CV 510$^{0}_{1}$
Modeling, Uncertainty, and Data for Engineers
(July – Nov 2025)

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Module Overview

Extremes
&
Risk Analysis

B.3

Extreme value
- Gumbel dist. (Q1C5.3)
- GEV dist. (generalized)
- Return period
- Design life
- Prob. of failure

Q2C7

B.4

Probabilistic risk analysis
- Hazard function
- Fragility function
- Risk assessment
- Prob. of failure

(Not from teach book)

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Flow

- Risk, reliability, resilience
- Typical probabilities
- Formalizing monkey attack
- Probabilistic monkey attack risk
- Probabilistic seismic risk

Risk, reliability, resilience

- p_f: probability of failure
- Risk: expected loss
- Reliability: Pr(system performs its function without failure)

$$\text{Reliability} = 1 - p_f$$

- Resilience: Ability of a system to withstand, recover, and restore functionality after disruption

Typical Probabilities

- Some target probabilities are:
  - Locally dramatic but rather harmless in a wider area
    e.g., building collapse
    $\sim\mathbf{10^{-4}}$ for the lifetime of the facility
  - More dramatic failure consequences, such as nuclear power plants
    $\leq \mathbf{10}^{-5}$/year core meltdown probability (Japan)
    $\leq \mathbf{10}^{-6}$/year large release probability Chernobyl 1986; Fukushima 2011
  - Drawing an ace of spade in a randomized deck of cards
    1/52 = $19 \cdot 10^{-3}$
  - Rule of thumb,
    Probability of death/person/year > 10^{-3} $\Rightarrow$ Immediate Action
    Probability of death/person/year < 10^{-6} $\Rightarrow$ events so unusual that little can reasonably be done

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Typical Probabilities

- Rule of thumb, Probability of death/person/year
  - > 10^{-3} $\Rightarrow$ Immediate Action
  - < 10^{-6} $\Rightarrow$ events so unusual that little can reasonably be done
- Examples:
  - Worker in unregulated mine in early 20th century
    Often > 10^{-2}/worker/year
  - Riding a motorbike without a helmet (in low-income countries)
    $\sim 10^{-4}$/person/hour
  - Riding a motorbike with a helmet (in low-income countries)
    $\sim 10^{-5}$/person/hour
  - Structural collapse of old and ill-maintained buildings $\sim 10^{-3}$
  - Commercial plane 1.2×10^{-8} /person-hour => 24×10^{-6}
  - Car 0.7×10^{-8} /person-hour => 200×10^{-6}
    (accounting for average exposure)

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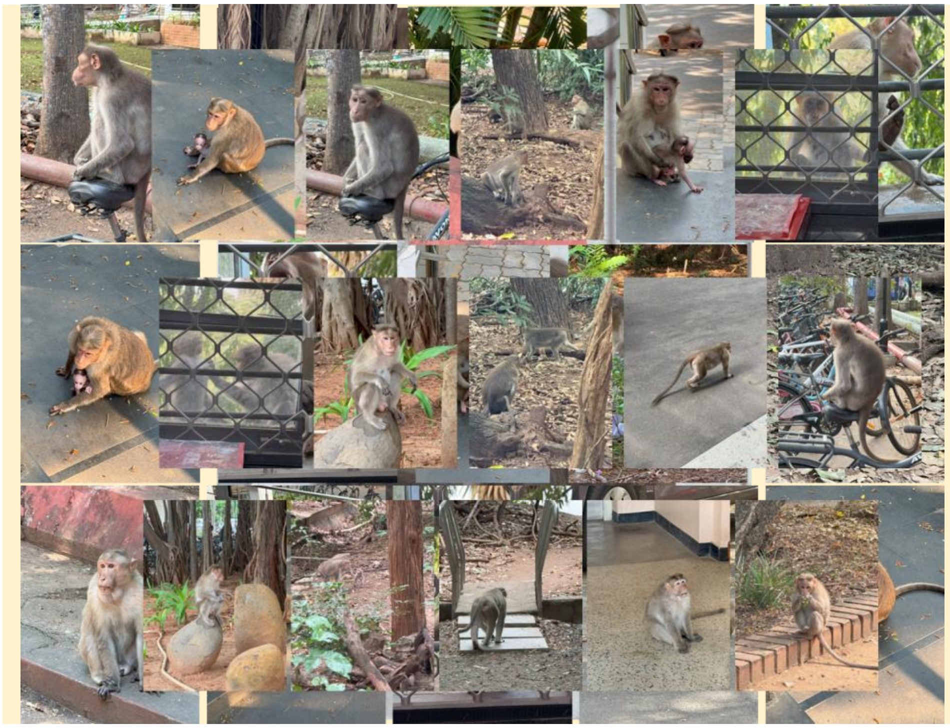

