
     0     0     0
     0     1     0
>> x > 1,  x >=-1
ans =
     0     1     1
     0     0     0
ans =
     0     1     1
     1     1     1
>> y = x>=-1, x > y
y =
     0     1     1
     1     1     1
ans =
     0     1     1
     0     0     0
```

We may combine logical tests, as in

```
>> x
x =
```

```
   -2.0000    3.1416    5.0000
   -5.0000   -3.0000   -1.0000
>> x > 3 & x < 4
ans =
     0     1     0
     0     0     0
>> x > 3 | x == -3
ans =
     0     1     1
     0     1     0
```

As one might expect, `&` represents **and** and (not so clearly) the vertical bar `|` means **or**; also `~` means **not** as in `~=` (not equal), `~(x>0)`, etc.

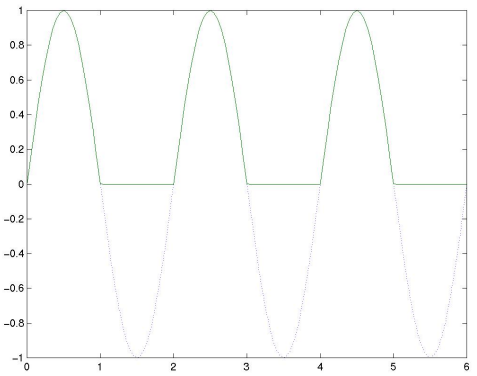
```
>> x > 3 | x == -3 | x <= -5
ans =
     0     1     1
     1     1     0
```

One of the uses of logical tests is to "mask out" certain elements of a matrix.

```
>> x, L =  x >= 0
x =
   -2.0000    3.1416    5.0000
   -5.0000   -3.0000   -1.0000
L =
     0     1     1
     0     1     1
>> pos = x.*L
pos =
         0    3.1416    5.0000
         0         0         0
```

so the matrix `pos` contains just those elements of `x` that are non–negative.

```
>> x = 0:0.05:6; y = sin(pi*x);
>> Y = (y>=0).*y;
>> plot(x,y,':',x,Y,'-' )
```



20.1 While Loops

There are some occasions when we want to repeat a section of Matlab code until some logical condition is satisfied, but we cannot tell in advance how many times we have to go around the loop. This we can do with a `while...end` construct.

Example 20.1 *What is the greatest value of n that can be used in the sum*

$$1^2 + 2^2 + \cdots + n^2$$

and get a value of less than 100?

```
>> S = 1; n = 2;
>> while S+ n^2 < 100
     S = S + n^2;  n = n+1;
   end
>> [n-1, S]
ans =
     6    91
```

The lines of code between `while` and `end` will only be executed if the condition `S+n^2 < 100` is true.

Exercise 20.1 *Replace `100` in the previous example by `10` and work through the lines of code by hand. You should get the answers $n = 2$ and $S = 5$.*

Exercise 20.2 *Type the code from Example20.1 into a script–file named `WhileSum.m` (See §10.)*

A more typical example is

Example 20.2 *Find the approximate value of the root of the equation $x = \cos x$. (See Example 13.1.)*

We may do this by making a guess $x_1 = \pi/4$, say, then computing the sequence of values

$$x_n = \cos x_{n-1}, \qquad n = 2, 3, 4, \ldots$$

and continuing until the difference, d, between two successive values $|x_n - x_{n-1}|$ is small enough.

Method 1:

```
>> x = zeros(1,20); x(1) = pi/4;
>> n = 1;  d = 1;
>> while d > 0.001
     n = n+1; x(n) = cos(x(n-1));
     d = abs( x(n) - x(n-1) );
   end
   n,x(1:n)
n =
    14
x =
Columns 1 through 6
0.7854 0.7071 0.7602 0.7247 0.7487 0.7326
Columns 7 through 12
0.7435 0.7361 0.7411 0.7377 0.7400 0.7385
Columns 13 through 14
0.7395 0.7388
```

There are a number of deficiencies with this program. The vector `x` stores the results of each iteration but we don't know in advance how many there may be. In any event, we are rarely interested in the intermediate values of `x`, only the last one. Another problem is that we may never satisfy the condition $d \leq 0.001$, in which case the program will run forever, so we should place a limit on the maximum number of iterations.
Incorporating these improvements leads to

Method 2:

```
>> xold = pi/4; n = 1;  d = 1;
>> while d > 0.001 &  n < 20
     n = n+1;  xnew = cos(xold);
     d = abs( xnew - xold );
     xold = xnew;
   end
>> [n, xnew, d]
ans =
   14.0000    0.7388    0.0007
```

We continue around the loop so long as $d > 0.001$ **and** $n < 20$. For greater precision we could use the condition $d > 0.0001$, and this gives

```
>> [n, xnew, d]
ans =
   19.0000    0.7391    0.0001
```

from which we may judge that the root required is $x = 0.739$ to 3 decimal places.
The general form of `while` statement is

```
while a logical test
                Commands to be executed
                when the condition is true
end
```

20.2 `if...then...else...end`

This allows us to execute different commands depending on the truth or falsity of some logical tests. To test whether or not π^e is greater than, or equal to, e^π:

```
>> a = pi^exp(1); c = exp(pi);
>> if a >= c
     b = sqrt(a^2 - c^2)
   end
```

so that `b` is assigned a value only if $a \geq c$. There is no output so we deduce that $a = \pi^e < c = e^\pi$.
A more common situation is

```
>> if a >= c
     b = sqrt(a^2 - c^2)
   else
     b = 0
   end
b =
     0
```

which ensures that `b` is always assigned a value and confirming that $a < c$.
A more extended form is

