WIA1002/WIB1002 Data Structure

Lab: Recursion (Applications)

Question 1 – (Related to Tutorial 3 question)

The recurrence relation in Tutorial #3 is given by the expression below:

$$F(s,t) = F(s-1,t) + F(s,t-1)$$

Write a recursive method that finds the value $F(4,10)$, $F(12,8)$, and $F(7,12)$.

Question 2 – Binomial expansion

Recall that

$$(x+y)^n = \sum_{k=0}^{n} \binom{n}{k} x^{n-k} y^k ,$$

where $\binom{n}{k}$ can be found recursively using the relation below:

$$\binom{n}{k} = \binom{n-1}{k-1} + \binom{n-1}{k},$$

with the base cases of $\binom{n}{0} = \binom{n}{n} = 1$. Write a method that performs binomial expression when n (i.e., an integer) is input from the user. For example, if the input is $n=3$, your method should print the following:

(x+y)^3 = 1x^3y^0 + 3x^2y^1 + 3x^1y^2 + 1x^0y^3.

Question 3: This question consists of 2 parts

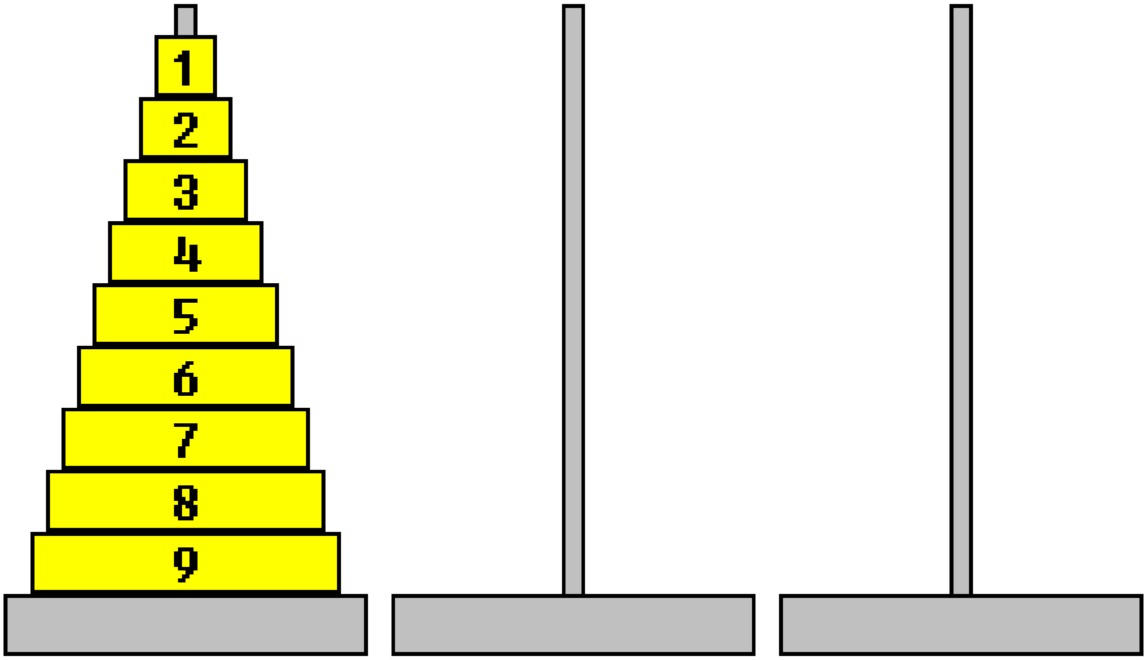

Figure 1: Tower of Hanoi with 3 pegs and 9 discs

Figure 1 shows the original state for the Tower of Hanoi with 3 pegs and 9 discs, where all the labeled discs are on peg A. The priest is assigned to move the discs from one peg to another. However, there are some rules that the priest must obey:

i. A disc can be moved to any peg.
ii. The priest can only move one disc at a time.
iii. A larger disc cannot be put on top of any smaller discs. For example, disc #3 cannot be on top of disc #2, or #1.

Continue…

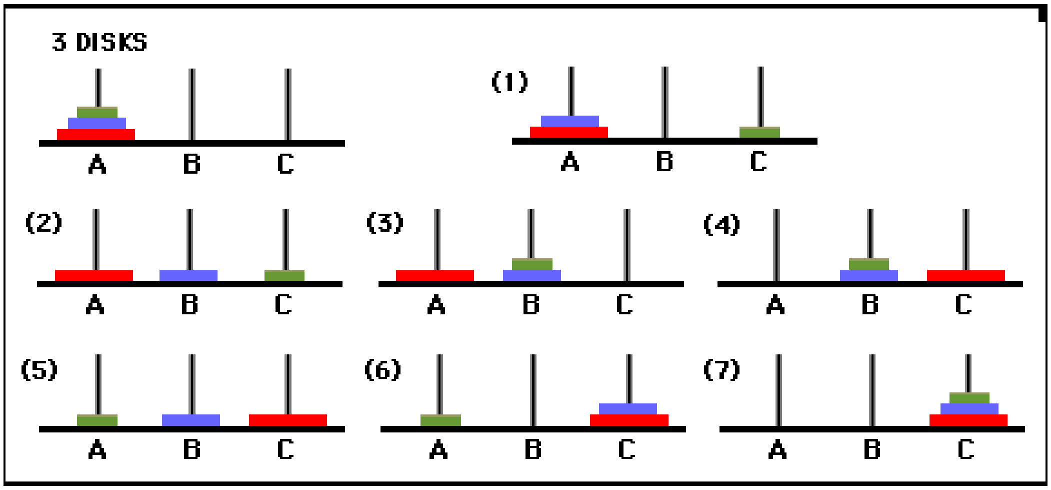

Fig. 2: Tower of Hanoi with 3 pegs and 3 discs

Consider the case of 3 discs (i.e., disc #1 -green, #2 -blue and #3 -red). Figure 2 illustrates the steps needed to move all 3 discs from peg A to peg C. The steps performed are as follows:

Step 1: Move disc #1 to Peg C
Step 2: Move disc #2 to Peg B
Step 3: Move disc #1 (from Peg C) to Peg B
Step 4: Move disc #3 (from Peg A) to Peg C
Step 5: Move disc #1 (from Peg B) to Peg A
Step 6: Move disc #2 (from Peg B) to Peg C
Step 7: Move disc #1 (from Peg A) to Peg C

Note that there are exactly 7 steps to move all 3 discs from Peg A to Peg B. Let n denote the number of discs, and $T(n)$ be the number of steps required. Then, $T(n)=7$ when $n=3$.

(a) Write a recurrence relation (i.e., mathematical expression) governing the number of steps involved in moving all n discs from one peg to another (say from Peg A to Peg C). Print your answer to the screen.
[Hint: Look at $T(1), T(2), T(3)$, etc. to find the pattern]

(b) Write a recursive method that computes the number of steps $T(n)$ required in moving all n discs from one peg to another (say from Peg A to Peg C). The value n should be input by the user (i.e., not hard coded). Note that you are NOT required to list the actual steps, but just return the value $T(n)$.

End of Lab