CV 510$^{0}_{1}$
Modeling, Uncertainty, and Data for Engineers

(July – Nov 2025)

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Flow

- Covariance
- Correlation
- Parametric distributions
- SciPy

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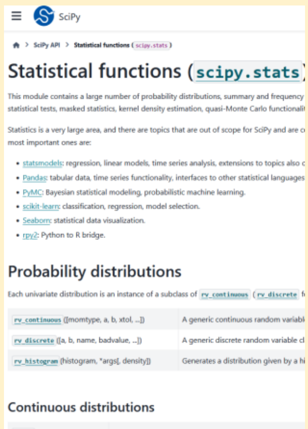
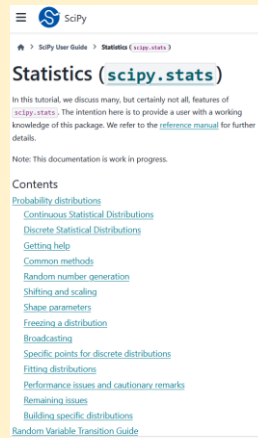
Announcement

- Practice `scipy.stats` for the following:
  - Using distributions → Shifting, scaling
  - Generating random numbers
  - Fitting distributions





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scipy

- [Ref.] https://docs.scipy.org/doc/scipy/reference/stats.html
- Tutorial https://docs.scipy.org/doc/scipy/tutorial/stats.html
- Probability distributions https://docs.scipy.org/doc/scipy/tutorial/stats/probability_distributions.html

- Run examples

  - Common Methods
  - Random number generator
  - Shifting-scaling: loc, scale
  - Fitting distributions

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Check-in with teach-book

https://mude.citg.tudelft.nl/book/2024/

- Q1 Topics (Chapters 5, 6, 2, and 3)
  - Q1C5 Univariate continuous distribution
    - Q1C5.1 PDF/CDF
    - Q1C5.2 Empirical Distributions
    - Q1C5.3 Parametric Distributions
    - Q1C5.4 Fitting a Distribution
  - Q1C6 Multivariate Distributions (briefly)
  - Q1C2 Propagation of Uncertainty
  - Q1C3 Observation Theory: least-sq., Hyp. Test, Conf. Intervals
- Q2 Topics (Chapter 7 and 8)
  - Q2C7 Extreme value theory: GEV, return period, POT
  - Q2C8 Risk and decision making (CBA)
- Fundamental Concepts
  - Chapter 6, 7, 8, 9. Probability basics, rv, z- and t-tables
- Programming
  - Fundamental Concepts → Chapter 10

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Module Overview

B.1

Uncertainty & Estimation

B.1.1

Probabilistic basics and RVs
- Probability laws
- Random variables
- Discrete, continuous RVs
- PMF/PDF/CDF
- Expectation
- Variance/cov./corr.

Q1C5, Funda6-7-8

B.1.2

Core distributions
- Gaussian
- Uniform
- Exponential
- Lognormal
- Symmetric/skewness
- *Tails*: thick/light

Q1C5, Funda9-10

B.1.3

Propagation laws & least-sq.
- Uncertainty propagation
- Least-square errors
- Parameter estimation

Q1C2-C3, Funda9-10

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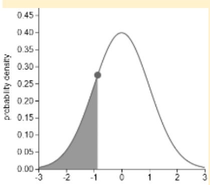
In-Class problem

- The compressive strength of a concrete mix is normally distributed with mean 40 MPa and standard deviation 5 MPa.

Out of 10,000 specimen produced, how many will generally have strength less than 35 MPa?

How many with strength less than 30 MPa?



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Gaussian

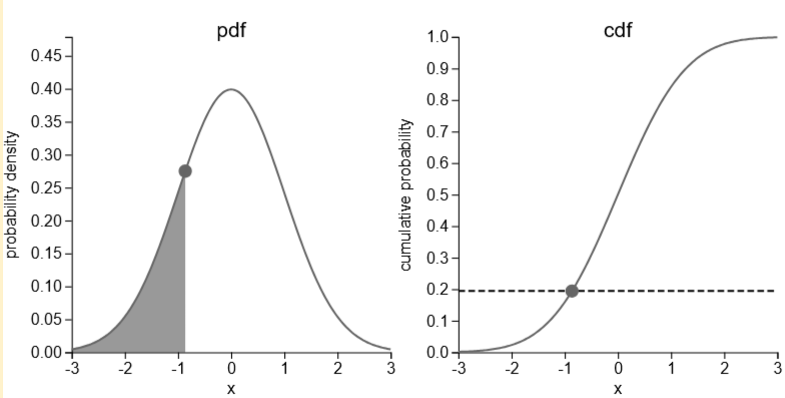

Fig. 5.1 Interactively visualize the relationship between the PDF and the CDF (the integral of the PDF value). #

