

1 Dry Run & Analyze: Time & Space complexity

i. $n = 4$

```
void PrintTriangle (int n) {
    for (int i = 0; i < n; i++)
        for (int j = 0; j <= i; j++)
            System.out.print ("*")
```

1st iteration:

$i = 0$ & $j = 0$

Print
*

For 2nd iteration:

$i = 1$ & $j = 0, 1$

* *

for 3rd iteration

$i = 2$ & $j = 0, 1, 2$

* * *

for 4th iteration

$i = 3$ & $j = 0, 1, 2, 3$

* * * *

*. Time complexity :-

i	j
0	0*

No. of iteration

0

i	j
1	0, 1*

1

i	j
2	0, 1, 2*

2

i	j
3	0, 1, 2, 3*

3

i	j
4	0, 1, 2, 3

4

\therefore sum of first n natural numbers

$$S = \frac{n(n+1)}{2}$$

$$f(n) = \frac{n^2 + n}{2}$$

① ignore constant

② take bigger

$$\therefore \underline{\underline{O(n^2)}}$$

2) Dry run for $n=8$. What's the number of iterations? Time complexity?

→ 1st iteration

$$i=1 \quad j=0$$

Print

$$i=1 \quad j=1$$

1,0

$$i=1 \quad j=2$$

1,1

$$i=1 \quad j=3$$

1,2

$$i=1 \quad j=4$$

1,3

$$i=1 \quad j=5$$

1,4

$$i=1 \quad j=6$$

1,5

$$i=1 \quad j=7$$

1,6

$$i=1 \quad j=8$$

1,7

2nd iteration.

$$i=2, \quad j=0$$

2,0

$$i=2, \quad j=1$$

2,1

$$i=2, \quad j=2$$

2,2

$$i=2, \quad j=3$$

2,3

$$i=2, \quad j=4$$

2,4

$$i=2, \quad j=5$$

2,5

$$i=2, \quad j=6$$

2,6

$$i=2, \quad j=7$$

2,7

Code: void PrintPattern (int n) {
 for (int i=1; i<=n; i*=2) {
 for (int j=0; j<n; j++) {
 System.out.println (i+j); } } }

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3rd iteration

i = 4, j = 0	4, 0
i = 4, j = 1	4, 1
i = 4, j = 2	4, 2
i = 4, j = 3	4, 3
i = 4, j = 4	4, 4
i = 4, j = 5	4, 5
i = 4, j = 6	4, 6
i = 4, j = 7	4, 7

4th iteration

i = 8, j = 0	8, 0
i = 8, j = 1	8, 1
i = 8, j = 2	8, 2
i = 8, j = 3	8, 3
i = 8, j = 4	8, 4
i = 8, j = 5	8, 5
i = 8, j = 6	8, 6
i = 8, j = 7	8, 7

Time complexity :-

Outer loop $\Rightarrow O(\log n)$ $\because i = 2^k$

Inner loop $\Rightarrow O(n)$

$O(n) \times O(\log n)$

$O(n \log n)$

3) Dry run for $n=20$. How many recursive calls? what values are printed.

```
void rechalf(int n) {
    if (n <= 0) return;
    System.out.println(n + " ");
    rechalf(n/2);
}
```

→ first check $n=20$

$\cancel{n \leq 0}$ false

function call

$rechalf(20/2) = 10$

∴ 2nd call

$n = 10$

check, $10 \leq 0$, false

function call

$rechalf(10/2) = 5$

∴ 3rd call

$n = 5$

check, $5 \leq 0$, false.

function call

$rechalf(5/2) = 2$

∴ 4th call

$n = 2$

check, $2 \leq 0$, False

∴ recursive call

$rechalf(2/2) = 1$

5th call

$$n = 1$$

check $1 <= 0$ false

recursive call

reCHalf(1|2); = 0

6th call

$$n = 0 \text{ true}$$

print ~~1~~

returns to previous call

Print

$$20 > 0 \text{ n } \cancel{2}$$

Print

$$20$$

$$10 > 0$$

$$10$$

$$5 > 0$$

$$5$$

$$2 > 0$$

$$2$$

$$1 > 0$$

$$1$$

$$0 \leq 0$$

print ~~1~~

\therefore Total Recursive call = 6

it printing - 20, 10, 5, 2, 1

- 4) Dry run for $n = 3$. How many total call are made? what's the time complexity?

```
void fun(int n){  
    if (n == 0) return;  
    fun(n-1);  
    fun(n-1);  
}
```

→ First check $fun(3)$

$$n = 3$$

$$(3 \neq 0)$$

false

recursive call

$$\text{fun}(3-1) = 2$$

call fun(2)

call fun(2)

∴ 2nd call fun(2)

$$n = 2$$

if ($2 \neq 0$) false

$$\text{fun}(2-1) = 1$$

call fun(1)

call fun(1)

∴ 3rd call fun(1)

$$n = 1 - 0$$

if ($1 \neq 0$) false

call fun(1-1) = 0

call fun(0)

4th call

$$n = 0 - 0$$

if ($0 = 0$) true

call fun(0), call fun(0)

Total number of function call fun(n)

is $2^{(n+1)} - 1$

$$n=3$$

$$= 2^{(3+1)} - 1$$

$$= 2^4 - 1$$

$$= 16 - 1$$

$$= \underline{\underline{15}}$$

Time Complexity:

$$2^n$$

$$1 + 2 + 4 + 8 + \dots + 2^n$$

$$2^{n+1} - 1$$

Time Complexity = $O(2^n)$

5) Dry run for $n=3$. How many total iterations? Time complexity?

```
void triplenested(int n){  
    for(int i=0; i<n; i++)  
        for(int j=0; j<n; j++)  
            for(int k=0; k<n; k++)  
                System.out.println(i+j+k);  
}
```

→ 1st iteration:

i	j	k	Print
0	0	0	0

2nd iteration

i	j	k	Print
0	0	1	1
0	0	2	2
0	1	0	1
0	1	1	2
0	1	2	3
0	2	0	2
0	2	1	3
0	2	2	4

2nd iteration

~~i=1~~

2nd iteration.

i	j	k	Point
1	0	0	1
1	0	1	2
1	0	2	3
1	1	0	2
1	1	1	3
1	1	2	4
1	2	0	3
1	2	1	4
1	2	2	5
2	0	0	2
2	0	1	3
2	0	2	4
2	1	0	3
2	1	1	4
2	1	2	5
2	2	0	4
2	2	1	5
2	2	2	6

Time complexity:

Each loop runs n time

$$n=3 \quad 3^3 = 27 \text{ times.}$$

$$\underline{\underline{O(n^3)}}$$