MHM Approximations of discrete fracture networks

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Outline

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   - Simulating multiscale problems with MHM
2. Apply MHM to Fracture Networks
   - Problem Statement
   - Add fractures to MHM-HDiv
   - Substructuring MHM-H(Div) + Fractures
3. Implementation
   - Fracture Preprocessor
   - Gmsh programming
   - Finite element simulation
4. Numerical Results
   - One fracture
   - A two fracture simulation
   - Verify the influence of the layout of the MHM domain

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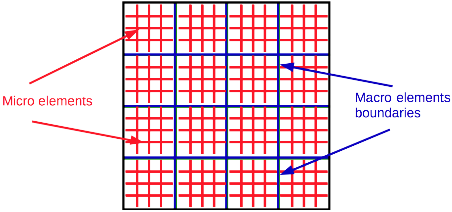
The Multiscale Hybid Method (MHM)

- MHM was conceived by Christopher Harder, F. Valentin and D. Paredes
- MHM is a numerical approximation technique conceived for simulating problems involving multiple scales
- It is a general concept that can be applied to different problems in computational mechanics
  - Flow through porous media
  - Elasticity
  - Maxwell equations

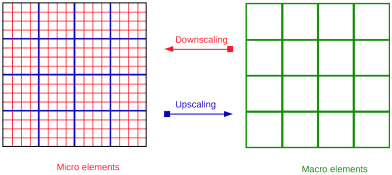
General concept of MHM

- MHM approximations are divided in two parts:
  - Approximation of the dual (flux) variable at the interface between *macro* elements
  - Approximation of the conservation law (flux and pressure) at the interior of each macro element



General concept of MHM

- MHM approximations contain two scaling operators:
- Downscaling (Coarse scale $\rightarrow$ Fine scale)
  - The fine scale behaviour is captured by the numerical simulation at the interior of the *macro* elements.
- Upscaling (Fine scale $\rightarrow$ Coarse scale)
  - The fine scale properties of the solution are transferred to the global problem associated with the fluxes.



Mixed approximation of Darcy's law

Find $(\mathbf{q}, p) \in H(div)(\Omega) \times L^2(\Omega)$ such that

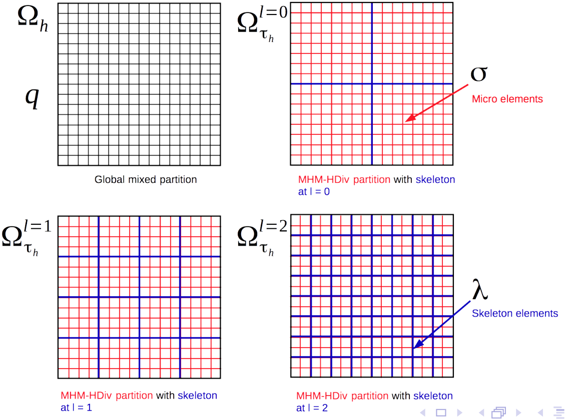
$$\int_\Omega K^{-1}\mathbf{q} \cdot \tau dV + \int_\Omega p\, div(\tau) dV = \int_{\partial\Omega_D} p_D(\tau \cdot n)\, ds$$

$$\int_\Omega div(\mathbf{q})\, z\, dV = \int_\Omega z\, f\, dV$$

for all $(\tau, z) \in H(div)(\Omega) \times L^2(\Omega)$ (+ b.c.)

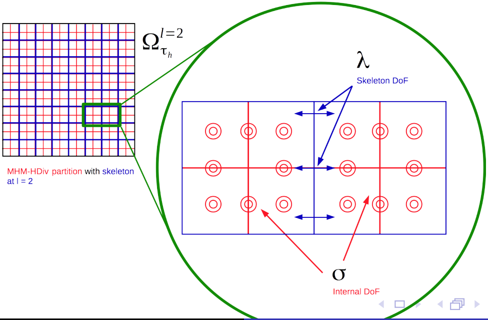
MHM with Mixed approximation

The flux functions into σ internal functions and functions associated with λ fluxes between *macro* elements, $\mathbf{q} = \sigma + \lambda$



MHM with Mixed Approximation - shape function restraints

- MHM with mixed approximation is equivalent to apply shape function restraints to the boundary fluxes.
- The multiplying coeficients of the fluxes (σ) are dependent on the multiplying coeficients of the boundary flux λ.
- The restraints ensure the strong continuity between the micro fuxes associated with two macro elements.