Teach-book Q1-C5 **Fig. 5.1** (interactive)

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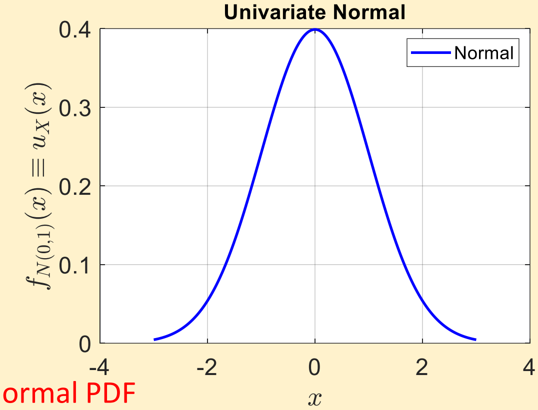
Gaussian

- Gaussian



Univariate normal PDF

$N(\mu, \sigma) = f_X(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2\right]$; $u(x) \equiv \varphi(x) = \frac{1}{\sqrt{2\pi}} \exp\left[-\frac{x^2}{2}\right]$

No closed-form equation for Normal CDF

Gaussian

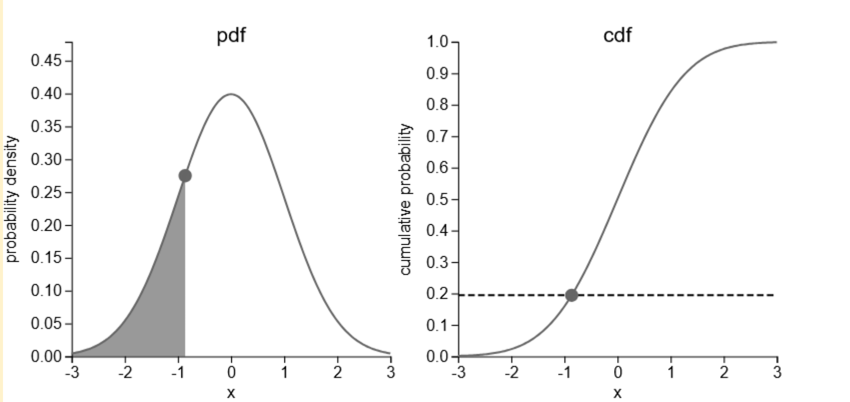


Fig. 5.1 Interactively visualize the relationship between the PDF and the CDF (the integral of the PDF value). #

Teach-book Q1-C5 **Fig. 5.1** (interactive)

Standard Normal Probabilities

Z-table

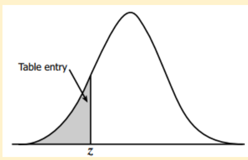


Table entry for z is the area under the standard normal curve to the left of z.

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
−3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002
−3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0003
−3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005
−3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
−3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
−2.9	.0019	.0018	.0018	.0017	.0016	.0016	.0015	.0015	.0014	.0014
−2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
−2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
−2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
−2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
−2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
−2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
−2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
−2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
−2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
−1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
−1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
−1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
−1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
−1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
−1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681
−1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
−1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
−1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
−1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
−0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
−0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
−0.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
−0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
−0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
−0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
−0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
−0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
−0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
−0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641

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Gaussian

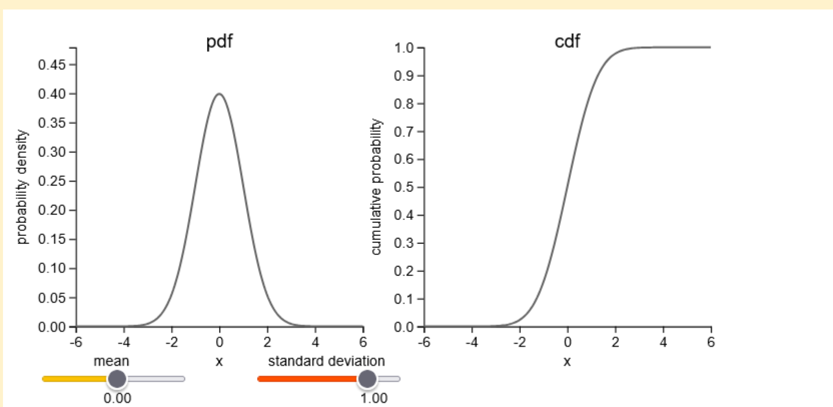

Fig. 5.3 Interactively change the mean and standard deviation of the Gaussian distribution to visualize the effect on the PDF and CDF.

