Brief Idea of Rain Gauge using a Capacitor

PH161 Group 8

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Assumptions and Formula

Assumptions

We assumed here, ideal gauge and permittivity. For example. We assumed small radius a as 1cm, large radius b as 5cm, and we have 0.1mm of water inside it while permittivity of water and air is 80.2 and 1, respectively.

Note.

$$C = 2\pi\epsilon_0 \frac{L}{ln\frac{a}{b}}, \quad a < b, \quad \epsilon_0 \approx 8.854 \times 10^{-12}$$

Further Assumptions

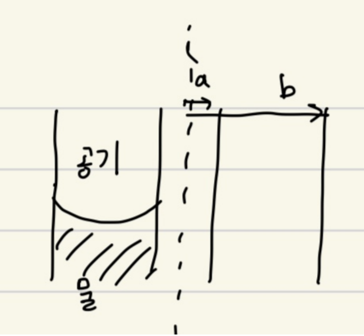


Figure: Our assumption 1)
a=1cm, b=5cm

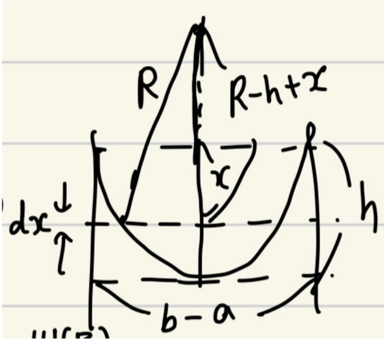


Figure: Our assumption 2) R=1m

Calculation Time

Area of Air

$$2\sqrt{R^2 - (R - h + x)^2} \tag{1}$$

Where K is relative permittivity.

Area of Rain

$$(b - a) - 2\sqrt{R^2 - (R - h + x)^2} \tag{2}$$

Calculation Time

$$dc_{air} = 2\pi\epsilon_0 \frac{dx}{ln\left(\frac{2\sqrt{R^2-(R-h+x)^2}+a}{a}\right)} \tag{3}$$

$$dc_{rain} = 2\pi\epsilon_0 K \frac{dx}{ln\left(\frac{(b-a)-2\sqrt{R^2-(R-h+x)^2}+a}{a}\right)} \tag{4}$$

Then let (1) in before page is α.

$$\begin{aligned}\frac{1}{dc_{tot}} &= \frac{1}{dc_{air}} + \frac{1}{dc_{rain}} = \frac{ln\left(\frac{\alpha+a}{a}\right)}{2\pi\epsilon_0 dx} + \frac{ln\left(\frac{b-\alpha}{a}\right)}{2\pi\epsilon_0 K dx} \\ &= \frac{Kln\left(\frac{\alpha+a}{a}\right) + ln\left(\frac{b-\alpha}{a}\right)}{2\pi\epsilon_0 K dx} = \frac{ln\left(\frac{(\alpha+a)^K(b-\alpha)}{a^{K+1}}\right)}{2\pi\epsilon_0 K dx}\end{aligned} \tag{5}$$

Calculation Time

$$\therefore dc_{tot} = \frac{2\pi\epsilon_0 K dx}{ln\left(\frac{(\alpha+a)^K(b-\alpha)}{a^{K+1}}\right)} \tag{6}$$

$$c_{tot} = \int_0^h dc_{tot} = \int_0^h \frac{2\pi\epsilon_0 K}{Kln(\alpha+1) + ln(b-\alpha) - (K+1)ln(a)} dx \tag{7}$$

Now since h is extremely small, we can ignore phenomena caused by meniscus.

Calculation Time

Now based on (7), previous page, we can calculate capacitance when height of accumulated rain is $x$cm. Suppose $a = 1$ and $b = 5$cm, and we have half-full gauge. Then

$$c_{tot} = 2\pi\epsilon_0 \frac{Rx + L - x}{ln\left(\frac{b}{a}\right)} = 2\pi \times 8.854 \times 10^{-12} \frac{79.2x + L}{ln\left(\frac{b}{a}\right)}.$$

By calculation, we can derive

$$c_{tot} = 1.386nF.$$

Type of Capacitor

Our Selection of Capacitor: Cylindrical Capacitor

Since cylinder is a typical appearance of rain gauge, we choose cylindrical capacitor.