```
>> if a >= c
     b = sqrt(a^2 - c^2)
   elseif a^c > c^a
     b = c^a/a^c
   else
     b = a^c/c^a
   end
b =
    0.2347
```

Exercise 20.3 *Which of the above statements assigned a value to* `b`*?*

The general form of the `if` statement is

```
if logical test 1
    Commands to be executed if test 1
    is true
elseif logical test 2
    Commands to be executed if test 2
    is true but test 1 is false
    ⋮

end
```

21 Further Built–in Functions

21.1 Rounding Numbers

There are a variety of ways of rounding and chopping real numbers to give integers. Use the definitions given in the table in §26 on page 42 in order to understand the output given below:

```
>> x = [-4 -1 1 4]*pi
x =
  -12.5664   -3.1416    3.1416   12.5664
>> round(x)
ans =
   -13    -3     3    13
>> fix(x)
ans =
   -12    -3     3    12
>> floor(x)
ans =
   -13    -4     3    12
>> ceil(x)
ans =
   -12    -3     4    13
>> sign(x),  rem(x,3)
ans =
    -1     0     1     1     1
ans =
   -0.5664  -0.1416   0.1416   0.5664
```

Do "`help round`" for help information.

21.2 The sum Function

The "`sum`" applied to a vector adds up its components (as in `sum(1:10)`) while, for a matrix, it adds up the components in **each column** and returns a row vector. `sum(sum(A))` then sums all the entries of `A`.

```
>>  A = [1:3; 4:6; 7:9]
A =
     1     2     3
     4     5     6
     7     8     9
>> s = sum(A), ss = sum(sum(A))
s =
    12    15    18
ss =
    45

>> x = pi/4*(1:3)';
>> A=[sin(x),sin(2*x),sin(3*x)]/sqrt(2)
>> A =
    0.5000    0.7071    0.5000
    0.7071    0.0000   -0.7071
    0.5000   -0.7071    0.5000

>> s1 = sum(A.^2), s2 = sum(sum(A.^2))
s1 =
    1.0000    1.0000    1.0000
s2 =
    3.0000
```

The sums of squares of the entries in each column of `A` are equal to 1 and the sum of squares of all the entries is equal to 3.

```
>> A*A'
ans =
    1.0000         0         0
         0    1.0000    0.0000
         0    0.0000    1.0000
>> A'*A
ans =
    1.0000         0         0
         0    1.0000    0.0000
         0    0.0000    1.0000
```

It appears that the products AA' and $A'A$ are both equal to the identity:

```
>> A*A' - eye(3)
ans =
   1.0e-15 *
   -0.2220         0         0
         0   -0.2220    0.0555
         0    0.0555   -0.2220
>> A'*A - eye(3)
ans =
   1.0e-15 *
```

```
   -0.2220         0         0
         0   -0.2220    0.0555
         0    0.0555   -0.2220
```

This is confirmed since the differences are at round–off error levels (less than 10^{-15}). A matrix with this property is called an *orthogonal* matrix.

21.3 max & min

These functions act in a similar way to `sum`. If `x` is a vector, then `max(x)` returns the largest element in `x`

```
>> x = [1.3 -2.4 0 2.3]
x =
    1.3000   -2.4000    0    2.3000
>>  max(x), max(abs(x))
ans =
    2.3000
ans =
    2.4000
>> [m, j] = max(x)
m =
    2.3000
j =
     4
```

When we ask for two outputs, the first gives us the maximum entry and the second the index of the maximum element.
For a matrix, `A`, `max(A)` returns a row vector containing the maximum element from each column. Thus, to find the largest element in `A`, we use `max(max(A))`.

21.4 Random Numbers

The function `rand(m,n)` produces an $m \times n$ matrix of random numbers, each of which is in the range 0 to 1. `rand` on its own produces a single random number.

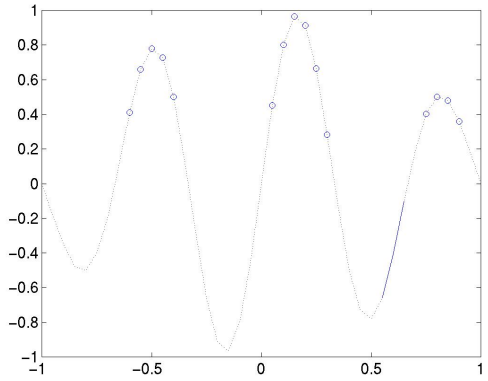
```
>> y =  rand, Y = rand(2,3)
y =
    0.9191
Y =
    0.6262    0.1575    0.2520
    0.7446    0.7764    0.6121
```

Repeating these commands will lead to different answers. Example 22.2 gives an illustration of the use of random numbers.

21.5 find for vectors

The function "`find`" returns a list of the positions (indices) of the elements of a vector satisfying a given condition. For example,

```
>> x = -1:.05:1;
>> y = sin(3*pi*x).*exp(-x.^2);
>> plot(x,y,':')
>> k = find(y > 0.2)
k =
Columns 1 through 12
9 10 11 12 13 22 23 24 25 26 27 36
Columns 13 through 15
  37  38  39
>> hold on, plot(x(k),y(k),'o')
>> km = find( x>0.5 & y<0)
km =
    32    33    34
>> plot(x(km),y(km),'-')
```



21.6 find for matrices

The `find`–function operates in much the same way for matrices:

```
>> A = [ -2 3 4 4; 0 5 -1 6; 6 8 0 1]
A =
    -2     3     4     4
     0     5    -1     6
```

```
     6     8     0     1
>> k = find(A==0)
k =
     2
     9
```

Thus, we find that `A` has elements equal to 0 in positions 2 and 9. To interpret this result we have to recognize that "`find`" first reshapes `A` into a column vector (see §15.1)—this is equivalent to numbering the elements of `A` by columns as in

$$\begin{array}{cccc} 1 & 4 & 7 & 10 \\ 2 & 5 & 8 & 11 \\ 3 & 6 & 9 & 12 \end{array}$$

```
>> n  = find(A <= 0)
n =
     1
     2
     8
     9
>> A(n)
ans =
    -2
     0
    -1
     0
```

Thus, `n` gives a list of the locations of the entries in `A` that are ≤ 0 and then `A(n)` gives us the values of the elements selected.

