CV 510$^{0}_{1}$
Modeling, Uncertainty, and Data for Engineers

(July – Nov 2025)

Dr. Prakash S Badal

1

Flow

- Announcement
- Module Outline

2

2

Announcement

- This week's classes
  - Additional class on Friday (12th Sep at 8 AM)

- Go through the previously uploaded lectures before the lecture hour.

Check-in with teach-book

https://mude.citg.tudelft.nl/book/2024/

- Q1 Topics (Chapters 5, 6, 2, and 3)
  - Q1C5 Univariate continuous distribution
    - Q1C5.1 PDF/CDF
    - Q1C5.2 Empirical Distributions
    - Q1C5.3 Parametric Distributions
    - Q1C5.4 Fitting a Distribution
  - Q1C6 Multivariate Distributions (briefly)
  - Q1C2 Propagation of Uncertainty
  - Q1C3 Observation Theory: least-sq., Hyp. Test, Conf. Intervals
- Q2 Topics (Chapter 7 and 8)
  - Q2C7 Extreme value theory: GEV, return period, POT
  - Q2C8 Risk and decision making (CBA)
- Fundamental Concepts
  - Chapter 6, 7, 8, 9. Probability basics, rv, z- and t-tables
- Programming
  - Fundamental Concepts → Chapter 10

Module Overview

B.1

Uncertainty & Estimation

B.1.1

Probabilistic basics and RVs
- Probability laws
- Random variables
- Discrete, continuous RVs
- PMF/PDF/CDF
- Expectation
- Variance/cov./corr.

B.1.2

Core distributions
- Gaussian
- Uniform
- Exponential
- Lognormal
- Symmetric/skewness
- *Tails*: thick/light

B.1.3

Propagation laws & least-sq.
- Uncertainty propagation
- Least-square errors
- Parameter estimation

5

B.1.2 Core distributions (Q1C5)

6

Random variable (rv): PMF, PDF

- For a discrete rv: *probability mass function* (PMF) is defined as:

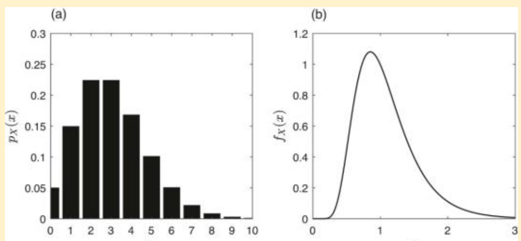
$$p_X(x) = \Pr(X = x)$$

PMF must satisfy: $0 \le p_X(x) \le 1$ and $\sum_x p_X(x) = 1$

- For a continuous rv: *probability density function* (PDF) is defined as:

$$f_X(x)\mathrm{d}x = \Pr(x < X \le x + \mathrm{d}x)$$

PDF must satisfy: $0 \le f_X(x)$ and $\int_{-\infty}^{\infty} f_X(x)\,\mathrm{d}x = 1$



© ADK22

7

7

Random variable (rv): CDF

- *Cumulative distribution function* (CDF) is defined as:

$$F_X(x) = \Pr(X \le x)$$

Probability that the rv X has a value less than x

- The CDF of a discrete RV X is

$$F_X(x) = \Pr(X \le x) = \sum_{x \le x_i} p_X(x_i)$$

- The CDF of a continuous RV X is

$$F_X(x) = \Pr(X \le x) = \int_{-\infty}^{x} f_X(x)\mathrm{d}x$$

Derivative of $F_X(x)$ for continuous rv:

