CV 510$^{0}_{1}$
Modeling, Uncertainty, and Data for Engineers
(July – Nov 2025)

Dr. Prakash S Badal

1

Module Overview

B.1

Uncertainty & Estimation

B.1.1

Probabilistic basics and RVs
- Probability laws
- Random variables
- Discrete, continuous RVs
- PMF/PDF/CDF
- Expectation
- Variance/cov./corr.

Q1C5, Funda6-7-8

B.1.2

Core distributions
- Gaussian
- Uniform
- Exponential
- Lognormal
- Symmetric/skewness
- *Tails*: thick/light

Q1C5, Funda9-10

B.1.3

Propagation laws & least-sq.
- Uncertainty propagation
- Least-square errors
- Parameter estimation

Q1C2-C3, Funda9-10

2

Flow

- Musings on Assignment-5
- Least-square errors
- Parameter estimation

Example

- If $Y = \ln X$, what are expectation and variance of Y? Use Taylor series approximations.

$\mathrm{E}[Y] \cong \ln \mu_X$ first order mean approximation

$\mathrm{E}[Y] \cong \ln \mu_X - \frac{\delta_X^2}{2}$ second order mean approximation

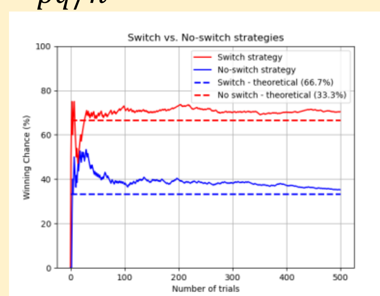
$\mathrm{Var}[Y] \cong \delta_X^2$ first order variance approximation

$\mathrm{E}[Y] \cong g(\mu_X) + \frac{1}{2}\left(\frac{\partial^2 g}{\partial x^2}\right)_{\mu_X} \sigma_X^2$ Second-order mean approximation

$\mathrm{Var}[Y] = \mathrm{E}[(Y-\mu_Y)^2] \cong \left(\left(\frac{\partial g}{\partial x}\right)_{\mu_X}\right)^2 \sigma_X^2$ First-order var. approx.

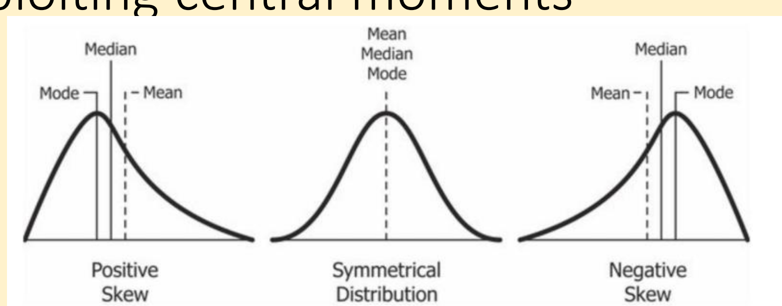
Monty Hall

- Repeated trials
  - "right action" only if taken repeatedly yields the "right outcome"
  - luck (transient-state) versus probability (stable-state)
  - n increases → variance reduces, $\sigma_n^2 = \sigma_1^2/n$ (prove!)
  - PMF of Bernoulli, $p(X) = \begin{cases} p, & \text{when } x = 1 \\ q = 1 - p, & \text{when } x = 0 \end{cases}$.
    - $\mathrm{E}[X] = p$, $\mathrm{Var}[X] = pq$
  - Repeated Bernoulli trials, $\sigma_{bernoulli,n}^2 = pq/n$
- Sampling always works!



5

Exploiting central moments



Right skew (skewness > 0)

Left skew (skewness < 0)

Central moments:

First:	$\mathrm{E}[(X-\mu)]$	zero
Second:	$\mathrm{E}[(X-\mu)^2]$	Variance
Third:	$\mathrm{E}[(X-\mu)^3]$	scaled Skewness (divide by σ^3 to get Skewness)
Fourth:	$\mathrm{E}[(X-\mu)^4]$	scaled Kurtosis (divide by σ^4 to get Kurtosis)

$$skewness = E\left[\left(\frac{X-\mu}{\sigma}\right)^3\right] = \frac{E[X^3] - 3\mu\sigma^2 - \mu^3}{\sigma^3}$$

