



# PANIMALAR ENGINEERING COLLEGE

(An Autonomous Institution, Affiliated to Anna University Chennai)

## QUESTION BANK

### Details of the Course

Name of the Department	: MATHEMATICS
Name of the Course	: COMPLEX VARIABLES AND TRANSFORMS
Course Code	: 23MA1206
Semester	: II
Common To Programme(s)	: CSE & IT

#### Instructions

**Blooms Level:** Blooms Level 1 & 2 is Lower Order (LO) Cognitive type, Blooms Level 3 & 4 is Intermediate Order Cognitive Type (IO) and Blooms Level 5 & 6 is Higher Order (HO) cognitive type.

**2 Marks:** For each unit five questions should be of lower order (LO) cognitive type and five Questions should be of Intermediate order (IO) cognitive type.

**13 /15 /16 Marks:** For each Unit four questions should be of lower order (LO) cognitive type i.e. remembrance type questions, five should be of intermediate order (IO) cognitive type i.e. understanding type questions and One Question should be on Higher Order (HO) Application / Design / Analysis / Evaluation / Creativity / Case study questions.

\* HO Order is not applicable if the Question Pattern does not have Part C. In Such cases consider HO as IO.

\*\* If the Mark for Part B & C is less than the maximum mark of the Question, Sub Divisions shall be added.

#### Course Outcome: (List the Course Outcomes of the Course)

**CO1:** Gradient, divergence and curl of a vector point function and related identities. Evaluation of line, surface and volume integrals using Gauss, Stokes and Green's theorems and their verification.

**CO2:** Understanding analytic functions, harmonic functions, conformal mapping.

**CO3:** Determine the types of singularities, residues, contour integration.

**CO4:** Determine the Fourier transforms for a function and evaluates special integrals.

**CO5:** Solve differential equations using laplace transforms.

**Bloom's Level:** BL1 - Remembering, BL2 - Understanding, BL3 - Applying, BL4 - Analyzing, BL5- Evaluating, BL6 - Creating.

### Diagrams, Table Values, Equations must be legible and clear.

UNIT- I – VECTOR CALCULUS				
PART A ( 2 Marks)		Bloom's Level	Course Outcome	Marks Allotted
1.	Find the directional derivative of $\phi = x^2yz + 4xz^2$ at $(1, -2, -1)$ in the direction of $2\vec{i} - \vec{j} - 2\vec{k}$ .	[BL1]	[CO1]	[2]
2.	Find a unit normal to the surface $xy = z^2$ at the point $(1, 1, -1)$ .	[BL2]	[CO1]	[2]
3.	Find $\nabla(r^n)$ .	[BL1]	[CO1]	[2]
4.	Find (i) $\nabla \cdot \vec{r}$ (ii) $\nabla \times \vec{r}$	[BL2]	[CO1]	[2]
5.	Prove that the vector $\vec{F} = z\vec{i} + x\vec{j} + y\vec{k}$ is solenoidal.	[BL2]	[CO1]	[2]

Couse Instructor  
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Assistant Professor

Course Coordinator  
Mr. N. Yathavan  
Assistant Professor(G-I)