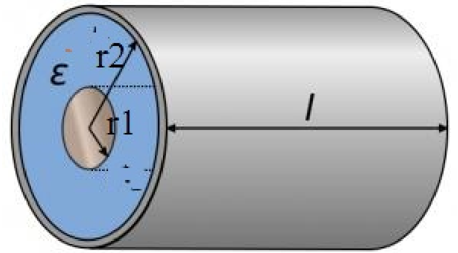


Figure: Example of Cylindrical Capacitor

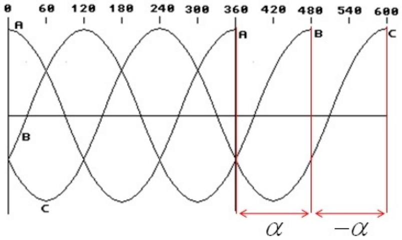
Type of Current

Our Considerations:

1. Alternating Current - We can use phase to measure
2. Direct Current - Unfortunately, we failed to find any advantages

Our Selection of Current: AC

We think Direct Current is not appropriate for rain gauge because it has trouble for continuous measurement, so we decided to use Alternating Current, AC.



How to make AC?

Our Considerations:

1. Function Generator - This device didn't fit to our project, since this 'violate' our purpose of rain gauge, to be protable and convenient.
2. Oscillator Circuit - Because of reasons above, we judged this is more 'fit' to our project.

Our Selection: OC

We take more weight in protability and convenience. So function generator was not a good choice. Also, by choose OC, we can make gauge with less money that FG.
Further, as we talk a bit later, we have to choose which kind of OC to use among various kind of OC, but we will leave it to our future....

About Circuit

NOTE!

We actually failed to decide 100%ly which kind of OC we should use since its various variations, but now, we are concerning Wien Bridge Oscillator.

Our Selection: Wien Bridge Oscillator

Basically, WBO, variation of Bridge Circuit, generates sine waves. It can generate various frequencies, so we chose it. But since it has some variations, we yet deciding which kind of circuit we'll use specifically.

How to measure?

We will use formula

$$\frac{V_{in}}{V_{out}} = \frac{Xc}{Z}$$

where V_{out} is voltage on the capacitor and Z is impedance.

About Circuit

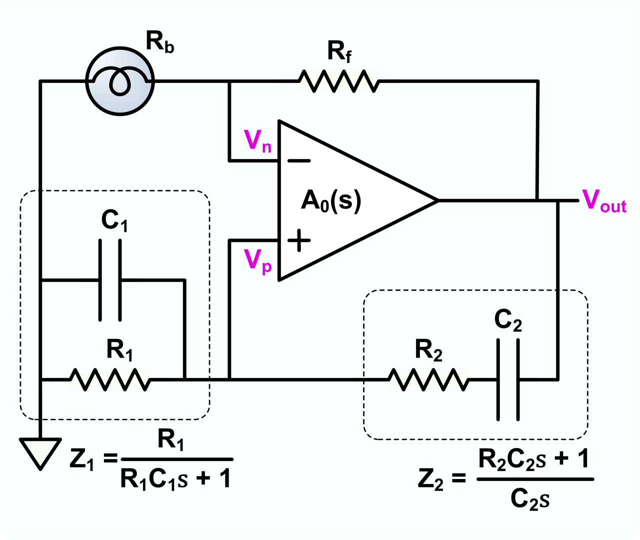


Figure: Wien Bridge Oscillator

About Accuracy

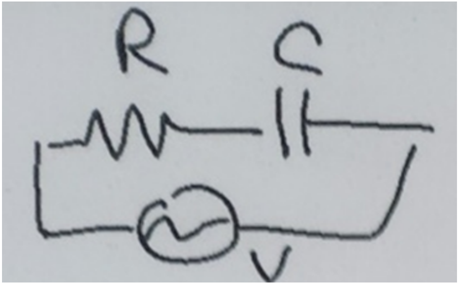


Figure: Our Assumption on this accuracy test

Note.

$$C \approx kl$$

where $k \approx 10^{-9}$ and l is fall (m).

About Accuracy

Let impedance of capacitor is $\frac{1}{wC}$.
Then, Voltage at R is:

$$\frac{VR}{\sqrt{R^2 + \frac{1}{C^2 w^2}}} = \frac{VRCw}{\sqrt{1 + R^2 C^2 w^2}}.$$

Approximate $RCW << 1$. Then,

$$\approx VRCw = V_R. \qquad \therefore I = \frac{V_R}{kRwV}.$$

Let:
V_R: Voltmeter's Error $\sim 1\%$
R: Circuit's Resistance and Elec. Capacity $\sim 5\%$
k: Shape of Capacitor $\sim 5\%$
$\therefore$ Tot. Error $\approx$7%.

References

- 지식저장고. (27 June 2017). [일반물리학] 18. 전기용량과 유전체 (1: 전기용량)
- Richard Wolfson. (25 June 2020). *Essential University Physics(4th ed.). Pearson*
- Wikipedia. Wien Bridge Oscillator.