Problem statement

Consider a fluid flow through porous media with a superposed fracture network:

$$\int_{T_2} \vec{\psi}_2 . K_2^{-1} \vec{\sigma}_2 d\Omega - \int_{T_2} div(\vec{\psi}_d)\, p_2 d\Omega + \int_{T_1} \Sigma(\vec{\psi}_2 . n) p_{\Gamma_2} d\omega = - \int_{\partial\Omega_D} p_D$$

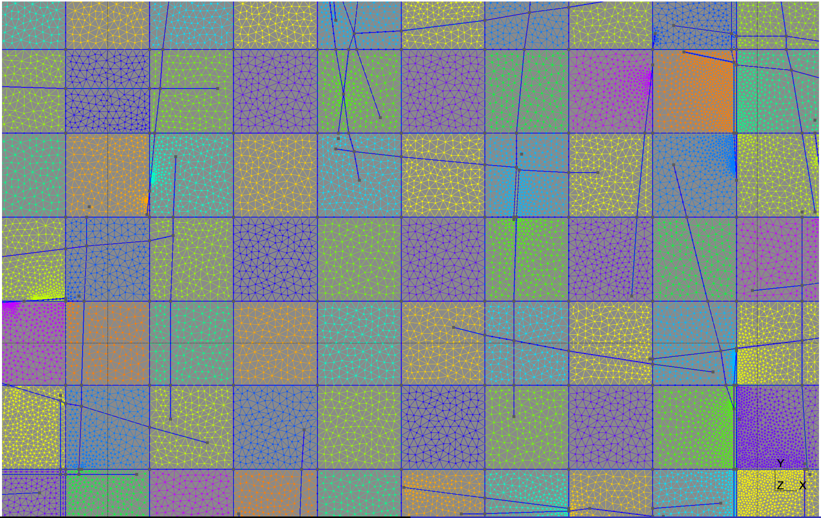
$$- \int_{T} div(\vec{\sigma}_2)\varphi_2 d\Omega = \int f\ \varphi_d d\Omega$$

$$\int_{T_1} \vec{\psi}_1 . K_1^{-1} \vec{\sigma}_1 dT_1 - \int_{T_1} div(\vec{\psi}_1)\, p_1 dT_1 + \int_{T_0} \Sigma(\vec{\psi}_1 . n)\, p_0 dT_0 = 0$$

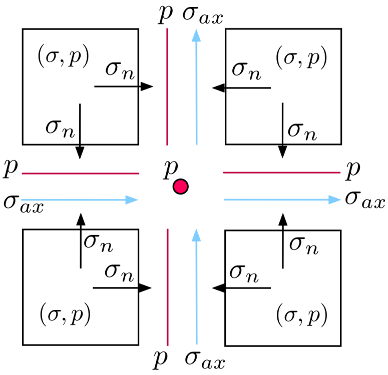
$$- \int_{T_1} div(\vec{\sigma}_1)\varphi_1 + \int_{\Gamma_2} \varphi_1 \Sigma \vec{\sigma}_2 . n = 0$$

$$\int_{T_0} \varphi_0 \Sigma(\vec{\sigma}_1 . n) dT_0 = 0$$

Problem statement

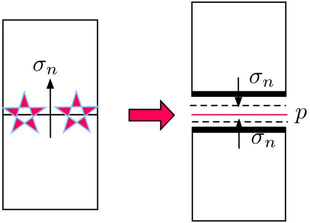


H(div) Configuration with fracture flow



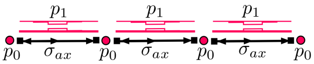
How to incorporate fractures in MHM-HDiv

- Starting from a H(div) approximation
- Split the continuous H(div) flux function - add a pressure Lagrange multiplier



Adding a one dimensional flux/pressure pair

- After creating the pressure Lagrange element
- Superpose a pressure flux pair
- Add an H(Div) wrapper
- Add a zero dimensional pressure Lagrange Multiplier



Statically condense internal equations

- order the equations with the internal degrees of freedom first

$$\begin{bmatrix} K_{11} & K_{12} \\ K_{21} & K_{22} \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} F_1 \\ F_2 \end{bmatrix}$$