Head of the Department  
Dr. G. Nagarajan  
Professor

6.	Prove that the vector $\vec{F} = yz\vec{i} + zx\vec{j} + xy\vec{k}$ is irrotational.	[BL1]	[CO1]	[2]
7.	Find 'a' such that $(3x - 2y + z)\vec{i} + (4x + ay - z)\vec{j} + (x - y + 2z)\vec{k}$ is solenoidal.	[BL3]	[CO1]	[2]
8.	Find the constants a,b,c so that $\vec{F} = (x + 2y + az)\vec{i} + (bx - 3y - z)\vec{j} + (4x + cy + 2z)\vec{k}$ is irrotational.	[BL3]	[CO1]	[2]
9.	If $\vec{F} = x^3\vec{i} + y^3\vec{j} + z^3\vec{k}$ then find $\text{div}(\text{curl } \vec{F})$ .	[BL2]	[CO1]	[2]
10.	State the physical interpretation of the line integral $\int_A^B \vec{F} \cdot d\vec{r}$ .	[BL2]	[CO1]	[2]
<b>Descriptive Questions (16 Marks)</b>				
1.	Verify Green's theorem for $\int_C (3x - 8y^2)dx + (4y - 6xy)dy$ , where C is the boundary of the region given by $x = 0, y = 0, x + y = 1$ .	[BL4]	[CO1]	[16]
2.	Verify Green's theorem for $\int_C (xy + y^2)dx + x^2dy$ , where C is the closed region bounded by $y = x, y = x^2$ .	[BL4]	[CO1]	[16]
3.	Evaluate $\int_C (y - \sin x)dx + \cos x dy$ by using Greens theorem where C is the triangle formed by the lines $y = 0, x = \frac{\pi}{2}$ and $y = \frac{2x}{\pi}$ .	[BL5]	[CO1]	[16]
4.	Verify Gauss divergence theorem for $\vec{F} = 4xz\vec{i} - y^2\vec{j} + yz\vec{k}$ over the cube bounded by $x = 0, x = 1, y = 0, y = 1, z = 0, z = 1$ .	[BL4]	[CO1]	[16]
5.	Verify Gauss divergence theorem for $\vec{F} = (x^2 - yz)\vec{i} + (y^2 - zx)\vec{j} + (z^2 - xy)\vec{k}$ taken over the rectangular parallelepiped bounded by $x = 0, x = a, y = 0, y = b, z = 0, z = c$	[BL4]	[CO1]	[16]
6.	Verify Gauss divergence theorem for $\vec{F} = x^2\vec{i} + y^2\vec{j} + z^2\vec{k}$ taken over the cube bounded by the planes $x=0, x=a, y=0, y=b, z=0, z=c$ .	[BL4]	[CO1]	[16]
7.	Verify Stoke's theorem, $\vec{F} = (x^2 + y^2)\vec{i} - 2xy\vec{j}$ , taken around the rectangular region bounded by the lines $x = \pm a, y = 0, y = b$	[BL4]	[CO1]	[16]

8.	Verify Stoke's theorem for $\vec{F} = (y - z)\vec{i} + (yz)\vec{j} - xz\vec{k}$ where S is the surface bounded by the planes $x = 0, x = 1, y = 0, y = 1, z = 0, z = 1$ not included in the xoy plane (above XOY Plane).	[BL4]	[CO1]	[16]
9.	(i) Prove that $\vec{F} = (6xy + z^3)\vec{i} + (3x^2 - z)\vec{j} + (3xz^2 - y)\vec{k}$ is irrotational vector and find the scalar potential such that $\vec{F} = \nabla\phi$ .  (ii) Find the angle between the surfaces $z = x^2 + y^2 - 3$ and $x^2 + y^2 + z^2 = 9$ at $(2, -1, 2)$ .	[BL5]  [BL4]	[CO1]	[8]  [8]
10.	Determine a and b such that the surfaces $ax^2 - byz = (a + 2)x$ and $4x^2y + z^3 = 4$ cut orthogonally at $(1, -1, 2)$ .	[BL5]	[CO1]	[16]

## UNIT- II –ANALYTIC FUNCTIONS

PART A ( 2 Marks)		Bloom's Level	Course Outcome	Marks Allotted
1.	State the necessary and sufficient conditions for f (z) to be analytic.	[BL1]	[CO2]	[2]
2.	Verify whether $f(z) = \bar{z}$ is analytic function or not.	[BL2]	[CO2]	[2]
3.	Show that $f(z) =  z ^2$ is differentiable at $z=0$ but not analytic.	[BL2]	[CO2]	[2]
4.	Check for the analyticity of $\log z$	[BL3]	[CO2]	[2]
5.	State any two properties of analytic functions.	[BL1]	[CO2]	[2]
6.	Show that $xy^2$ cannot be the real part of an analytic function.	[BL2]	[CO2]	[2]
7.	Find the value of m if $u = 2x^2 - my^2 + 3x$ is harmonic.	[BL3]	[CO2]	[2]
8.	Find the image of the circle $ z  = 3$ under the transformation $w = 2z$ .	[BL4]	[CO2]	[2]
9.	Find the invariant points of the bilinear transformation $\frac{z-1}{z+1}$	[BL2]	[CO2]	[2]
10.	Determine the critical points of the transformation $w^2 = (z - \alpha)(z - \beta)$ .	[BL3]	[CO2]	[2]