$$f_X(x) = \frac{\mathrm{d}F_X(x)}{\mathrm{d}x}$$

8

8

Gaussian

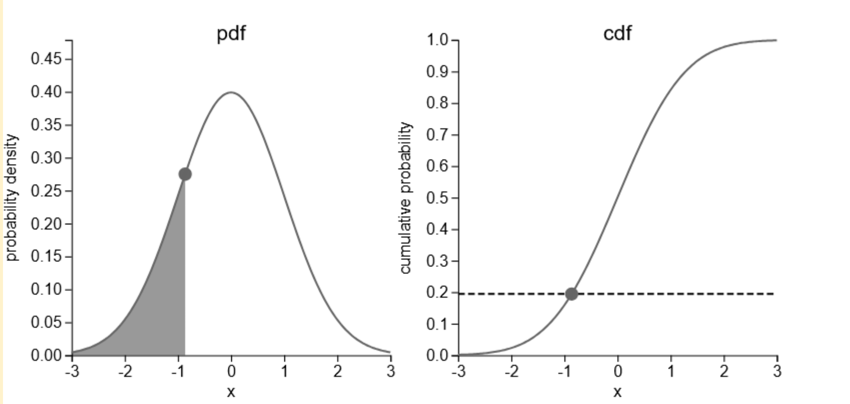


Fig. 5.1 Interactively visualize the relationship between the PDF and the CDF (the integral of the PDF value). #

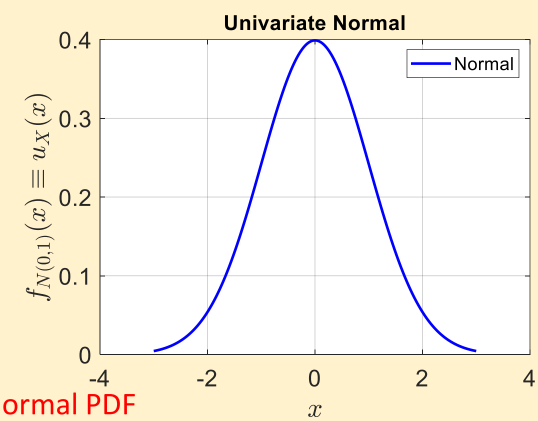
Teach-book Q1-C5 **Fig. 5.1** (interactive)

9

9

Gaussian

- Gaussian



Univariate normal PDF

$$N(\mu,\sigma) = f_X(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2\right]; \qquad u(x) \equiv \varphi(x) = \frac{1}{\sqrt{2\pi}} \exp\left[-\frac{x^2}{2}\right]$$

No closed-form equation for Normal CDF

10

10

Gaussian

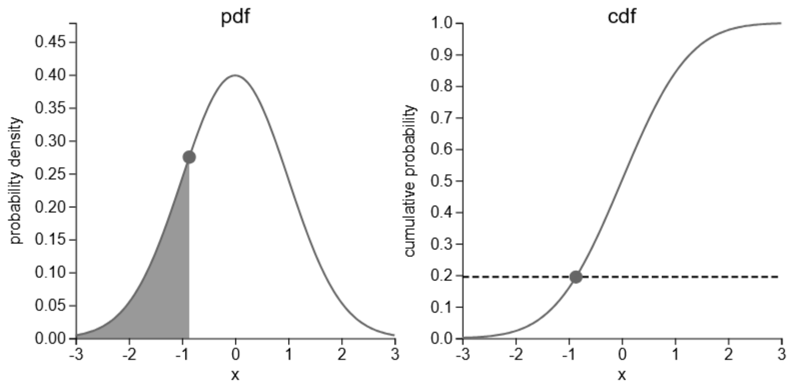


Fig. 5.1 Interactively visualize the relationship between the PDF and the CDF (the integral of the PDF value). #

Teach-book Q1-C5 **Fig. 5.1** (interactive)

Standard Normal Probabilities

Z-table

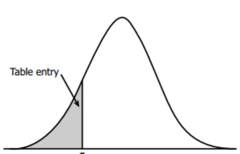


Table entry for z is the area under the standard normal curve to the left of z.

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
−3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002
−3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0003
−3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005
−3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
−3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
−2.9	.0019	.0018	.0018	.0017	.0016	.0016	.0015	.0015	.0014	.0014
−2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
−2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
−2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
−2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
−2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
−2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
−2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
−2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
−2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
−1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
−1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
−1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
−1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
−1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
−1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681
−1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
−1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
−1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
−1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
−0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
−0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
−0.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
−0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
−0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
−0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
−0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
−0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
−0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
−0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641

