

0.1 Fields

Note:-

- This is summary note for 2023 Fall, KAIST MAS212 - Linear Algebra course.
- I assumed that you are familiar enough to MAS109 - Introduction to Linear Algebra course, or just some notation used in basic linear algebra like matrix and row operations.
- I also assumed that you are familiar enough to some mathematical logic symbols.
- We used Kenneth Hoffman / Ray Kunze - Linear Algebra 2nd ed.

Definition 0.1.1: Field

Algebraic structure \mathbb{F} satisfying given properties are called field:

1. Addition is commutative: $\forall \{x, y\} \subset \mathbb{F} \ (x + y = y + z)$
2. Addition is associative: $\forall \{x, y, z\} \subset \mathbb{F} \ (x + (y + z) = (x + y) + z)$
3. $\forall x \in \mathbb{F} \ \exists ! 0 \in \mathbb{F} \ (x + 0 = x)$
4. $\forall x \in \mathbb{F} \ \exists ! (-x) \in \mathbb{F} \ (x + (-x) = 0)$
5. Multiplication is commutative: $\forall \{x, y\} \subset \mathbb{F} \ (xy = yx)$
6. Multiplication is associative: $\forall \{x, y, z\} \subset \mathbb{F} \ (x(yz) = (xy)z)$
7. $\forall x \in \mathbb{F} \ \exists ! 1 \in \mathbb{F} \ (x1 = x)$
8. $\forall x \in \mathbb{F} \ \exists ! x^{-1} = 1/x \in \mathbb{F} \ (xx^{-1} = 1)$
9. $\forall \{x, y, z\} \in \mathbb{F} \ (x(y + z) = xy + xz)$

0.2 Systems of Linear Equations

This Chapter is Intentionally Skipped at Lectures.

0.3 Matrices and Elementary Row Operations

This Chapter is Intentionally Skipped at Lectures.

0.4 Row-Reduced Echelon Matrices

This Chapter is Intentionally Skipped at Lectures.

0.5 Matrix Multiplication

Definition 0.5.1: Matrix Multiplication

$$C := [C_{ij}]. \quad C_{ij} := \sum_{r=1}^n A_{ir} B_{rj}.$$

Theorem 0.5.1

Matrix multiplication is associative, but not commutative.

0.6 Invertible Matrices

Definition 0.6.1: Invertible Matrices

P is invertible $\iff \exists! Q \ (PQ = QP = I)$.