```
>> m = find( A' == 0)
m =
     5
    11
```

Since we are dealing with `A'`, the entries are numbered by rows.

22 Function m–files

We can extend the number of Matlab built-in functions by writing our own. They are special cases of m–files (§7).

Example 22.1 *The area, A, of a triangle with sides of length a, b and c is given by*

$$A = \sqrt{s(s-a)(s-b)(s-c)},$$

where $s = (a+b+c)/2$. Write a Matlab function that will accept the values a, b and c as inputs and return the value of A as output.

The main steps to follow when defining a Matlab function are:

1. Decide on a name for the function, making sure that it does not conflict with a name that is already used by Matlab. In this example the name of the function is to be `area`, so its definition will be saved in a file called `area.m`

2. The first line of the file must have the format:

   `function` [*list of outputs*]
   = *function_name*(*list of inputs*)

   For our example, the output (A) is a function of the three variables (inputs) a, b and c so the first line should read

   ```
   function [A] = area(a,b,c)
   ```

3. Document the function. That is, describe briefly the purpose of the function and how it can be used. These lines should be preceded by `%` which signify that they are comment lines that will be ignored when the function is evaluated.

4. Finally include the code that defines the function. This should be interspersed with sufficient comments to enable another user to understand the processes involved.

The complete file might look like:

```
function  [A] = area(a,b,c)
% Compute the area of a triangle whose
% sides have length a, b and c.
% Inputs:
%    a,b,c:  Lengths of sides
% Output:
%        A: area of triangle
% Usage:
%      Area = area(2,3,4);
% Written by dfg, Oct 14, 1996.
s = (a+b+c)/2;
A = sqrt(s*(s-a)*(s-b)*(s-c));
%%%%%%%%% end of area %%%%%%%%%%
```

The command
```
>> help area
```
will produce the leading comments from the file:

```
Compute the area of a triangle whose
sides have length a, b and c.
Inputs:
   a,b,c:  Lengths of sides
Output:
       A: area of triangle
Usage:
     Area = area(2,3,4);
Written by dfg, Oct 14, 1996.
```

To evaluate the area of a triangle with side of length 10, 15, 20:

```
>> Area = area(10,15,20)
Area =
   72.6184
```

where the result of the computation is assigned to the variable `Area`—the use of a capitalised variable name is critical here, otherwise there would be confusion between the variable name and the function name. If we inadvertently use a variable name that coincides with a function name, as in

```
>> sin = sin(pi/6)
sin =
    0.5000
>> sin(pi/2)
??? Subscript indices must either be
    real positive integers or logicals.
```

Matlab now considers the name `sin` to refer to a variable and `pi/2` in the command `sin(pi/2)` is interpreted as an index to a vector. To reclaim the function name we clear the variable `sin` from memory with
```
>> clear sin
```
The variable `s` used in the definition of the `area` function above is a "local variable": its value is local to the function and cannot be used outside:

```
>> s
??? Undefined function or variable s.
```

If we were interested in the value of s as well as A, then the first line of the file should be changed to

```
function [A,s] = area(a,b,c)
```

where there are two output variables.
This function can be called in several different ways:

1. No outputs assigned

   ```
   >> area(10,15,20)
   ans =
      72.6184
   ```

   gives only the area (first of the output variables from the file) assigned to `ans`; the second output is ignored.

2. One output assigned

   ```
   >> Area = area(10,15,20)
   Area =
      72.6184
   ```

   again the second output is ignored.

3. Two outputs assigned

   ```
   >> [Area, hlen] = area(10,15,20)
   Area =
      72.6184
   hlen =
      22.5000
   ```

The previous examples illustrate the fact that a function may have a different number of outputs. It is also possible to write function files that accepts a variable number of inputs. For example, in the context of our `area` function, to calculate the area of a right angled triangle it is only necessary to specify the lengths of two of the sides since the third (the hypotenuse) can be calculated by Pythagoras's theorem. So our amended function operates as previously described but, when only two input arguments are supplied, it assumes the triangle to be right angled. It does this by using the reserved variable `nargin` that holds the number of input arguments. The revised function, called `area2`, might then resemble the following code:

```
function  [A] = area2(a,b,c)
% Compute the area of a triangle whose
% sides have length a, b and c.
% Inputs: either
%    a,b,c:  Lengths of 3 sides
% or
%    a, b: two shortest sides of a
%            right angled triangle
% Output:
%        A: area of triangle
% Usage:
%      Area = area2(2,3,4);
% or
%      Area = area2(3,4);
% Written by dfg, Oct 14, 1996.
% Extended  Oct 25, 2012
if nargin <2
    error( 'Not enough arguments')
elseif nargin==2
    c = sqrt(a^2+b^2);
end
s = (a+b+c)/2;
A = sqrt(s*(s-a)*(s-b)*(s-c));
%%%%%%%%% end of area2 %%%%%%%%%%
```

The command `error` issues an error message to the screen and could be usefully employed in the following exercise.

Exercise 22.1 *Explain the output obtained from the command*