Descriptive Questions (16 Marks)				
1.	<p>(i) If <math>f(z)</math> is a regular function of <math>z</math>, prove that</p> $\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)  f(z) ^2 = 4 f'(z) ^2.$ <p>(ii) Find the analytic function <math>f(z)</math> whose real part is</p> $\frac{\sin 2x}{\cos h 2y + \cos 2x}$	[BL4]	[CO2]	[8]  [8]
2.	<p>(i) If <math>f(z)</math> is a regular function of <math>z</math>, prove that</p> $\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right) \log  f(z)  = 0$ <p>(ii) Find the analytic function <math>f(z) = u + iv</math> if</p> $u - v = e^x(\cos y - \sin y)$	[BL4]	[CO2]	[8]  [8]
3.	Show that the function $u = \frac{1}{2} \log(x^2 + y^2)$ is harmonic and determine its conjugate harmonic. Also find $f(z)$ .	[BL5]	[CO2]	[16]
4.	<p>(i) Find the analytic function <math>f(z) = u + iv</math> whose imaginary part is <math>e^{-x}(x \cos y + y \sin y)</math>.</p> <p>(ii) Prove that the real and imaginary parts of an analytic function satisfy the Laplace equation in two dimension <math>\nabla^2 u = 0</math> &amp; <math>\nabla^2 v = 0</math>.</p>	[BL5]	[CO2]	[8]  [8]
5.	<p>(i) Prove that <math>u = x^2 - y^2</math> and <math>v = \frac{-y}{x^2 + y^2}</math> both satisfy the Laplace equation but <math>u + iv</math> is not analytic.</p> <p>(ii) Find the bilinear transformation which maps the points <math>z = -1, 0, 1</math> onto the points <math>w = -1, -i, 1</math>.</p>	[BL5]	[CO2]	[8]  [8]
6.	Show that the transformation $w = \frac{1}{z}$ transforms all circles and straight lines in the $z$ -plane into circles or straight lines in the $w$ -plane.	[BL5]	[CO2]	[16]
7.	Find the image of the infinite strips (i) $\frac{1}{4} < y < \frac{1}{2}$ (ii) $0 < y < \frac{1}{2}$ , under the transformation $w = \frac{1}{z}$ .	[BL4]	[CO2]	[16]
8.	<p>(i) Find the image of <math> z - 2i  = 2</math> under the transformation <math>w = \frac{1}{z}</math>.</p> <p>(ii) Find the bilinear transformation which maps the points <math>z = 1, i, -1</math> onto the points <math>w = i, 0, -i</math>.</p>	[BL4]	[CO2]	[8]  [8]

9.	(i) Find the image of $1 < x < 2$ under the transformation $w = \frac{1}{z}$ .  (ii) Find the bilinear transformation that maps the points $z = 1, i, -1$ onto the points $w = 0, 1, \infty$ .	[BL5]  [BL4]	[CO2]	[8]  [8]
10.	Find the bilinear transformation that maps the points $z = 0, -1, i$ into the points $w = i, 0, \infty$ respectively. Also find the image of the unit circle $ z  = 1$ .	[BL5]	[CO2]	[16]