- reduce the equations

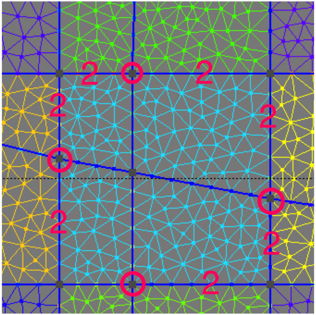
$$\left[K_{22} - K_{21} K_{11}^{-1} K_{12}\right] [u_2] = \left[F_2 - K_{21} K_{11}^{-1} F_1\right]$$

- Compute the internal degrees of freedom

$$[u_1] = \left[K_{11}^{-1}\right] [F_1] - [K_{12}] [u_2]$$

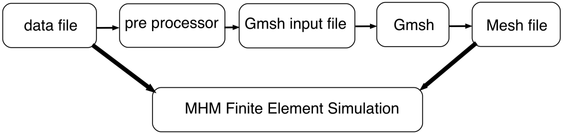
Reducing the equation of an MHM subdomain

- There are 20 external equations



Generating an MHM+fracture approximation

- MHM+frac simulations are generated by the interaction of three computational tools
- Input data
  - is a unique file which defines the fracture configuration



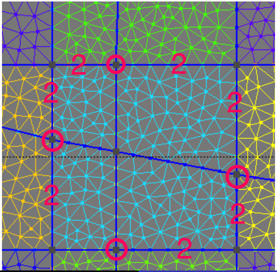
Fracture pre processor

- Transforms a fracture definition file into a Gmsh input file

```
 0 0 9 9 - Domain Size
13 - Number of MHM domains
 0.33 0.003 - Element size and Singular Element size
#
 1 1 - simulation type (0 = Steady State, 1 = time evolution) initial_pressure
#
 5 - number of materials
 "Darcy" 2 1 flow 1 1  - name dimension material_id material_type rho Perm
 "BCIN" 1 -1 boundary 0 1 - name dimension material_id bc_type value
 "BCOUT" 1 -2 boundary 0 0 - name dimension material_id bc_type value
 "BCNOFLOW" 1 -3 boundary 1 0 - name dimension material_id bc_type value
 "BC" 1 -4 boundary 1 0 - name dimension material_id bc_type value
#
 2 - number of timestep series
 0.1 20 - delt nsteps
 1. 20 - delt nsteps
```

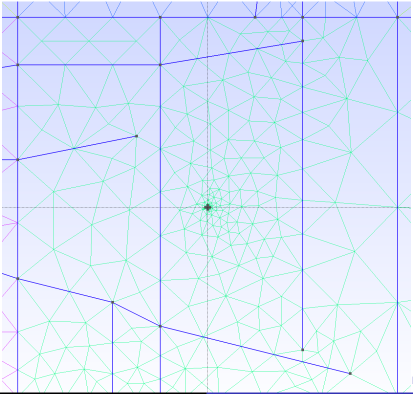
Gmsh programming

- Gmsh allow to associate *physical groups* to points, lines, surfaces and volumes
- The fracture preprocessor generates commands associating physical groups with sections of fractures, MHM boundaries and MHM subdomains
- Internal fractures are *embedded* in MHM domains



Gmsh programming

- The input file to Gmsh can be modified
  - to incorporate well
  - to modify boundary conditions

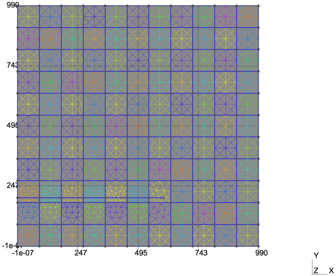


Gmsh will crash

- when fracture lines overlap
- when a point of a fracture is too close to another fracture
- when a fracture line coincides with the boundary of an MHM domain

Putting it all toghether

- the fracture definition file and Gmsh output file are used to construct an MHM simulation
- Elements and MHM boundaries are grouped using TPZRefPattern objects

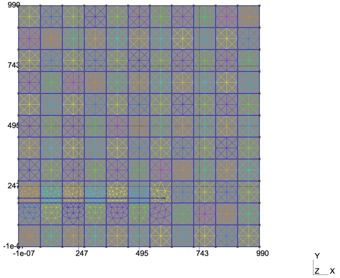


Numerical Results

- Some simulations are qualitative
- Some are comparative
- Some verify the robustness of the approximation technique

