CV 510$^{0}_{1}$
Modeling, Uncertainty, and Data for Engineers
(July – Nov 2025)

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Module Overview

Extremes & Risk Analysis

B.3

Extreme value
- Gumbel dist. (Q1C5.3)
- GEV dist. (generalized)
- Return period
- Design life
- Prob. of failure

Q2C7

B.4

Probabilistic risk analysis
- Hazard function
- Fragility function
- Risk assessment

(Not from teach book)

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CLT and CI: (estimating mean)

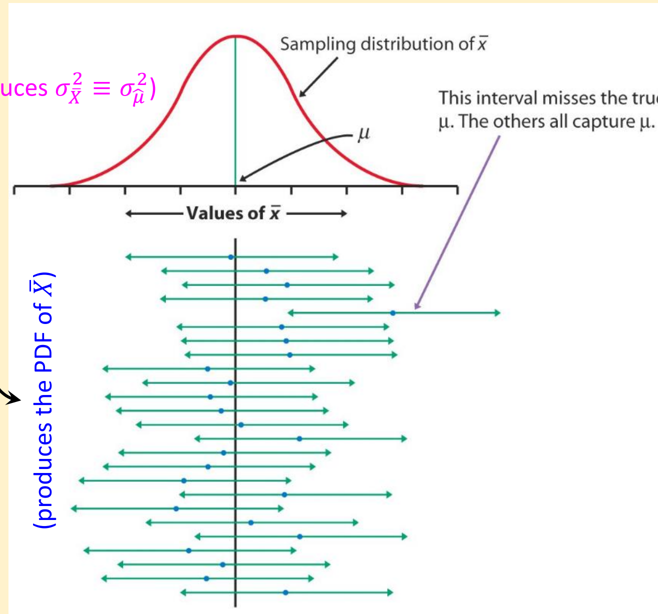
(reduces $\sigma_{\bar{X}}^2 \equiv \sigma_{\hat{\mu}}^2$)

Number of specimen in a sample (used for averaging)

vs.

Number of reps

(produces the PDF of $\bar{X}$)



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Flow

- Extreme value analysis
- Return period
- Design life
- Prob. of failure

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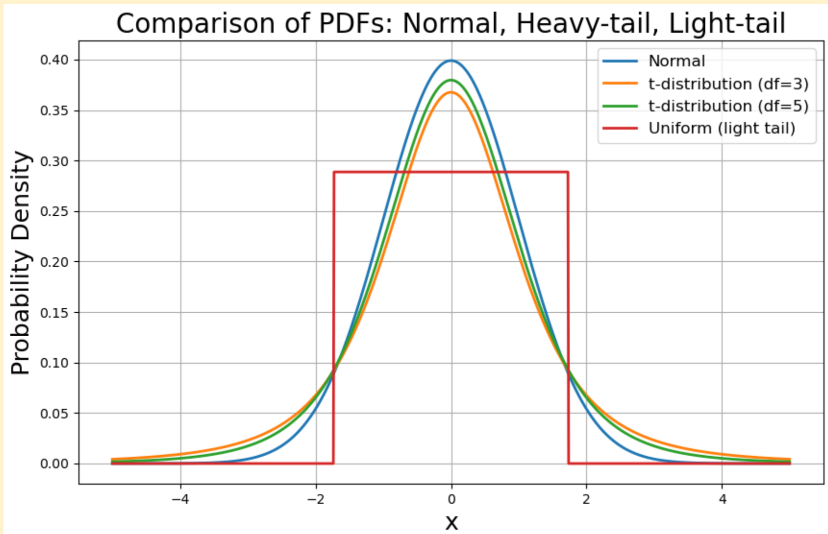
Tail of distributions