Scale and location parameters
Teach-book Q1-C5 **Fig. 5.3** (interactive)

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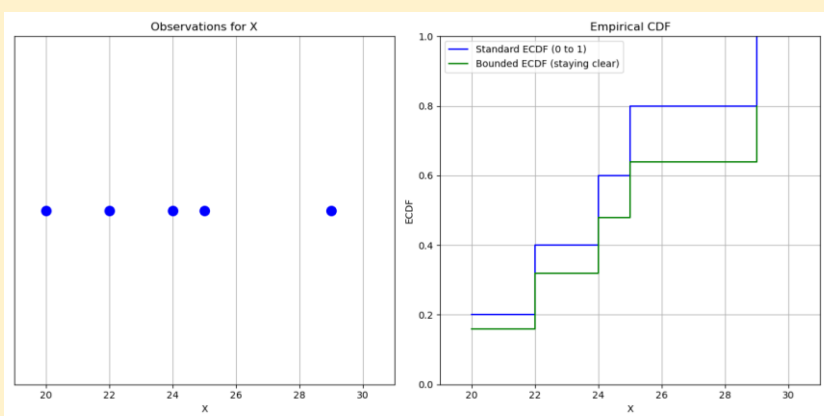
Inverse CDF

- For designing a structure, we often need

a value that is not exceeded with more than p probability:

$$x = F^{-1}(p)$$

Empirical distribution



Standard ECDF: $F_n(x) = \frac{i}{n}$ goes from 0 to 1

Bounded ECDF: $F_{n,b}(x) = \frac{i}{n+1}$ stays clear of 0 and 1

useful in Q-Q/probability plotting, avoids $-\infty$ or $+\infty$.

scipy

- Tutorial (not reference) https://docs.scipy.org/doc/scipy/tutorial/stats/probability_distributions.html
- https://docs.scipy.org/doc/scipy/tutorial/stats.html
  - Run examples

- Common Methods
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Uniform Distribution

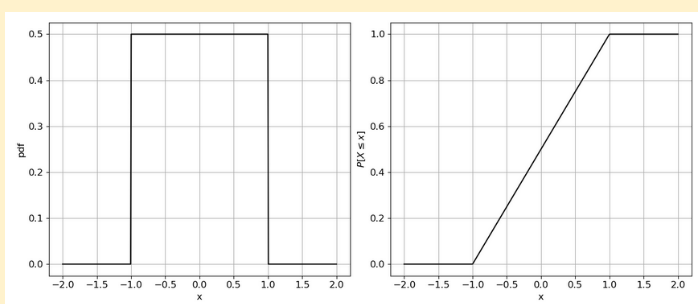

Fig. 5.11 Uniform distribution function: (left) PDF, and (right) CDF.

PDF

$$f_X(x) = \begin{cases} \dfrac{1}{b-a}, & a \le x \le b \\ 0 & \text{otherwise} \end{cases}$$

CDF

$$F_X(x) = \begin{cases} 0 & \text{for } x < a \\ \dfrac{x-a}{b-a} & \text{for } a \le x \le b \\ 1 & \text{for } x > b \end{cases}$$

Properties $\mathrm{E}[X] = \dfrac{1}{2}(a+b); \quad \mathrm{Var}[X] = \dfrac{1}{12}(b-a)^2$

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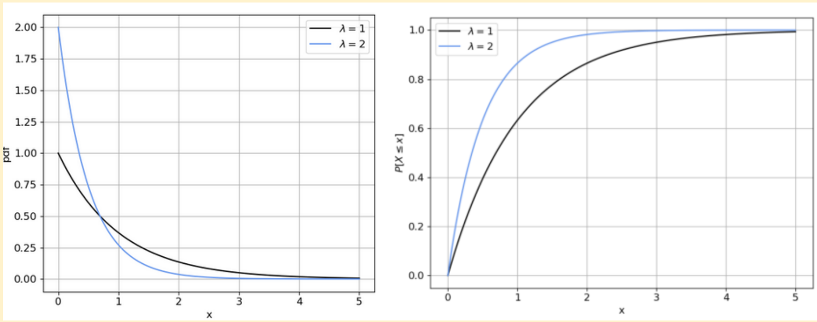
Exponential Distribution