### UNIT- III –COMPLEX INTEGRATION

PART A ( 2 Marks)		Bloom's Level	Course Outcome	Marks Allotted
1.	State Cauchy's Integral theorem.	[BL1]	[CO3]	[2]
2.	Evaluate $\int_C \frac{3z^2 + 7z + 1}{z + 1} dz$ , where C is $ z  = \frac{1}{2}$ .	[BL4]	[CO3]	[2]
3.	Evaluate $\int_C \frac{z}{(z-2)} dz$ where C is the unit circle $ z  = 3$ .	[BL4]	[CO3]	[2]
4.	Expand $f(z) = e^z$ as a Taylor series about the point $z = 0$ .	[BL3]	[CO3]	[2]
5.	Obtain the Taylor series expansion of $\log(1+z)$ about $z = 0$ .	[BL3]	[CO3]	[2]
6.	State Cauchy's Residue theorem.	[BL1]	[CO3]	[2]
7.	Find the residue of $\frac{1-e^{2z}}{z^4}$ at $z=0$	[BL3]	[CO3]	[2]
8.	Define singular points.	[BL1]	[CO3]	[2]
9.	Classify the singularities for the function $f(z) = \frac{\sin z - z}{z^3}$ at $z=0$ .	[BL2]	[CO3]	[2]
10.	Define essential singularities and give an example.	[BL1]	[CO3]	[2]
Descriptive Questions (16 Marks)				
1.	(i) Evaluate $\int_C \frac{z+4}{z^2+2z+5} dz$ , where C is the circle $ z + 1 + i  = 2$ by Cauchy's Integral formula.  (ii) Expand as a Laurent's series of the function $f(z) = \frac{z}{z^2 - 3z + 2}$ in the region $1 <  z  < 2$ and $ z  > 2$ .	[BL4]	[CO3]	[8]  [8]

2.	<p>(i) Using Cauchy's integral formula evaluate <math>\int_c \frac{\sin \pi z^2 + \cos \pi z^2}{(z-2)(z-3)} dz</math> where C is <math> z  = 4</math>.</p> <p>(ii) Evaluate <math>\int_c \frac{z-1}{(z+1)^2(z-2)} dz</math> where c is the circle <math> z-i  = 2</math>, using Cauchy's Residue theorem.</p>	[BL5]	[CO3]	[8]
3.	<p>(i) Find the Laurent's series expansion of the function <math>f(z) = \frac{7z-2}{z(z-2)(z+1)}</math> valid in the region <math>1 &lt;  z+1  &lt; 3</math>.</p> <p>(ii) Evaluate <math>\int_c \frac{z dz}{(z-1)(z-2)^2}</math> where C is <math> z-2  = \frac{1}{2}</math> using Cauchy's integral formula.</p>	[BL5]	[CO3]	[8]
4.	<p>Find the Laurent's series expansion of the function <math>f(z) = \frac{z^2-1}{z^2+5z+6}</math> valid in the regions a) <math>2 &lt;  z  &lt; 3</math> b) <math> z  &lt; 2</math> c) <math> z  &gt; 3</math></p>	[BL5]	[CO3]	[16]
5.	Using contour integration, evaluate $\int_0^{2\pi} \frac{d\theta}{13+5\sin\theta}$ .	[BL5]	[CO3]	[16]
6.	Evaluate $\int_0^{2\pi} \frac{d\theta}{a+b\cos\theta}$ , $a > b$ by using contour integration.	[BL5]	[CO3]	[16]
7.	Evaluate $\int_0^{2\pi} \frac{d\theta}{(1-2a\cos\theta+a^2)}$ , $ a  < 1$ using contour integration.	[BL5]	[CO3]	[16]
8.	Evaluate $\int_{-\infty}^{\infty} \frac{x^2}{(x^2+a^2)(x^2+b^2)} dx$ $a>0, b>0$ using contour integration.	[BL5]	[CO3]	[16]
9.	Evaluate $\int_{-\infty}^{\infty} \frac{x^2-x-2}{x^4+10x+9} dx$ , by using contour integration.	[BL5]	[CO3]	[16]
10.	Evaluate $\int_0^{\infty} \frac{1}{(1+x^2)^2} dx$ , by using contour integration.	[BL5]	[CO3]	[16]