- *Excess Kurtosis:* $E[(X-\mu)^4]/\sigma^4 - 3$



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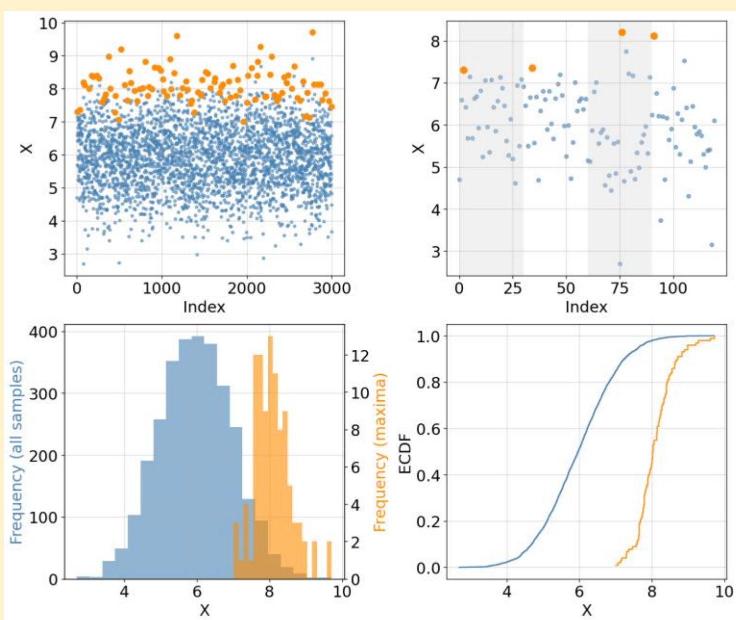
Extreme value analysis



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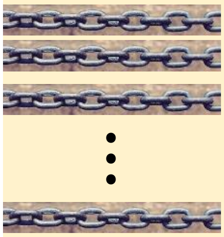
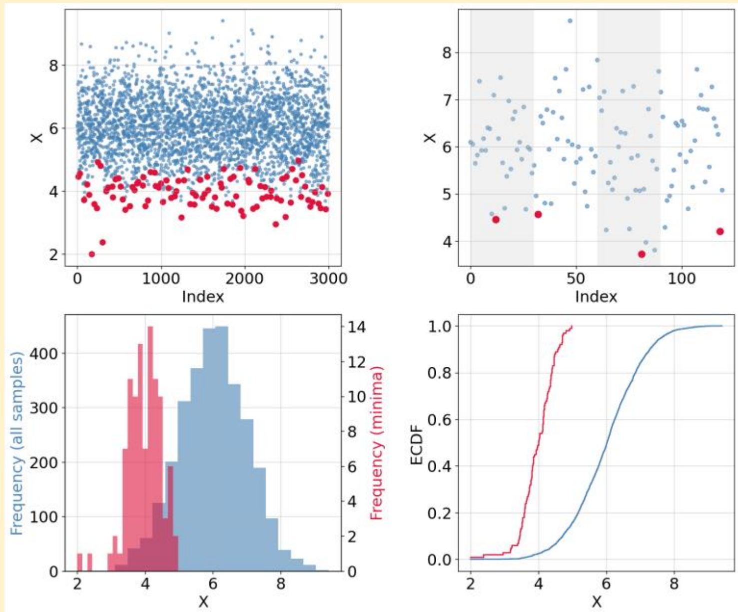
Extreme value analysis





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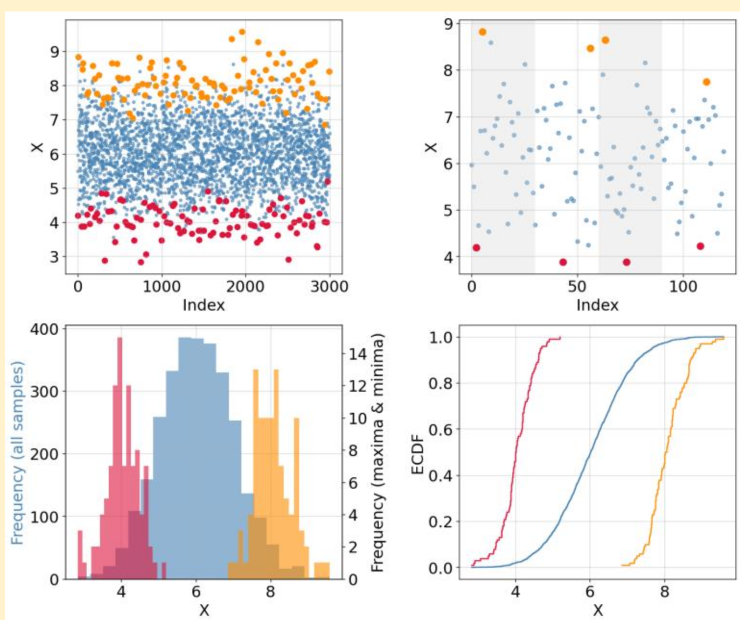
Extreme value analysis