PDF

$$f_X(x) = \begin{cases} \lambda e^{-\lambda x}, & x \geq 0, \lambda > 0 \\ 0 & \text{otherwise} \end{cases}$$

CDF

$$F_X(x) = 1 - e^{\lambda x}, \qquad x > 0$$

Properties $\mathrm{E}[X] = \dfrac{1}{\lambda}; \qquad \mathrm{Var}[X] = \dfrac{1}{\lambda^2}$

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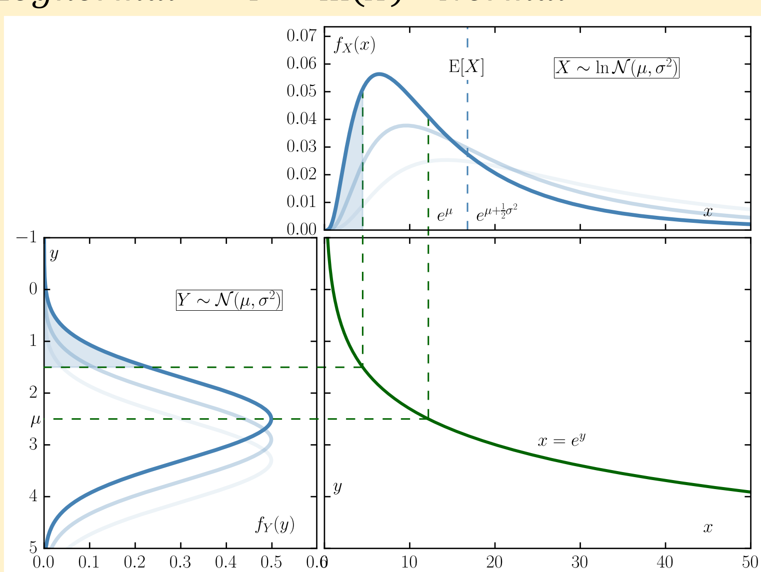
Lognormal

- $X \sim Lognormal \Leftrightarrow Y = \ln(X) \sim Normal$



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Gumbel distributions

- When we are interested in

the smallest

or

the largest of a set of rv's,

e.g., a chain of links: smallest strength.

Flood level under a bridge: highest flood level during its lifetime.

$$Y_1 = \min(X_1, X_2, \dots, X_n)\,,$$

$$Y_n = \max(X_1, X_2, \dots, X_n)\,.$$

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Gumbel distributions

$$Y_1 = \min(X_1, X_2, \dots, X_n)\,,$$
$$Y_n = \max(X_1, X_2, \dots, X_n)\,.$$

The CDF of Y_1 is:

$F_{Y_1}(y) = \Pr(Y_1 \le y) = 1 - \Pr(Y_1 > y) = 1 - \prod_{i=1}^{n} \Pr(X_i > y)$

CDF $F_{Y_1}(y) = 1 - [1 - F_X(y)]^n$

PDF $f_{Y_1}(y) = 1 - n f_X(y)[1 - F_X(y)]^{n-1}$

Similarly, the CDF of Y_n is:

$F_{Y_n}(y) = \Pr(Y_2 \le y) = \prod_{i=1}^{n} \Pr(X_i < y)$

CDF $F_{Y_n}(y) = [F_X(y)]^n$

PDF $f_{Y_n}(y) = n f_X(y)[F_X(y)]^{n-1}$

Exact when distributions are known. Integration needed for mean/var. Hence, asymptotic distributions: GEV

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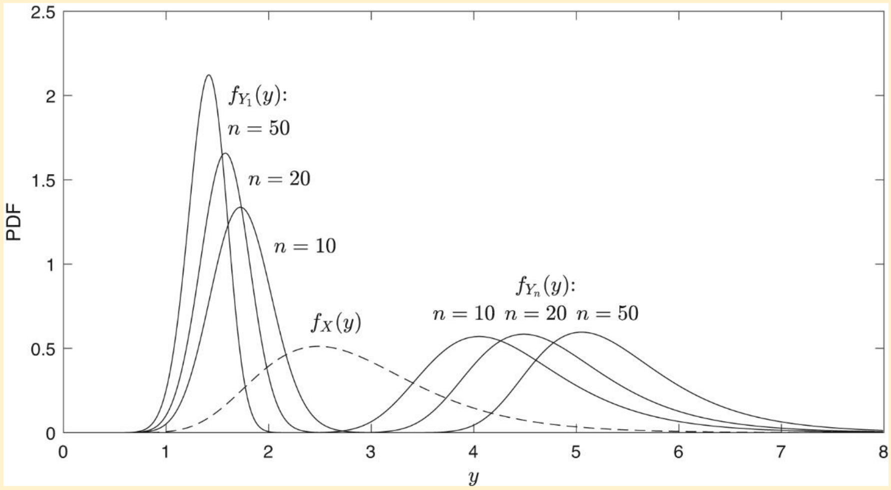
Gumbel distributions