```
area(4,5,10)
```

Devise a test to warn the user of this type of situation.

Exercise 22.2 *Extend the* `area` *function so that it also calculates the area of an equilateral triangle when only one input argument is supplied, as in* `area(2)`*.*

Example 22.2 *Write a function–file that will simulate n throws of a pair of dice.*

This requires random numbers that are integers in the range 1 to 6 which can be produced with

```
floor(1 + 6*rand)
```

Recall that `floor` takes the largest integer that is smaller than a given real number (see §21.1 and Table 2, page 42).

File: `dice.m`

```
function  [d] = dice(n)
% simulates "n" throws of a pair of dice
% Input:     n, the number of throws
% Output:  an n times 2 matrix, each
%          row referring to one throw.
%
% Useage:  T = dice(3)
    d = floor(1 + 6*rand(n,2));
%% end of dice

>> dice(3)
ans =
     6     1
     2     3
     4     1
>> sum(dice(100))/100
ans =
    3.8500    3.4300
```

The last command gives the average value over 100 throws (it should theoretically have the value 3.5).

Example 22.3 *Construct a function that will return the nth Fibonnaci number f_n, where $f_1 = 0, f_2 = 1$ and*

$$f_n = f_{n-1} + f_{n-2}, \qquad n = 3, 4, 5, \ldots.$$

(See Example 18.1.) The function has:

- **Input**: Non-negative integer n
- **Output**: f_n

We shall describe four possible functions and try to assess which provides the best solution.

Method 1: File `Fib1.m`

```
function f = Fib1(n)
% Returns the nth number in the
% Fibonacci sequence.
F  = zeros(1,n);
F(2) = 1;
   for i = 3:n
     F(i) = F(i-1) + F(i-2);
   end
f = F(n);
```

This code resembles that given in Example 18.1. We have simply enclosed it in a function m–file and given it the appropriate header. The most significant change is the line `F=zeros(1,n)` which serves to both define the value of `F(1)` and to allocate sufficient memory to store a vector to hold the first n Fibonacci numbers. Had we not done this then the length of the vector `F` would be extended on each trip around the loop and more memory would have to be allocated for its storage. The time penalties this would incur would not be significant in this example (since, with modest values of n, it computes in a tiny fraction of a second) but could be important when dealing with large arrays in codes that are run many times over.

Method 2: File `Fib2.m`
The first version was rather wasteful of memory, all the entries in the sequence where saved even though we only required the last one for output. The second version removes the need to use a vector.

```
function f = Fib2(n)
% Returns the nth number in the
% Fibonacci sequence.
if n==1
   f = 0;
elseif n==2
   f = 1;
else
   f1 = 0; f2 = 1;
   for i = 3:n
     f = f1 + f2;
     f1 = f2; f2 = f;
   end
end
```

Method 3: File: `Fib3.m`
This version makes use of an idea called "recursive programming"—the function makes calls to itself.

```
function f = Fib3(n)
% Returns the nth number in the
% Fibonacci sequence.
if n==1
   f = 0;
elseif n==2
   f = 1;
else
   f = Fib3(n-1) + Fib3(n-2);
end
```

Method 4: File `Fib4.m`
The final version uses matrix powers. The vector `y` has two components, $\underline{y} = \begin{bmatrix} f_n \\ f_{n+1} \end{bmatrix}$.

```
function f = Fib4(n)
% Returns the nth number in the
% Fibonacci sequence.
A = [0 1;1 1];
y = A^n*[1;0];
f=y(1);
```

Assessment: One may think that, on grounds of style, the 3rd is best (it avoids the use of loops) followed by the second (it avoids the use of a vector). The situation is somewhat different when it comes to speed of execution. When $n = 20$ the time taken by each of the methods is (in seconds)

Method	Time
1	5.8052×10^{-5}
2	2.5534×10^{-5}
3	1.4972×10^{-1}
4	5.4041×10^{-5}

What is immediately obvious from these timings is that Method 3 is significantly slower than the others. Moreover, the time increases dramatically as n is increased and is totally impractical. Methods 1, 2, and 4 execute very rapidly and the times increase quite slowly as n is increased. When times are averaged over many hundreds of runs it becomes clear that Method 2 is the most efficient followed by Method 1.

23 Plotting Surfaces

A surface is defined mathematically by a function $f(x, y)$—corresponding to each value of (x, y) we compute the height of the function by

$$z = f(x, y).$$

In order to plot this we have to decide on the ranges of x and y—suppose $2 \leq x \leq 4$ and $1 \leq y \leq 3$. This gives us a square in the (x, y)–plane. Next, we need to choose a grid on this domain; Figure 8 shows the grid with intervals 0.5 in each direction. Finally, we have to eval-

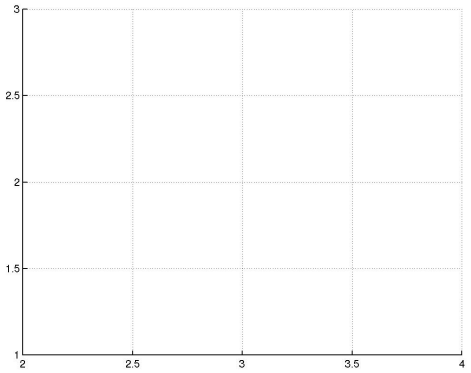


Fig. 8: An example of a 2D grid

uate the function at each point of the grid and "plot" it.

Suppose we choose a grid with intervals 0.5 in each direction for illustration. The x– and y–coordinates of the grid lines are

```
x = 2:0.5:4;   y = 1:0.5:3;
```

in Matlab notation. We construct the grid with `meshgrid`:

```
>> [X,Y] = meshgrid(2:.5:4, 1:.5:3);
>> X
X =
   2.0000   2.5000   3.0000   3.5000   4.0000
   2.0000   2.5000   3.0000   3.5000   4.0000
   2.0000   2.5000   3.0000   3.5000   4.0000
   2.0000   2.5000   3.0000   3.5000   4.0000
   2.0000   2.5000   3.0000   3.5000   4.0000
>> Y
Y =
   1.0000   1.0000   1.0000   1.0000   1.0000
   1.5000   1.5000   1.5000   1.5000   1.5000
   2.0000   2.0000   2.0000   2.0000   2.0000
   2.5000   2.5000   2.5000   2.5000   2.5000
   3.0000   3.0000   3.0000   3.0000   3.0000
```

If we think of the ith point along from the left and the jth point up from the bottom of the grid) as corresponding to the (i, j)th entry in a matrix, then `(X(i,j), Y(i,j))` are the coordinates of the point. We then need to evaluate the function f using `X` and `Y` in place of x and y, respectively. As in Example 14.1, elementwise operations (powers, products, etc.) are usually appropriate.

Example 23.1 *Plot the surface defined by the function*

$$f(x, y) = (x - 3)^2 - (y - 2)^2$$

for $2 \leq x \leq 4$ *and* $1 \leq y \leq 3$.

```
>> [X,Y] = meshgrid(2:.2:4, 1:.2:3);
>> Z = (X-3).^2-(Y-2).^2;
>> mesh(X,Y,Z)
>> title('Saddle'), xlabel('x'),ylabel('y')
```

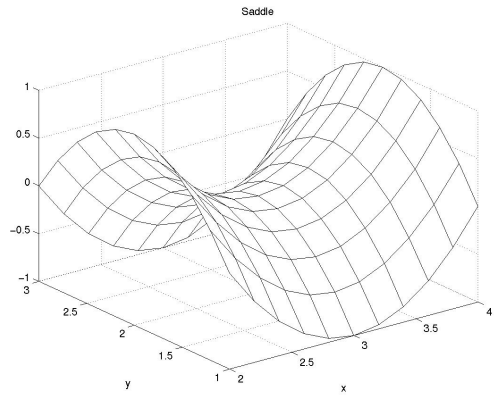


Fig. 9: Plot of Saddle function.

Exercise 23.1 *Repeat the previous example replacing* `mesh` *by* `surf` *and then by* `surfl`. *Consult the help pages to find out more about these functions.*

Example 23.2 *Plot the surface defined by the function*

$$f = -xye^{-2(x^2+y^2)}$$

on the domain $-2 \leq x \leq 2, -2 \leq y \leq 2$. *Find the values and locations of the maxima and minima of the function.*

```
>> [X,Y] = meshgrid(-2:.1:2,-2:.2:2);
>> f =  -X.*Y.*exp(-2*(X.^2+Y.^2));
>> figure (1)
>> mesh(X,Y,f), xlabel('x'), ylabel('y'), grid
>> figure (2),  contour(X,Y,f)
>> xlabel('x'), ylabel('y'), grid, hold on
```

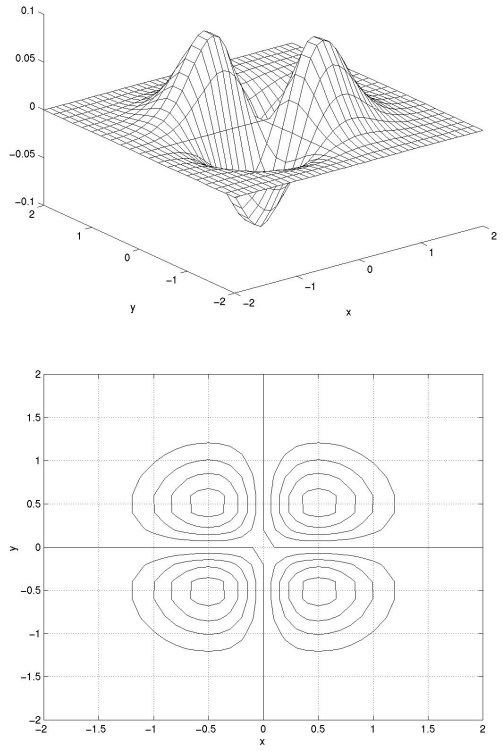


Fig. 10: "`mesh`" and "`contour`" plots.

To locate the maxima of the "`f`" values on the grid:

```
>> fmax = max(max(f))
fmax =
    0.0886
>> kmax = find(f==fmax)
kmax =
   323
   539
>> Pos = [X(kmax), Y(kmax)]
Pos =
   -0.5000    0.6000
    0.5000   -0.6000
>> plot(X(kmax),Y(kmax),'*')
>> text(X(kmax),Y(kmax),' Maximum')
```

See §13.10 for formatting text and labels.

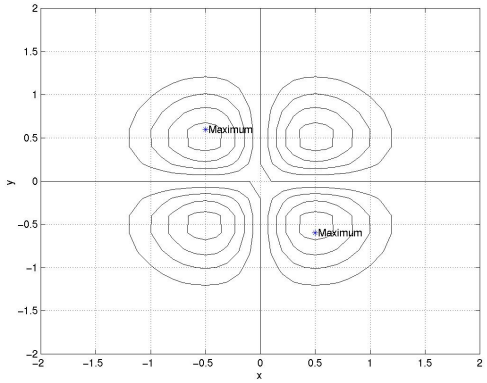


Fig. 11: `contour` plot showing maxima.

24 Reading/Writing Data Files

Direct input of data from keyboard becomes impractical when

- the amount of data is large and
- the same data is analysed repeatedly.

In these situations input and output is preferably accomplished via data files. We have already described in Section 9 the use of the commands `save` and `load` that, respectively, write and read the values of variables to disk files.
When data are written to or read from a file it is crucially important that a correct data format is used. The data format is the key to interpreting the contents of a file and must be known in order to correctly interpret the data in an input file. There are two types of data files: formatted and unformatted. Formatted data files uses format strings to define exactly how and in what positions of a record the data is stored. Unformatted storage, on the other hand, only specifies the number format. The files used in this section are available from the web site

`http://www.maths.dundee.ac.uk/software/matlab.shtml`

Those that are unformatted are in a satisfactory form for the Windows version on Matlab (version 6.1) but not on Version 5.3 under Unix.

Exercise 24.1 *Suppose the numeric data is stored in a file '`table.dat`' in the form of a table, as shown below.*