Gaussian

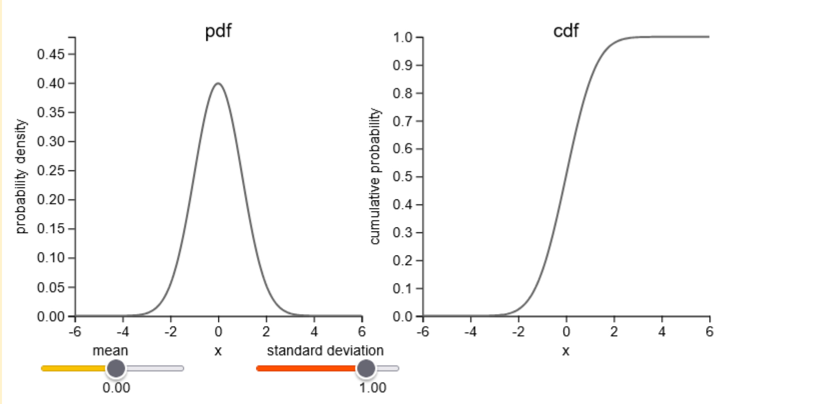


Fig. 5.3 Interactively change the mean and standard deviation of the Gaussian distribution to visualize the effect on the PDF and CDF.

Scale and location parameters
Teach-book Q1-C5 **Fig. 5.3** (interactive)

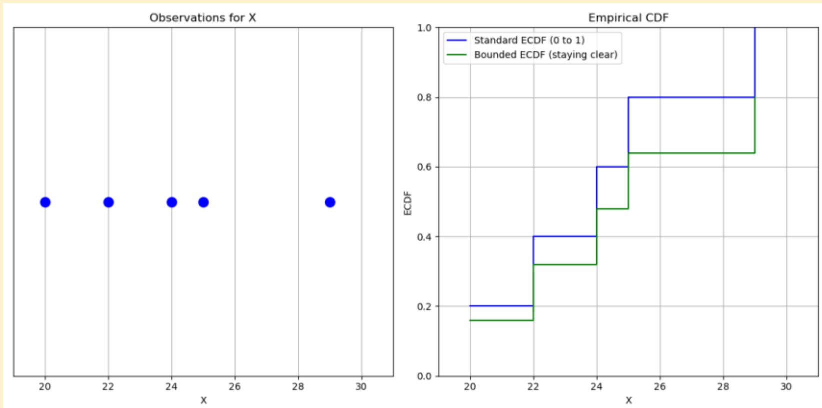
Inverse CDF

- For designing a structure, we often need

a value that is not exceeded with more than p probability:

$$x = F^{-1}(p)$$

Empirical distribution



Standard ECDF: $F_n(x) = \frac{i}{n}$ goes from 0 to 1

Bounded ECDF: $F_{n,b}(x) = \frac{i}{n+1}$ stays clear of 0 and 1

useful in Q-Q/probability plotting, avoids $-\infty$ or $+\infty$.

15

15

scipy

- Tutorial (not reference) https://docs.scipy.org/doc/scipy/tutorial/stats/probability_distributions.html
- https://docs.scipy.org/doc/scipy/tutorial/stats.html
  - Run examples

- Common Methods
- Random number generator
- Shifting-scaling: loc, scale
- Fitting distributions

16

16

Uniform Distribution

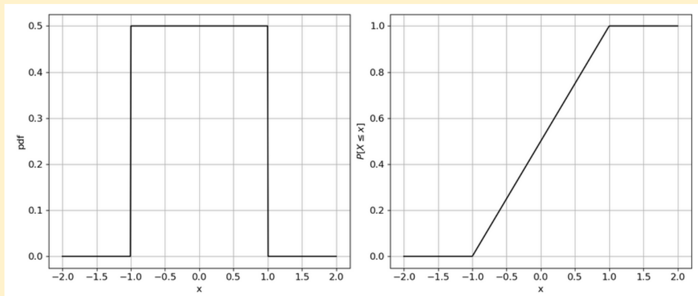


Fig. 5.11 Uniform distribution function: (left) PDF, and (right) CDF.