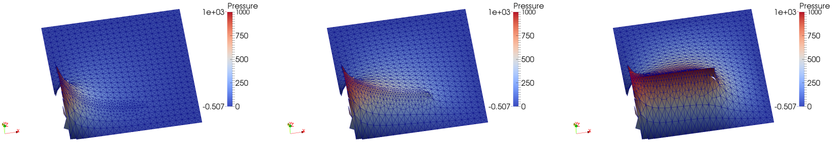
A single fracture in a reservoir

- Study the effect of the permeability of the fracture
- Domain 900 x 900, permeability K=1



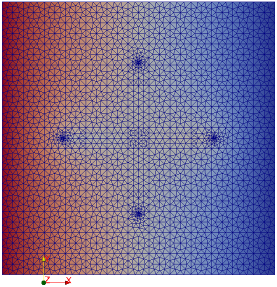
Pressure profile as function of fracture permeability

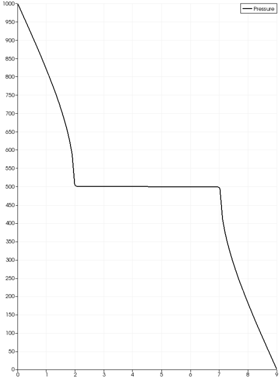
- Fracture permeability K=1, K=1000, K=100000



A two fracture simulation

- 9 x 9 reservoir. A vertical and horizontal fracture from 2 to 7, permeability K=1, total fracture permeability K=100





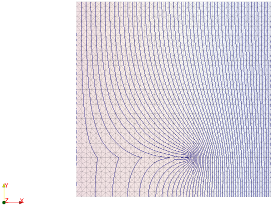
A two fracture simulation

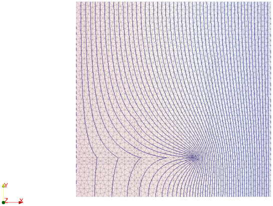
- Comparative results with a finite volume simulator.
- MHM linear resulted in 841 equations

Author	Fluid Flux
Wenchao	1.2892
MHM quadratic	1.285
MHM linear	1.284

A fracture on the skeleton mesh

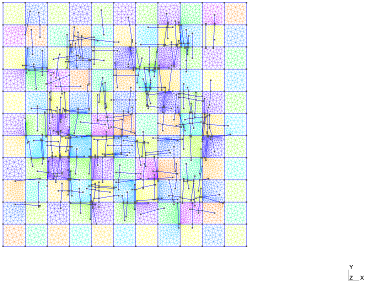
- Depending on the MHM layout a fracture may coincide with a MHM boundary
- Boundary flux integrated was 1452 versus 1461





A complex fracture network

- Around 250 fracture in random patterns

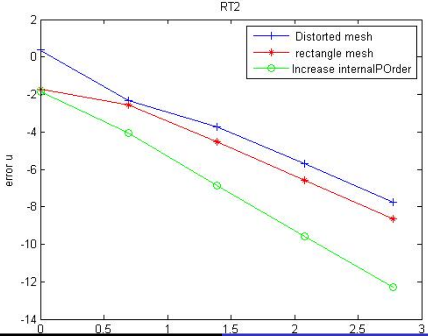


Objectives

- Verify if the H(Div) approximation spaces can be used for the construction of robust and convergent tensor space approximations
- Verify if enhanced convergence rates for the primal variable are applicable to tensor space approximations
- Work developed with
    - Prof Jun Hu and Shudan Tian
    - Prof Sonia M Gomes and Pablo Carvalho

Results obtained

- The tensor space approximations in standard configuration yield optimal convergence rates, even for distorted meshes
- Increasing the internal order of approximation improves the convergence rate, but optimal rates are not obtained



Summary

- Discrete fracture networks in conjunction with MHM were developed and implemented
- The combination of a general data file with Gmsh resulted in a very general approach
- MHM can also be very efficient for the simulation of time dependent problems
- Tensor spaces have been implemented and certified

Future Work

- Can MHM-$H(div)$ be used as a preconditioner for the full scale solution?
- Can the approach for fracture networks be extended to three dimensional configurations?
- Can the convergence rates of the tensor spaces be justified by theoretical analysis?

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