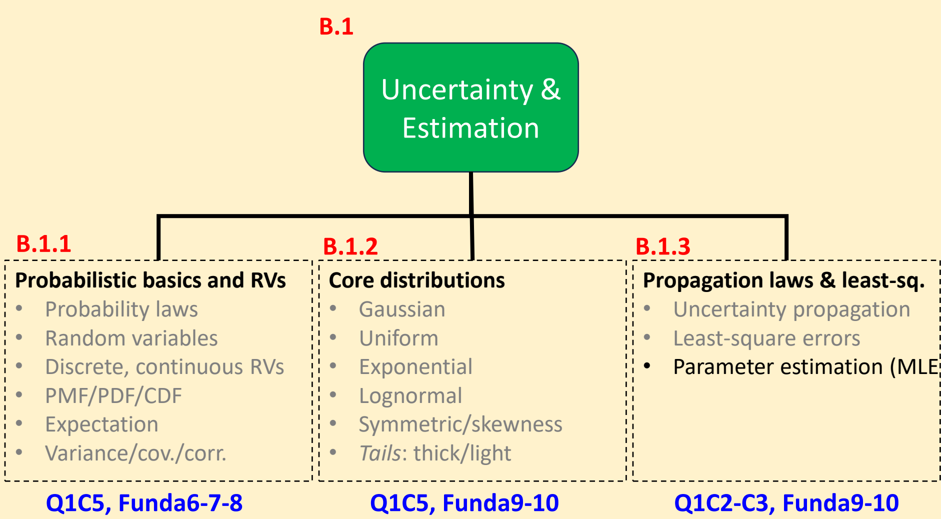
# CV 510$^{0}_{1}$
# Modeling, Uncertainty, and Data for Engineers

(July – Nov 2025)

Dr. Prakash S Badal

1

## Module Overview




2

# Flow (so far and week ahead)

- So far:
  - Probability axioms, rules, PMF/PDF/CDF
  - Variance, covariance, correlation
  - Distributions: uniform, Gaussian, exponential, Gumbel, lognormal
  - Standard normal; reading z-table
  - Uncertainty propagation: linear, nonlinear function (FO and SO approximation)
  - Fitting a distribution Linear regression (estimation): least-square method
- This week:
  - Estimation: Method of moments (MoM), Maximum likelihood estimation (MLE)
  - Inference: Confidence interval (CI)
  - Inference: Hypothesis testing (HT)
  - Goodness of fit (GoF): $\chi^2$, KS
- Check-in with teachbook
  - MLE: Q1C3.6 and Q1C5.4.2
  - CI: Q1C3.5
  - HT: Q1C3.9
  - GoF: Q1C5.4

3

3

# Parameter estimation: method of moments

- Obtain the moments (e.g., mean, variance) from two sources:
  - Observations:

$$\mathrm{E}[X] = \mu_X = \int_{-\infty}^{+\infty} x f_X(x)\mathrm{d}x$$

$$\mathrm{Var}[X] = \sigma_X^2 = \mathrm{E}[(X - \mu_X)^2] = \mathrm{E}[X^2] - \mu_X^2$$

  - Parameters:

Depends on the distribution type

- Equate them to solve for the parameters

4

4

# Distributions and parameters

| Dist. | PDF | CDF | Mean & Variance |
|---|---|---|---|
| **Normal/ Gaussian** | $f(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2\right]$ | $F(x) = \Phi(x)$ Use z-table | $\mathrm{E}[X] = \mu$ <br> $\mathrm{Var}[X] = \sigma^2$ |
| **Uniform** | $f(x) = \begin{cases} \frac{1}{b-a} & \text{for } x \in [a,b] \\ 0 & \text{otherwise} \end{cases}$ | $F(x) = \begin{cases} 0 & \text{for } x < a \\ \frac{x-a}{b-a} & \text{for } x \in [a,b] \\ 1 & \text{otherwise} \end{cases}$ | $\mathrm{E}[X] = \frac{1}{2}(a+b)$ <br> $\mathrm{Var}[X] = \frac{1}{12}(b-a)^2$ |
| **Exponential** | $f(x) = \lambda \exp(-\lambda x)$ | $F(x) = 1 - \exp(-\lambda x)$ | $\mathrm{E}[X] = \frac{1}{\lambda}$ <br> $\mathrm{Var}[X] = \frac{1}{\lambda^2}$ |
| **Gumbel** | $f(x) = \frac{1}{\beta}\exp[-z + \exp(-z)]$, where $z = \frac{x-\alpha}{\beta}$ | $F(x) = \exp[-\exp(-z)]$ | $\mathrm{E}[X] = \alpha + \beta\gamma$ <br> $\mathrm{Var}[X] = \frac{\pi^2}{6}\beta^2$ <br> $\gamma = 0.577$ |
| **Lognormal** | $f(x) = \frac{1}{x\sigma\sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{\ln x - \mu}{\sigma}\right)^2\right]$ | $F(x) = \Phi\left(\frac{\ln x - \mu}{\sigma}\right)$ Use z-table | $\mathrm{E}[X] = \exp\left(\mu + \frac{\sigma^2}{2}\right)$ <br> $\mathrm{Var}[X] = [\exp(\sigma^2) - 1]\exp(2\mu + \sigma^2)$ |

5

# Example

- Assume that Earthquakes in Rome expressed as MCS index follow Gumbel type-I distribution. Earthquake intensity and number of earthquakes between 1000 CE – 1980 CE are:

| MSC intensity, $x_i$ | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|
| Number, $f_i$ | 113 | 132 | 56 | 22 | 4 | 2 |

| $\sum x = \sum x_i f_i$ | $\sum x^2 = \sum x_i^2 f_i$ | $\sum f_i$ |
|---|---|---|
| 994 | 3328 | 329 |

Find the parameters of the distribution.