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Gumbel distributions

Positive tail

PDF

$$f_{Y_n}(y) = \frac{1}{\beta} e^{-\left(\frac{y-\mu}{\beta} + e^{-\left(\frac{y-\mu}{\beta}\right)}\right)}$$

CDF

$$F_{Y_n}(y) = e^{-e^{-\left(\frac{y-\mu}{\beta}\right)}}$$

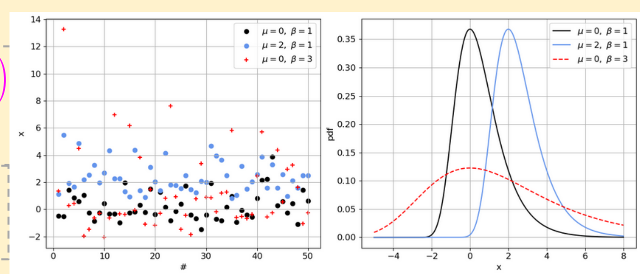

Fig. 5.14 Gumbel distribution function: (left) random samples, and (right) PDF.

Negative tail

PDF

$$f_{Y_1}(y) = \frac{1}{\beta} e^{\left(\frac{y-\mu}{\beta} - e^{\left(\frac{y-\mu}{\beta}\right)}\right)}$$

CDF

$$F_{Y_1}(y) = e^{-e^{\left(\frac{y-\mu}{\beta}\right)}}$$

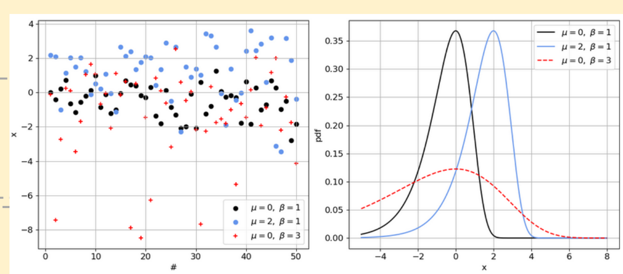

Fig. 5.16 Left-tailed Gumbel distribution function: (left) random samples, and (right) PDF.

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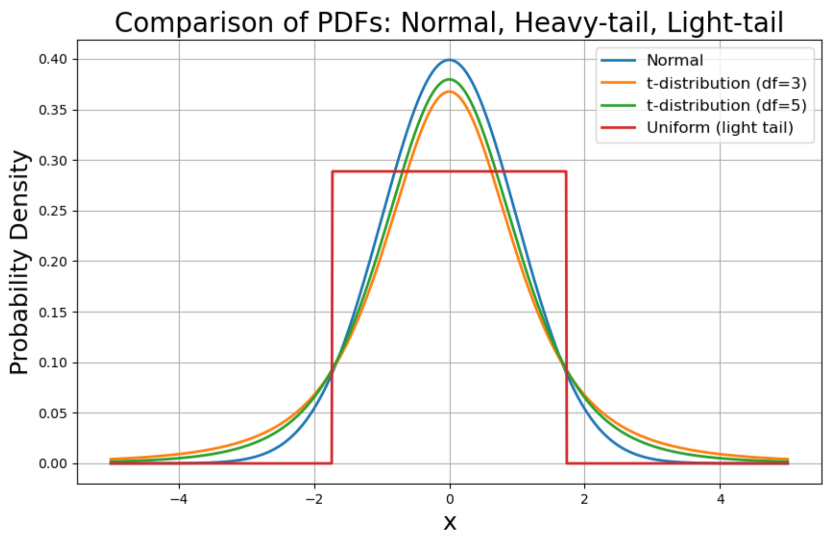
Core distributions

- *Tails*: thick/light



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Next

B.1.3 Propagation laws & least-squares (Q1C3)

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Questions, comments, or concerns?

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