```
100   2256
200   4564
300   3653
400   6798
500   6432
```

The three commands,

```
>> fid = fopen('table.dat','r');
>> a = fscanf(fid,'%3d%4d');
>> fclose(fid);
```

respectively

1. open a file for reading (this is designated by the string '`r`'). The variable `fid` is assigned a unique integer which identifies the file used (a file identifier). We use this number in all subsequent references to the file.
2. read pairs of numbers from the file whose identifier is `fid`, one with 3 digits and one with 4 digits, and
3. close the file with file identifier `fid`.

This produces a **column vector** `a` with elements, `100 2256 200 4564 ...500 6432`. This vector can be converted to 5×2 matrix by the command `A = reshape(a,2,5)';`.

24.1 Formatted Files

Some computer codes and measurement instruments produce results in formatted data files. In order to read these results into Matlab for further analysis the data format of the files must be known. Formatted files in ASCII format are written to and read from with the commands `fprintf` and `fscanf`.

> `fprintf(fid, 'format', variables)` writes variables an a format specified in string 'format' to the file with identifier fid
>
> `a = fscanf(fid, 'format',size)` assigns to variable `a` data read from file with identifier `fid` under format '`format`'.

Exercise 24.2 *Study the available information and help on* `fscanf` *and* `fprintf` *commands. What is the meaning of the format string,* `'%3d\n'` *?*

Example 24.1 *Suppose a sound pressure measurement system produces a record with 512 time – pressure readings stored on a file '*`sound.dat`*'. Each reading is listed on a separate line according to a data format specified by the string,* `'%8.6f %8.6f'`.

A set of commands reading time-sound pressure data from '`sound.dat`' is,

Step 1: Assign a namestring to a file identifier.

```
>> fid1 = fopen('sound.dat','r');
```

The string 'r' indicates that data is to be read (not written) from the file.

Step 2: Read the data to a vector named 'data' and close the file,

```
>> data = fscanf(fid1, '%f %f');
>> fclose(fid1);
```

Step 3: Partition the data in separate time and sound pressure vectors,

```
>> t = data(1:2:length(data));
>> press = data(2:2:length(data));
```

The pressure signal can be plotted in a `lin-lin` diagram,

```
>> plot(t, press);
```

The result is shown in Figure 12.

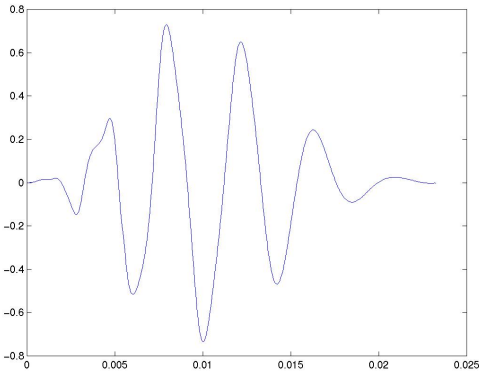


Fig. 12: Graph of "sound data" from Example 24.1

24.2 Unformatted Files

Unformatted or binary data files are used when small-sized files are required. In order to interpret an unformatted data file the data precision must be specified. The precision is specified as a string, e.g., '`float32`', controlling the number of bits read for each value and the interpretation of those bits as character, integer or floating point values. Precision '`float32`', for instance, specifies each value in the data to be stored as a floating point number in 32 memory bits.

Example 24.2 *Suppose a system for vibration measurement stores measured acceleration values as floating point numbers using 32 memory bits. The data is stored on file 'vib.dat. The following commands illustrate how the data may be read into Matlab for analysis.*

Step 1: Assign a file identifier, `fid`, to the string specifying the file name.

```
>> fid = fopen('vib.dat','rb');
```

The string `'rb'` specifies that binary numbers are to be read from the file.

Step 2 Read all data stored on file '`vib.dat`' into a vector vib.

```
>> vib = fread(fid, 'float32');
>> fclose(fid);
>> size(vib)
ans =
      131072
```

The `size(vib)` command determines the size, i.e., the number of rows and columns of the vibration data vector.

In order to plot the vibration signal with a correct time scale, the sampling frequency (the number of instrument readings taken per second) used by the measurement system must be known. In this case it is known to be 24000 Hz so that there is a time interval of 1/24000 seconds between two samples.

Step 3: Create a column vector containing the correct time scale.

