Department of Civil Engineering, IIT Madras
CV 5100 (Modeling, Uncertainty and Data for Engineers)

Sample Questions (1 Oct, 2025)

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1. **(Probability)** For two random variables X_1 and X_2, show that the covariance $\mathrm{Cov}[X_1, X_2] = \mathrm{E}[X_1 X_2] - \mathrm{E}[X_1]E[X_2]$. What are the maximum and minimum values that $\mathrm{Cov}[X_1, X_2]$ can take? [2+1]

2. **(Uncertainty propagation)** Suppose X_1 and X_2 are two random variables with means μ_1 and μ_2, respectively and standard deviations σ_1 and σ_2. If correlation coefficient between X_1 and X_2 is ρ_{12}. Derive an expression for variance of a random variable $Y = a_1 X_1 + a_2 X_2 + c$, where a_1, a_2, and c are deterministic constants. [2]

3. **(Approximation)** Let X be a random variable with mean $\mu_X = 3$ and standard deviation $\sigma_X = 4$. If $Y = 5 \ln X + \sin(2\pi X) - e^{-3X}$. Find (a) the second-order approximation of the mean of Y and (b) first-order approximation of the variance of Y. [2+2]

4. **(Moments)** Name first four moments of a random distribution. How are each of them used? [2]

5. **(Least-square)** For a linear model $y = ax^2 + bx + c$, if five observations for (x, y) are given as $(x_1, y_1), (x_2, y_2), (x_3, y_3), (x_4, y_4)$, and (x_5, y_5). Write the matrix form for least-square estimate of parameters $[a, b, c]^{\mathrm{T}}$. [2]

6. **(Normal table)** A machine produces rods with diameters normally distributed, mean = 10 mm, standard deviation = 0.2 mm. What proportion of rods are expected to be below 9.7 mm? [2]

7. **(Central limit theorem)** Prove that standard error of the mean of a sample of size n is inversely proportional to $\sqrt{n}$. [2]

8. **(Confidence interval)** For the population with standard deviation $\sigma = 10$, if the mean of a sample of size $n = 25$ is 52, construct a 95% confidence interval for the population mean. [2]

9. **(Hypothesis testing)** Define Type I error and Type II error in your own words. Which one is usually controlled more strictly? [2]

10. **(Hypothesis testing)** A city government wants to know if more than 25% of commuters would use a new bike-sharing system. They plan to survey a random sample of 500 commuters. If the survey shows strong evidence that the proportion of interested commuters is greater than 25%, they will allocate funds to build bike stations. Otherwise, they will not proceed with the program. (a) State the null and alternative hypotheses. (b) Explain what a Type I error would mean here. (c) Explain what a Type II error would mean here. [3]

11. **(Hypothesis testing)** A quality control engineer tests whether the mean length of bolts is 5 cm. She collects $n = 36$ samples and observes mean = 5.1, standard deviation = 0.3. Perform a hypothesis test at significance level of $\alpha = 0.05$, stating H_0 and H_1, Z-statistic, and conclusion. [2]

12. **(Hypothesis testing)** A researcher is testing whether adding a new chemical increases the mean yield strength of a steel alloy, known to have standard deviation $\sigma =$ 40 MPa. The current process has mean yield strength 500 MPa. They will take a sample of n = 16 specimens. The researcher uses a one-tailed test at significance level $\alpha = 0.05$.

(a) State the two hypotheses.
(b) Find the critical value of $\overline{X}$ above which H_0 will be rejected. Interpret what a Type I error means in this context.
(c) Suppose the true mean with the chemical is 550 MPa. Compute the probability of a Type II error under this alternative. Interpret what this probability means for the researcher.

[1+2+2]
