

CV 510₁

Modeling, Uncertainty, and Data for Engineers

(July – Nov 2025)

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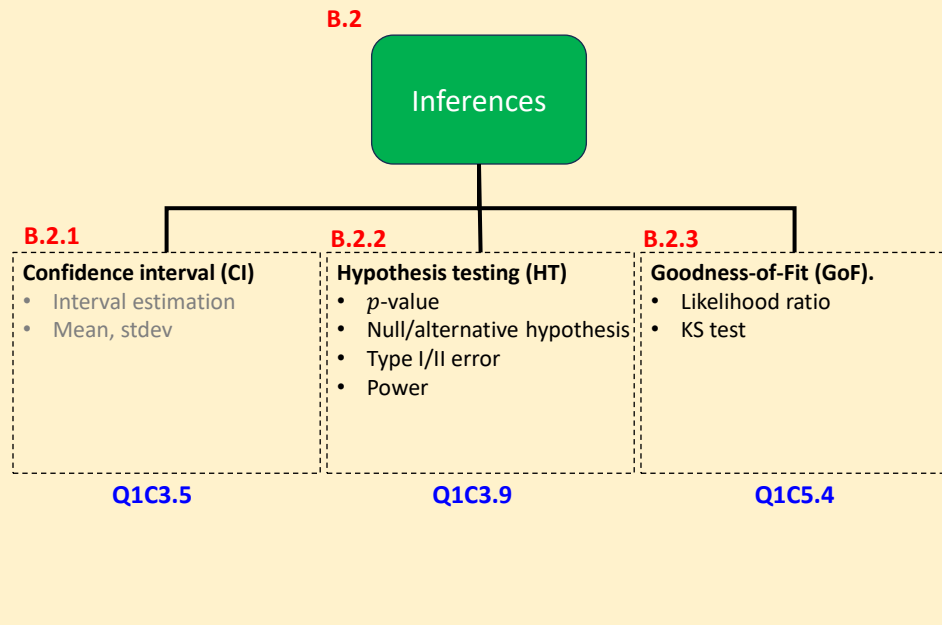
Parameter estimation

- Next few class:
 - Estimation: MoM, MLE
 - Inference: Confidence interval (CI)
 - **Inference**: Hypothesis testing (HT)
 - **Goodness of fit** (GoF): χ^2 , KS

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Module Overview



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Flow

- CLT recap: demo
- Null/alternative hypothesis
- Type I/II error
- Power of test
- p -value

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Central limit theorem: recap

- For large sample size n , **sample mean** approaches

a **normal distribution**,

with **mean μ** (same as population mean), and

variance σ^2/n (less than population Var.).

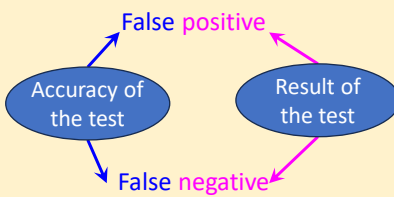
Spreadsheet demo!

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Spot the error*

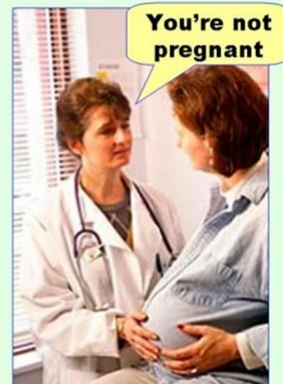
*: not an example from real life



Type I error
(false positive)



Type II error
(false negative)



People are generally not pregnant!

Default

You are not pregnant

H_0

Alternative

You are pregnant

H_1

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Justice system in a democracy

- How can someone be arrested if they really are **presumed innocent**?
- Why is a defendant pronounced “**Not Guilty**” instead of innocent?
- Why do citizens put up with a system that allows **criminals to go free on technicalities**?

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Common challenges in justice, research (life?)

- A **strong urge** to believe in unusual event/discovery
 - A charged person (aka “**defendant**”) is the **convict**
 - This new medicine **cures** Alzheimer’s/cancer
 - New concrete mixing yields **higher strength/durability**
- Huge repercussions of wrong decision

You acted on your belief, BUT The belief turned out to be FALSE	You did NOT act on your belief, and the belief turned out to be TRUE
Innocent being punished	guilty going free
Patients getting the wrong medicine (side effect/death?)	Patients suffering/dying, despite a discovered medicine
Buildings constructed using inferior material	Wasted resources, unsustainable

- **No sure-shot way** to prove the unusual event
- **Limited data**

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Hypothesis testing is the solution!

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Hypothesis testing (& justice system)

No numerical values in courts, but they share four common features:

1. **The alternative hypothesis:** This is why a *criminal is arrested*.
 - The police, of course, do not think that the criminal is innocent.
 - The researchers think that their treatment is effective. H_1 or H_A .
2. **The null hypothesis:** The *presumption of innocence*.
 - The suspect or treatment didn't do anything. H_0 is the logical opposite of H_1 .
3. **A standard of justice:** A *reasonable doubt*. A test score!
 - No possibility of absolute proof. So, a standard has to be set.
 - Reject the null hypothesis beyond a reasonable doubt.
4. **A data sample:** Evaluation of *partial information*.
 - Eye-witnesses/fingerprints/DNA analysis/experimental/numerical data of treatment.
 - Getting the "whole truth and nothing but the truth" is often impossible.

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Hypothesis testing (& justice system)

Both statistical testing and the justice system:

1. Concentrate on rejecting the null hypothesis

- It's much easier
- Rejection of presumption of innocence \equiv defendant is pronounced guilty

2. Consider a failure to reject the null hypothesis

- As "Not guilty" verdict.
- "medicine does not treat cancer/concrete is not stronger".
- Proving H_0 (the null hypothesis of innocence) will take endless evidence.

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Hypothesis testing (& justice system)

Neither statistical testing and the justice system are perfect:

- Sometimes, the jury makes an error.
 - An innocent person goes to jail
 - Statisticians call it a Type I error
- Sometimes, a guilty person is set free
 - Statisticians call it a Type II error

Which one is worse?

- Citizens find Type II error disturbing but not as horrifying as Type I errors.

$$\text{Type I error} \equiv \left\{ \begin{array}{c} \text{an innocent person goes to jail} \\ + \\ \text{a guilty person walks free} \end{array} \right\}$$

- In a sense, a Type I error is twice as bad as a Type II error

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Errors in Hypothesis testing (& justice system)

- An innocent person goes to jail (Type I error)
- A guilty person is set free (Type II error)

$$\text{Type I error} \equiv \left\{ \begin{array}{c} \text{an innocent person goes to jail} \\ + \\ \text{a guilty person walks free} \end{array} \right\}$$

HT (& justice system) puts a lot of emphasis on avoiding Type I error.

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Errors in Hypothesis testing (& justice system)

HT & justice lady put a lot of emphasis on avoiding Type I error.

Product example:

- Null hypothesis, H_0 : a product satisfies customer requirements.
- If H_0 is rejected, do not sell the product to customers.

Type I error: Rejecting a good batch by mistake. A very expensive error.

Type II error: Failing to reject a bad batch and shipping to customers (losing customer and tarnishing company's image)

Which one is worse? Type II error

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Questions, comments,
or concerns?

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