6

6

Exploiting central moments

S. No.	Distribution	Skewness	Kurtosis (excess)	#param.
1	Normal $\mathcal{N}(\mu, \sigma)$	0	3 (0)	2
2	Uniform $[a, b]$	0	1.8 (-1.2)	2
3	Exponential $\mathrm{Exp}(\lambda)$	2	9 (+6)	1
4	Lognormal $\mathcal{LN}(\lambda, \zeta)$	Bad-looking fun of ζ	Bad-looking fun of ζ (above -3)	2
5	Gumbel type-1 (largest)	≈ 1.14	5.4 (+2.4)	2
6	Gumbel type-2 (smallest)	≈ -1.14	5.4 (+2.4)	2
7	t-distribution	0	$\frac{3(n-2)}{n-4}$; excess of $\frac{6}{n-4}$	2, #dof
8	Weibull	Param-dependent	Param-dependent	3
9	Beta	Param-dependent	Param-dependent	4 (2 shape, loc, scale)

7

Linear models: intro

- Linear function relationship

For a linear functional relationship,

$$\mathrm{E}[Y \mid X = x] = Ax$$

A: deterministic constant

In other words,

$$Y = Ax + \epsilon$$

such that

$$\mathrm{E}[\epsilon] = 0$$

$\varepsilon \equiv$ random error: zero mean, finite variance

In a deterministic linear model, x is given:

$$\sigma_Y = \sigma_\epsilon$$

8

8

Linear models: example

- Linear function relationship

$$E[Y_i] = x_1 + x_2 t_i$$

t_i: observation time; epoch; given; deterministic

x_1: intercept
x_2: rate of change

Linear functional model is

$$E\left(\begin{bmatrix} Y_1 \\ Y_2 \\ \vdots \\ Y_n \end{bmatrix}\right) = \underbrace{\begin{bmatrix} 1 & t_1 \\ 1 & t_2 \\ \vdots & \vdots \\ 1 & t_n \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}}_{x}$$

9

9

Linear models: example

- Is this a linear model

$$E[Y_i] = x_1 + x_2 t_i + x_3 t_i^2$$

Write its linear functional relationship

$$E\left(\begin{bmatrix} Y_1 \\ Y_2 \\ \vdots \\ Y_n \end{bmatrix}\right) = \underbrace{\begin{bmatrix} 1 & t_1 & t_1^2 \\ 1 & t_2 & t_2^2 \\ \vdots & \vdots & \vdots \\ 1 & t_n & t_n^2 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}}_{x}$$

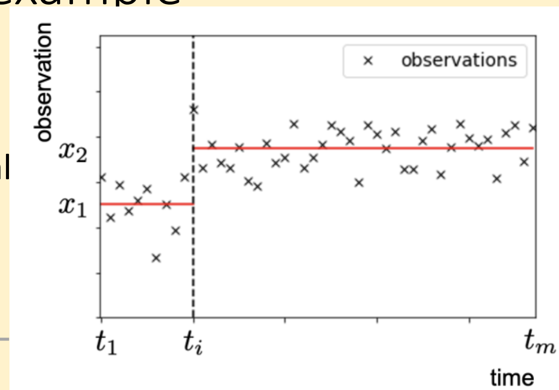
A linear model in x

10

10

Linear models: example

- Is this a linear model



Write its linear functional relationship

$$E\left(\begin{bmatrix} Y_1 \\ \vdots \\ Y_{i-1} \\ Y_i \\ \vdots \\ Y_n \end{bmatrix}\right) = \underbrace{\begin{bmatrix} 1 & 0 \\ \vdots & \vdots \\ 1 & 0 \\ 0 & 1 \\ \vdots & \vdots \\ 0 & 1 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}}_{x}$$

A linear model in x

11

11

Linear models: example

- Is this a linear model

Step function, $s = x_2 - x_1$

Write its linear functional relationship

$$E\left(\begin{bmatrix} Y_1 \\ \vdots \\ Y_{i-1} \\ Y_i \\ \vdots \\ Y_n \end{bmatrix}\right) = \underbrace{\begin{bmatrix} 1 & 0 \\ \vdots & \vdots \\ 1 & 0 \\ 1 & 1 \\ \vdots & \vdots \\ 1 & 1 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} x_1 \\ s \end{bmatrix}}_{x}$$

A linear model in x

12

12

Questions, comments, or concerns?

15

15