```
>> dt = 1/24000;
>> t = dt*(1:length(vib))';
```

Step 4: Plot the vibration signal in a `lin-lin` diagram

```
>> plot(t,vib);
>> title('Vibration signal');
>> xlabel('Time,[s]');
>> ylabel('Acceleration, [m/s^2]');
```

25 Graphic User Interfaces

The efficiency of programs that are used often and by several different people can be improved by simplifying the input and output data management. The use of Graphic User Interfaces (GUI), which provides facilities such as menus, pushbuttons, sliders etc, allow programs to be used without any knowledge of Matlab. They also provides means for efficient data management.

A graphic user interface is a Matlab script file customized for repeated analysis of a specific type of problem. There are two ways to design a graphic user interface. The simplest method is to use a tool especially designed for the purpose. Matlab provides such a tool and it is invoked by typing '`guide`' at the Matlab prompt. Maximum flexibility and control over the programming is, however, obtained by using the basic user interface commands. The following text demonstrates the use of some basic commands.

Example 25.1 *Suppose a sound pressure spectrum is to be plotted in a graph. There are four alternative plot formats; lin-lin, lin-log, log-lin and log-log. The graphic user interface below reads the pressure data stored on a binary file selected by the user, plots it in a lin-lin format as a function of frequency and lets the user switch between the four plot formats.*

We use two m–files. The first (`specplot.m`) is the main driver file which builds the graphics window. It calls the second file (`firstplot.m`) which allows the user to select among the possible `*.bin` files in the current directory.

```
%  File: specplot.m
%
% GUI for plotting a user selected frequency spectrum
% in four alternative plot formats, lin-lin,
% lin-log, log-lin and log-log.
%
% Author: U Carlsson, 2001-08-22

      % Create figure window for graphs
figWindow = figure('Name','Plot alternatives');
      % Create file input selection button
fileinpBtn = uicontrol('Style','pushbutton',...
  'string','File','position',[5,395,40,20],...
  'callback','[fdat,pdat] = firstplot;');
      % Press 'File' calls function 'firstplot'

% Create pushbuttons for switching between four
% different plot formats. Set up the axis stings.
X = 'Frequency, [Hz]';
Y = 'Pressure amplitude, [Pa]';
linlinBtn = uicontrol('style','pushbutton',...
  'string','lin-lin',...
  'position',[200,395,40,20],'callback',...
  'plot(fdat,pdat);xlabel(X);ylabel(Y);');
linlogBtn = uicontrol('style','pushbutton',...
```

```
  'string','lin-log',...
  'position',[240,395,40,20],...
  'callback',...
  'semilogy(fdat,pdat);xlabel(X);ylabel(Y);');
loglinBtn = uicontrol('style','pushbutton',...
  'string','log-lin',...
  'position',[280,395,40,20],...
  'callback',...
  'semilogx(fdat,pdat);xlabel(X);ylabel(Y);');
loglogBtn = uicontrol('style','pushbutton',...
  'string','log-log',...
  'position',[320,395,40,20],...
  'callback',...
  'loglog(fdat,pdat);xlabel(X);   ylabel(Y);')

% Create exit pushbutton with red text.

exitBtn = uicontrol('Style','pushbutton',...
  'string','EXIT','position',[510,395,40,20],...
  'foregroundcolor',[1 0 0],'callback','close');
```

Executing this GUI from the command line (`>> specplot`) brings the following screen.

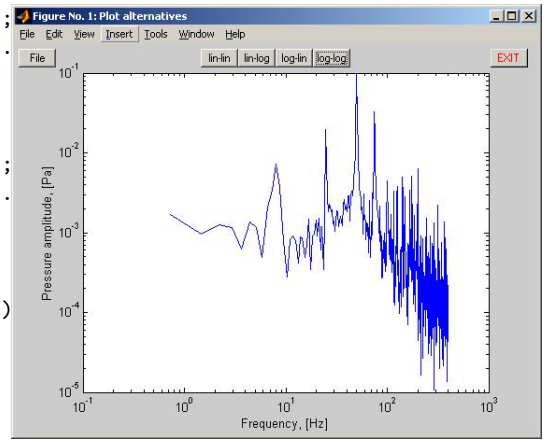


Fig. 13: Graph of "vibration data" from Example 25.1

```
% Script file: firstplot.m
% Brings template for file selection. Reads
% selected filename and path and plots
% spectrum in a  lin-lin diagram.
% Output data are frequency and pressure
% amplitude vectors: 'fdat' and  'pdat'.
% Author: U Carlsson, 2001-08-22

function [fdat,pdat] = firstplot

% Call Matlab function 'uigetfile' that
% brings file selction template.