- Observations:

$\mathrm{E}[X] = \mu_X = 994/329 = 3.02$

$\mathrm{Var}[X] = \mathrm{E}[X^2] - \mu_X^2 = 10.12 - 3.02^2 \approx 0.99$

$\mathrm{E}[X^2] = 3328/329 \approx 10.12$

- Parameters for Gumbel type-I:

$\mathrm{E}[X] = \alpha + \beta\gamma$

$\mathrm{Var}[X] = \frac{\pi^2}{6}\beta^2$

- Equate them to solve for the parameters

Solve to find, $\alpha \approx 2.57, \beta \approx 0.77$

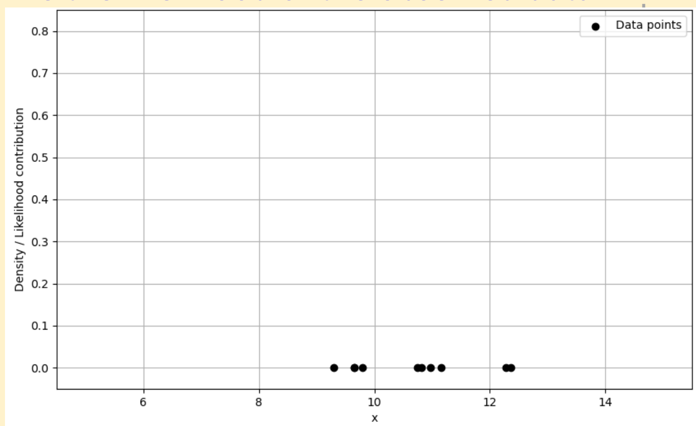
6

6

# Maximum likelihood estimation

**Basic idea** **Objective of MLE:**

To determine the parameters of a model
such that
they maximize the likelihood of the observed data.




7

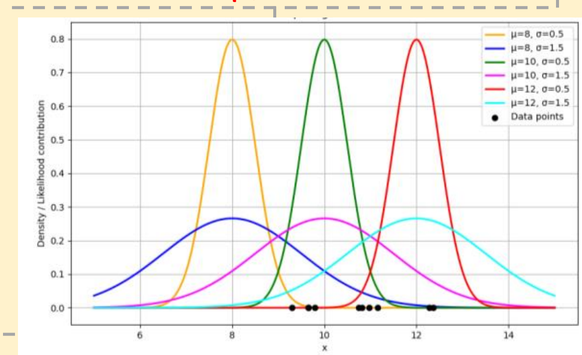
# Maximum likelihood estimation

Find $\mathcal{L}(\mu, \sigma | x_i) = f(x_i | \mu, \sigma)$ for each dart, $x_i$.
Maximize the product of likelihood,

$$\mathcal{L}(\mu, \sigma | \mathbf{x}) = \prod_{i}^{n} f(x_i | \mu, \sigma)$$

$$\hat{\mu}, \hat{\sigma} = \arg\max_{\mu,\sigma} \mathcal{L}(\mu, \sigma | \mathbf{x})$$

$$\hat{\mu}, \hat{\sigma} = \arg\max_{\mu,\sigma} \ln[\mathcal{L}(\mu, \sigma | \mathbf{x})]$$




8

# Maximum likelihood estimation

Find $\mathcal{L}(\mu, \sigma | x_i) = f(x_i | \mu, \sigma)$ for each dart, $x_i$.

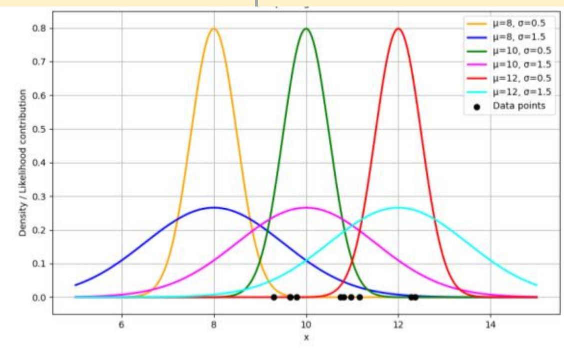
Maximize the product of likelihood,

$$\mathcal{L}(\mu, \sigma | \mathbf{x}) = \prod_{i}^{n} f(x_i | \mu, \sigma)$$

$$\hat{\mu}, \hat{\sigma} = \arg\max_{\mu,\sigma} \mathcal{L}(\mu, \sigma | \mathbf{x})$$

$$\hat{\mu}, \hat{\sigma} = \arg\max_{\mu,\sigma} \ln[\mathcal{L}(\mu, \sigma | \mathbf{x})]$$

Excel Demo!




9

Questions, comments, or concerns?

10

10