## UNIT- IV -FOURIER TRANSFORMS

PART A ( 2 Marks)		Bloom's Level	Course Outcome	Marks Allotted
1.	State Fourier integral theorem.	[BL1]	[CO4]	[2]
2.	Write the Fourier transform pair.	[BL1]	[CO4]	[2]
3.	State Parseval's identity on Fourier transform.	[BL1]	[CO4]	[2]
4.	Write down the Fourier sine transform pair.	[BL1]	[CO4]	[2]
5.	Write down the Fourier cosine transform pair.	[BL1]	[CO4]	[2]
6.	Find the Fourier sine transform of $f(x) = e^{-ax}$ where $a > 0$	[BL2]	[CO4]	[2]
7.	Find the Fourier cosine transform of $f(x) = e^{-ax}$ where $a > 0$	[BL2]	[CO4]	[2]
8.	Find the Fourier sine transform of $\frac{1}{x}$ .	[BL2]	[CO4]	[2]
9.	Does Fourier Sine transform of $f(x) = k$ $0 \leq x \leq \infty$ exist? Justify your answer.	[BL3]	[CO4]	[2]
10.	Define self-reciprocal with respect to Fourier transform.	[BL1]	[CO4]	[2]
Descriptive Questions (16 Marks)				
1.	Find the Fourier transform of $f(x) = \begin{cases} a -  x  & \text{for }  x  < a \\ 0 & \text{for }  x  > a > 0 \end{cases}$ . Hence show that $\int_0^\infty \left(\frac{\sin t}{t}\right)^2 dt = \frac{\pi}{2}$ and $\int_0^\infty \left(\frac{\sin t}{t}\right)^4 dt = \frac{\pi}{3}$ . <a href="#">pg.no.4.41</a>	[BL5]	[CO4]	[16]
2.	Find the Fourier transform of $(x) = \begin{cases} a^2 - x^2 & \text{for }  x  < a \\ 0 & \text{for }  x  > a > 0 \end{cases}$ . Hence deduce that $\int_0^\infty \left(\frac{\sin t - t \cos t}{t^3}\right) dt = \frac{\pi}{4}$ and $\int_0^\infty \left(\frac{\sin t - t \cos t}{t^3}\right)^2 dt = \frac{\pi}{15}$ . <a href="#">pg.no.4.52</a>	[BL5]	[CO4]	[16]
3.	Find the Fourier transform of $f(x) = \begin{cases} 1 & \text{for }  x  < a \\ 0 & \text{for }  x  > a \end{cases}$ and hence find the value of $\int_0^\infty \left(\frac{\sin x}{x}\right) dx$ and $\int_0^\infty \left(\frac{\sin t}{t}\right)^2 dt$ .	[BL4]	[CO4]	[16]
4.	Find the Fourier transform of $f(x) = \begin{cases} 1 - x^2 & \text{for }  x  < 1 \\ 0 & \text{for }  x  \geq 1 \end{cases}$ . Hence prove that $\int_0^\infty \left(\frac{\sin s - s \cos s}{s^3}\right) \cos\left(\frac{s}{2}\right) ds = \frac{3\pi}{16}$ and $\int_0^\infty \left(\frac{\sin s - s \cos s}{s^3}\right)^2 ds = \frac{\pi}{15}$ .	[BL4]	[CO4]	[16]