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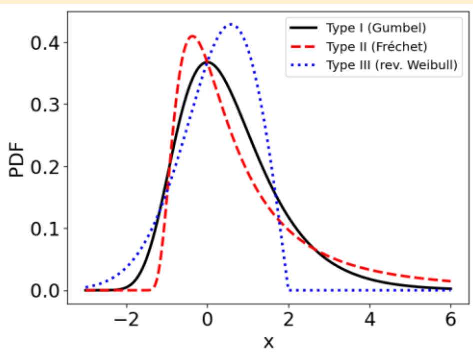
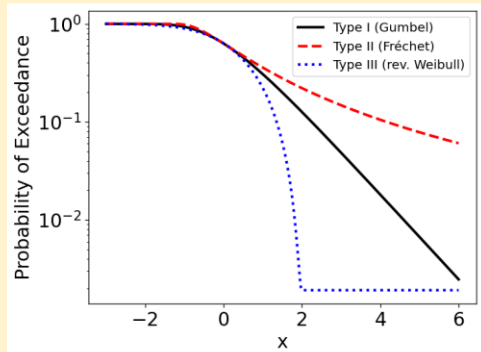
Selection of GEV distribution type

Tail type	Extreme value type	Parent distribution
Medium-/baseline tailed	Gumbel	Normal
Heavy-/fat-tailed	Fréchet	t-distribution
Light-/thin-tailed	Reversed Weibull	Uniform, beta





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Extreme value analysis

- Magnitude of extremes

Generalized extreme value (GEV) distributions
- Smallest/largest values?
- Look out for the tails? Thin/thick? Bounded?
- Estimate excess Kurtosis
- Pick one of the GEV models

- Frequency of extremes

Poisson process
- Count of extremes → Estimate event rate (λ)
- Independent or clustered?
  - Proceed with Poisson models
  - Decluster using peak-over-threshold/block maxima

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Return period

- Four key concepts
  - Prob. of exceedance in 1 year: $p_{f,1y}$
  - Return period: $T_r = 1/p_{f,1y}$
  - Design life: T_d, lifetime of an infrastructure
  - Prob. of exceedance in T_d: $p_{f,Td}$
- Poisson distribution

Probability of observing x extreme events in T period,

$\Pr(x, \lambda, T) = \frac{(\lambda T)^x e^{-\lambda T}}{x!}$ where $x = 0,1,2, \ldots$ and $\lambda > 0$

$\mathrm{E}[x] = \mathrm{Var}[x] = \lambda T$

Assumptions: Independent trials; large number of trials; small p

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Return period

- A design event has 10% probability of exceedance in 50 years ($p_{f,50y} = 0.10$). What is the annual expected number of events λ and the return period T_r in years?

$$\Pr(x, \lambda, T) = \frac{(\lambda T)^x e^{-\lambda T}}{x!}$$

$$\Pr(\text{exceedance}) = 1 - \Pr(\text{Non} - \text{exceedance})$$

$$\Pr(\text{exceedance in } T \text{ years}) = 1 - e^{-\lambda T}$$

$$p = 1 - e^{-\lambda T} \Rightarrow \lambda = -\frac{\ln(1-p)}{T}$$

$\Rightarrow \lambda = -\frac{\ln(1-0.1)}{50} \approx 0.00211$ per year

Return period, $T_r = \frac{1}{\lambda} \approx 475$ years

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Return period

- Given a design life of $T_d = 50$ years and a design event with annual probability ($p_{f,1y}$ =1/475), what is the probability of failure during the design life, $p_{f,Td}$?

$$\Pr(x, \lambda, T) = \frac{(\lambda T)^x e^{-\lambda T}}{x!}$$

$$\Pr(\text{failure in } T_d) = 1 - \Pr(\text{Nonfailure in } T_d)$$

$$\Pr(\text{failure in } T_d) = 1 - \left(1 - p_{f,1y}\right)^{T_d}$$

$$\Rightarrow p_{f,Td} = 1 - \left(1 - \frac{1}{475}\right)^{T_d} \approx 10\%$$

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Return period

- What is the probability of a 475-year return period event occurring at least once in 475 years?

$$\Pr(x, \lambda, T) = \frac{(\lambda T)^x e^{-\lambda T}}{x!}$$

$$\lambda = \frac{1}{475}; T = 475 \text{ years}$$

$$\Pr(\text{no occurrence}) = \Pr(x = 0) = e^{-1} \approx 0.368$$

$$\Pr(\text{at least once}) = 1 - \Pr(x = 0) \approx 0.632 = 63.2\%$$

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Questions, comments, or concerns?

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