[filename,pathname] = uigetfile('*.bin',...
  'Select binary data-file:');
% Change directory
cd(pathname);
% Open file for reading binary floating
% point numbers.
fid = fopen(filename,'rb');
data = fread(fid,'float32');
% Close file
fclose(fid);
% Partition data vector in frequency and
% pressure vectors
pdat = data(2:2:length(data));
fdat = data(1:2:length(data));
% Plot pressure signal in a lin-lin diagram
plot(fdat,pdat);
% Define suitable axis labels
xlabel('Frequency, [Hz]');
ylabel('Pressure amplitude, [Pa]');
```

Example 25.1 illustrates how the '`callback`' property allows the programmer to define what actions should result when buttons are pushed etc. These actions may consist of single Matlab commands or complicated sequences of operations defined in various subroutines.

Exercise 25.1 *Five different sound recordings are stored on binary data files,* `sound1.bin`*,* `sound2.bin`*, . . . ,* `sound5.bin`*. The storage precision is '*`float32`*' and the sounds are recorded with sample frequency 12000 Hz.*
Write a graphic user interface that, opens an interface window and

- *lets the user select one of the five sounds,*
- *plots the selected sound pressure signal as a function of time in a lin-lin diagram,*
- *lets the user listen to the sound by pushing a 'SOUND' button and finally*
- *closes the session by pressing a 'CLOSE' button.*

26 Command Summary

The command
`>> help`
will give a list of categories for which help is available (e.g. `matlab/general` covers the topics listed in Table 3.

Further information regarding the commands listed in this section may then be obtained by using:
`>> help topic`
try, for example,
`>> help help`

`abs`	Absolute value
`sqrt`	Square root function
`sign`	Signum function
`conj`	Conjugate of a complex number
`imag`	Imaginary part of a complex number
`real`	Real part of a complex number
`angle`	Phase angle of a complex number
`cos`	Cosine function, radians
`sin`	Sine function, radians
`tan`	Tangent function, radians
`cosd`	Cosine function, degrees
`sind`	Sine function, degrees
`tand`	Tangent function, degrees
`exp`	Exponential function
`log`	Natural logarithm
`log10`	Logarithm base 10
`cosh`	Hyperbolic cosine function
`sinh`	Hyperbolic sine function
`tanh`	Hyperbolic tangent function
`acos`	Inverse cosine, result in radians
`acosd`	Inverse cosine, result in degrees
`acosh`	Inverse hyperbolic cosine
`asin`	Inverse sine, result in radians
`asind`	Inverse sine, result in degrees
`asinh`	Inverse hyperbolic sine
`atan`	Inverse tan, result in radians
`atand`	Inverse tan, result in degrees
`atan2`	Two–argument form of inverse tan
`atanh`	Inverse hyperbolic tan
`round`	Round to nearest integer
`floor`	Round towards minus infinity
`fix`	Round towards zero
`ceil`	Round towards plus infinity
`rem`	Remainder after division

Table 2: Elementary Functions

Managing commands and functions.	
`help`	On-line documentation.
`doc`	Load hypertext documentation.
`what`	Directory listing of M-, MAT- and MEX-files.
`type`	List M-file.
`lookfor`	Keyword search through the HELP entries.
`which`	Locate functions and files.
`demo`	Run demos.
Managing variables and the workspace.	
`who`	List current variables.
`whos`	List current variables, long form.
`load`	Retrieve variables from disk.
`save`	Save workspace variables to disk.
`clear`	Clear variables and functions from memory.
`size`	Size of matrix.
`length`	Length of vector.
`disp`	Display matrix or text.
Working with files and the operating system.	
`cd`	Change current working directory.
`dir`	Directory listing.
`delete`	Delete file.
`!`	Execute operating system command.
`unix`	Execute operating system command & return result.
`diary`	Save text of MATLAB session.
Controlling the command window.	
`cedit`	Set command line edit/recall facility parameters.
`clc`	Clear command window.
`home`	Send cursor home.
`format`	Set output format.
`echo`	Echo commands inside script files.
`more`	Control paged output in command window.
Quitting from MATLAB.	
`quit`	Terminate MATLAB.

Table 3: General purpose commands.

Matrix analysis.	
`cond`	Matrix condition number.
`norm`	Matrix or vector norm.
`rcond`	LINPACK reciprocal condition estimator.
`rank`	Number of linearly independent rows or columns.
`det`	Determinant.
`trace`	Sum of diagonal elements.
`null`	Null space.
`orth`	Orthogonalization.
`rref`	Reduced row echelon form.
Linear equations.	
\ and `/`	Linear equation solution; use "help slash".
`chol`	Cholesky factorization.
`lu`	Factors from Gaussian elimination.
`inv`	Matrix inverse.
`qr`	Orthogonal- triangular decomposition.
`qrdelete`	Delete a column from the QR factorization.
`qrinsert`	Insert a column in the QR factorization.
`nnls`	Non–negative least- squares.
`pinv`	Pseudoinverse.
`lscov`	Least squares in the presence of known covariance.
Eigenvalues and singular values.	
`eig`	Eigenvalues and eigenvectors.
`poly`	Characteristic polynomial.
`polyeig`	Polynomial eigenvalue problem.
`hess`	Hessenberg form.
`qz`	Generalized eigenvalues.
`rsf2csf`	Real block diagonal form to complex diagonal form.
`cdf2rdf`	Complex diagonal form to real block diagonal form.
`schur`	Schur decomposition.
`balance`	Diagonal scaling to improve eigenvalue accuracy.
`svd`	Singular value decomposition.
Matrix functions.	
`expm`	Matrix exponential.
`expm1`	M- file implementation of expm.
`expm2`	Matrix exponential via Taylor series.
`expm3`	Matrix exponential via eigenvalues and eigenvectors.
`logm`	Matrix logarithm.
`sqrtm`	Matrix square root.
`funm`	Evaluate general matrix function.

Table 4: Matrix functions—numerical linear algebra.

Graphics & plotting.	
`figure`	Create Figure (graph window).
`clf`	Clear current figure.
`close`	Close figure.
`subplot`	Create axes in tiled positions.
`axis`	Control axis scaling and appearance.
`hold`	Hold current graph.
`figure`	Create figure window.
`text`	Create text.
`print`	Save graph to file.
`plot`	Linear plot.
`loglog`	Log-log scale plot.
`semilogx`	Semi-log scale plot.
`semilogy`	Semi-log scale plot.
Specialized X-Y graphs.	
`polar`	Polar coordinate plot.
`bar`	Bar graph.
`stem`	Discrete sequence or "stem" plot.
`stairs`	Stairstep plot.
`errorbar`	Error bar plot.
`hist`	Histogram plot.
`rose`	Angle histogram plot.
`compass`	Compass plot.
`feather`	Feather plot.
`fplot`	Plot function.
`comet`	Comet-like trajectory.
Graph annotation.	
`title`	Graph title.
`xlabel`	X-axis label.
`ylabel`	Y-axis label.
`text`	Text annotation.
`gtext`	Mouse placement of text.
`grid`	Grid lines.
`contour`	Contour plot.
`mesh`	3-D mesh surface.
`surf`	3-D shaded surface.
`waterfall`	Waterfall plot.
`view`	3-D graph viewpoint specification.
`zlabel`	Z-axis label for 3-D plots.
`gtext`	Mouse placement of text.
`grid`	Grid lines.

Table 5: Graphics & plot commands.

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