PDF

$$f_X(x) = \begin{cases} \frac{1}{b-a}, & a \le x \le b \\ 0 & \text{otherwise} \end{cases}$$

CDF

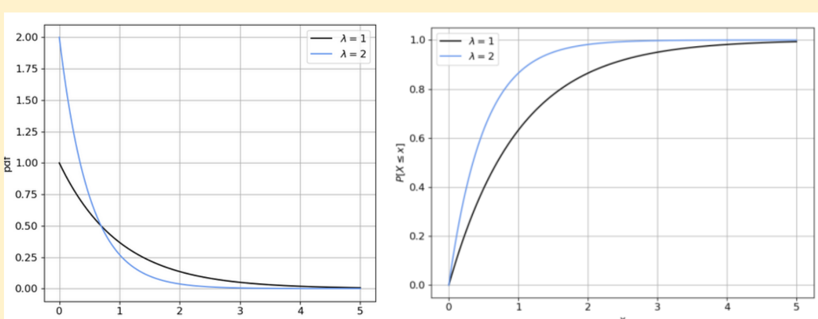
$$F_X(x) = \begin{cases} 0 & \text{for } x < a \\ \frac{x-a}{b-a} & \text{for } a \le x \le b \\ 1 & \text{for } x > b \end{cases}$$

Properties $\mathrm{E}[X] = \frac{1}{2}(a+b); \quad \mathrm{Var}[X] = \frac{1}{12}(b-a)^2$

17

17

Exponential Distribution



PDF

$$f_X(x) = \begin{cases} \lambda e^{-\lambda x}, & x \ge 0, \lambda > 0 \\ 0 & \text{otherwise} \end{cases}$$

CDF

$$F_X(x) = 1 - e^{\lambda x}, \qquad x > 0$$

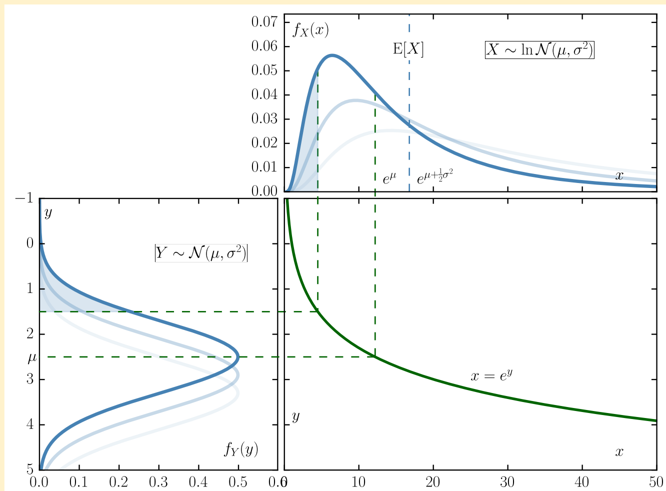
Properties $\mathrm{E}[X] = \frac{1}{\lambda}; \quad \mathrm{Var}[X] = \frac{1}{\lambda^2}$

18

18

Lognormal

- $X \sim Lognormal \Leftrightarrow Y = \ln(X) \sim Normal$

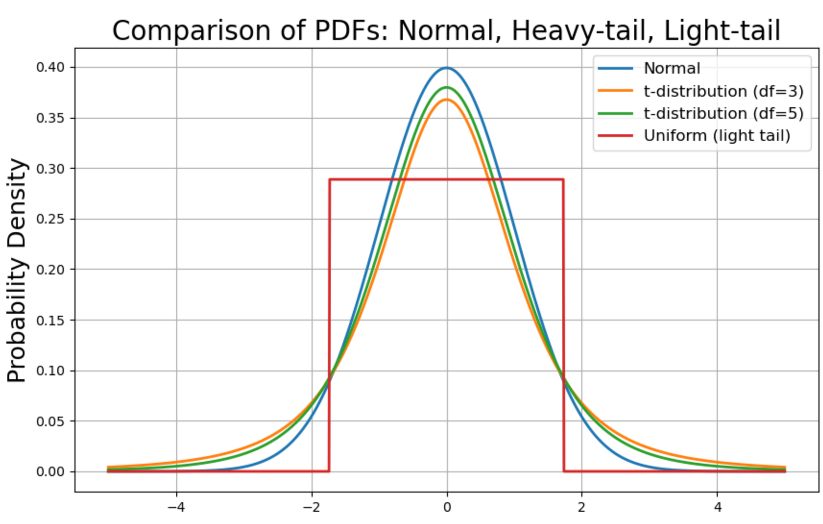


19

19

Core distributions

- *Tails*: thick/light



20

Questions, comments, or concerns?

22