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Risk from monkey attack on campus

- The risk from the monkey hazard involves:

- Person's familiarity (or lack thereof) with the hazard (immeasurable; Not useful)
- Severity of attack: How bad was the attack? (intensity measure)

  charged scratched bitten mauled

Let's call it "**Monkey Attack Scale,** $\boldsymbol{MAS}$".

- Rate of monkey attack: How often? (hazard rate)
- Damage to person: Time in the hospital? (damage measure)

None	Hours	days	week-or-more
none	minor	major	severe

- Consequence of damage: \$, downtime? (consequence function)

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Probabilistic monkey risk assessment

- Three components

Given: location & person ($\mathcal{L}$ and $\mathcal{P}$)

- Monkey hazard, $\mathcal{H}_{MAS} \coloneqq \Pr(MAS = mas|\mathcal{L})$
  - Occurrence rate, $g(MAS|\mathcal{L})$ Depends on location
- Person's fragility, $\mathcal{F}_{DM,\mathcal{P}}(mas) \coloneqq \Pr(DM = dm|mas, \mathcal{P}, \mathcal{L})$
  - Probability density, $p(DM|MAS, \mathcal{P}, \mathcal{L})$ Depends on person (age, weight)
- Consequence of damage, e.g., downtime, cost:
  $\mathcal{C}_{DT}(dm) \coloneqq \Pr(DT = dt|dm)$ or $\mathcal{C}_{COST}(dm) \coloneqq \Pr(COST = cost|dm)$
  - Probability density, $p(DV|DM, \mathcal{P}, \mathcal{L})$

$$g[DV|\mathcal{L}, \mathcal{P}] = \iint p(DV|DM, \mathcal{P}, \mathcal{L})\, p(DM|MAS, \mathcal{P}, \mathcal{L}) g(MAS|\mathcal{L}) \mathrm{d}MAS\, \mathrm{d}DM$$

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Probabilistic seismic risk assessment

- Three components

Given: location & building design ($\mathcal{L}$ and $\mathcal{D}$)

- Seismic hazard, $\mathcal{H}_{sa} \coloneqq k_0 s_a^{-k}$
  - Occurrence rate, $g(S_a|\mathcal{L})$ Depends on location/seismicity
- Building's fragility, drift demand, $\mathcal{F}_{col,\mathcal{D}}(s_a) \coloneqq \Pr(col|s_a, \mathcal{D})$
  - Probability density, $p(DM|S_a, \mathcal{L}, \mathcal{D})$ Depends on building (age, weight)
- Consequence of damage, e.g., downtime, repair cost ratio:
  $\mathcal{C}_{DT}(dm) \coloneqq \Pr(DT = dt|dm)$ or $\mathcal{C}_{RCR}(dm) \coloneqq \Pr(RCR = rcr|dm)$
  - Probability density, $p(DV|DM, \mathcal{L}, \mathcal{D})$

$$g[DV|\mathcal{L}, \mathcal{D}] = \iint p(DV|DM, \mathcal{L}, \mathcal{D})\, p(DM|IM, \mathcal{L}, \mathcal{D}) g(IM|\mathcal{L}) \mathrm{d}IM\, \mathrm{d}DM$$

$$g[DV|\mathcal{L}, \mathcal{D}] = \iiint p(DV|DM, \mathcal{L}, \mathcal{D}) p(DM|EDP, \mathcal{L}, \mathcal{D}) p(EDP|IM, \mathcal{L}, \mathcal{D}) g(IM|\mathcal{L}) \mathrm{d}IM\, \mathrm{d}DM$$

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Questions, comments, or concerns?

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