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02/03/22
STAT632 HW# 1

- Concept Questions

① (a) $\hat{y}_i = b_0 + b_1 x_i$ = equation for best fitting line or the least squares regression line.

so, $\hat{y} = -1.11 + 2.26x$

(b)

$H_0: B_1 = 0$

$H_A: B_1 \neq 0$

Based on the p-value (df: 50-1=49) of the intercept (p=.0103), we reject the $H_0$ at $\alpha = .05$

(c) p-value for the intercept, where df=n-1 (50-1=49)
= .0103

(d) $t = \dfrac{2.2606}{0.0981} \approx 23.04$

(e) The 95% confidence interval for $B_1$:

$[\hat{B}_1 \pm t_{\alpha/2, n-2} \, SE]$

$= [2.2606 \pm (2 \cdot .0106)(.0981)]$

$= 2.2606 \pm .1972$

$= (2.0634, 2.4578)$

(2)

(a) $\hat{\beta} = \dfrac{\sum_{i=1}^{n} x_i y_i}{\sum_{i=1}^{n} x_i^2}$

$\hat{y}_i = \hat{\beta} x_i$

↳ $Y_i = \beta x_i + e_i$, where $RSS = \sum_{i=1}^{n} (y_i - \hat{y}_i)^2 = \sum_{i=1}^{n} (y_i - \hat{\beta} x_i)^2$.

Set partial Derivative to 0:

$$\frac{d}{d\beta} RSS = -2 \sum_{i=1}^{n} x_i (y_i - \hat{\beta} x_i) = 0$$

$$\updownarrow$$

$$\sum_{i=1}^{n} x_i y_i - \hat{\beta} \sum_{i=1}^{n} x_i^2 = 0$$

$$\updownarrow$$

$$\hat{\beta} = \frac{\sum_{i=1}^{n} x_i y_i}{\sum_{i=1}^{n} x_i}$$

(b) $E(\hat{\beta}) = \beta$

↳ $E(\hat{\beta}|X) = E\left(\dfrac{\sum_{i=1}^{n} x_i y_i}{\sum_{i=1}^{n} x_i^2}\right) = \dfrac{\sum_{i=1}^{n} x_i E(y_i)}{\sum_{i=1}^{n} x_i^2} = \beta \cdot \dfrac{\cancel{\sum_{i=1}^{n} x_i^2}}{\cancel{\sum_{i=1}^{n} x_i^2}} = \beta$

(c) $var(\hat{\beta})$

↳ $var(\hat{\beta}|X) = Var\left(\dfrac{\sum_{i=1}^{n} x_i y_i}{\sum_{i=1}^{n} x_i^2}\right) = \dfrac{\sum_{i=1}^{n} \cancel{x_i^2} \sigma^2}{\left(\sum_{i=1}^{n} x_i^2\right)^{\cancel{2}}} = \dfrac{\sigma^2}{\sum_{i=1}^{n} x_i^2}$