5.	Show that $e^{\frac{-x^2}{2}}$ is a self-reciprocal with respect to Fourier transform.	[BL5]	[CO4]	[16]
6.	Find the Fourier sine transform of $f(x) = e^{-ax}$ where $a>0$ and hence deduce that $\int_0^\infty \frac{s \sin sx}{a^2+s^2} ds = \frac{\pi}{2} e^{-ax}$ .	[BL4]	[CO4]	[16]
7.	Find the Fourier cosine transform of $f(x) = e^{-ax}$ where $a>0$ and hence deduce that $\int_0^\infty \frac{\cos xt}{a^2+t^2} dt = \frac{\pi}{2a} e^{-a x }$	[BL4]	[CO4]	[16]
8.	Find the Fourier sine transform and Fourier cosine transforms of $x e^{-ax}$ .	[BL4]	[CO4]	[16]
9.	Using Parseval's identity, evaluate the following integrals (i) $\int_0^\infty \frac{dx}{(x^2+a^2)(x^2+b^2)}$ (ii) $\int_0^\infty \frac{x^2 dx}{(x^2+a^2)(x^2+b^2)}$	[BL5]	[CO4]	[16]
10.	Using Parseval's identity, evaluate the following integrals (i) $\int_0^\infty \frac{dx}{(x^2+a^2)^2}$ (ii) $\int_0^\infty \frac{x^2 dx}{(x^2+a^2)^2}$	[BL5]	[CO4]	[16]

UNIT- V -LAPLACE TRANSFORM				
PART A ( 2 Marks)		Bloom's Level	Course Outcome	Marks Allotted
1.	State the conditions under which the Laplace transform of $f(t)$ exist.	[BL1]	[CO5]	[2]
2.	State and prove first shifting theorem.	[BL1]	[CO5]	[2]
3.	State and prove change of scale property.	[BL1]	[CO5]	[2]
4.	Is the linearity property applicable to $L\left(\frac{1-\cos t}{t}\right)$	[BL2]	[CO5]	[2]
5.	Does $L\left(\frac{\cos at}{t}\right)$ exist?	[BL3]	[CO5]	[2]
6.	Find the $L^{-1}\left[\frac{1}{(s+1)^2+1}\right]$	[BL2]	[CO5]	[2]

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7.	Find the inverse Laplace transform of $\log\left(\frac{s+1}{s-1}\right)$	[BL3]	[CO5]	[2]
8.	Find the Laplace transform of unit step function.	[BL2]	[CO5]	[2]
9.	State Initial and Final value theorems.	[BL3]	[CO5]	[2]
10.	State convolution theorem.	[BL1]	[CO5]	[2]
<b>Descriptive Questions (16 Marks)</b>				
1.	(i) Find the Laplace transform of the triangular wave function $f(t) = \begin{cases} t & 0 \leq t \leq a \\ 2a-t & a \leq t \leq 2a \end{cases}$ with $f(t) = f(t+2a)$ <span style="color: blue;">ex.no.5.8.4</span> (ii) Find $L[t^2 e^{-t} \cos t]$ <span style="color: red;">eg.no.5.7.12</span> <span style="color: red;">pg.no.5.83</span>	[BL4]	[CO5]	[8]  [8]
2.	(i) Find the Laplace transform of the function $f(t)$ with period $\frac{2\pi}{\omega}$ for $f(t) = \begin{cases} \sin \omega t & \text{for } 0 < t < \frac{\pi}{\omega} \\ 0 & \text{for } \frac{\pi}{\omega} < t < \frac{2\pi}{\omega} \end{cases}$ (ii) Find $L^{-1}\left(\frac{s}{(s^2+a^2)^2}\right)$ using convolution theorem.	[BL4]	[CO5]	[8]  [8]
3.	(i) Find the Laplace transform of the square wave function $f(t) = \begin{cases} E & 0 \leq t \leq \frac{a}{2} \\ -E & \frac{a}{2} \leq t \leq a \end{cases}$ with $f(t) = f(t+a)$ . (ii) Find $L^{-1}\left(\frac{s}{(s^2+a^2)(s^2+b^2)}\right)$ using convolution theorem.	[BL4]	[CO5]	[8]  [8]
4.	(i) Find the Laplace transform of the square wave function $f(t)$ given by $f(t) = \begin{cases} K, & 0 \leq t \leq a \\ -K, & a \leq t \leq 2a \end{cases}$ and $f(t+2a) = f(t)$ (ii) Find $L\left(\frac{\cos at - \cos bt}{t}\right)$	[BL5]  [BL4]	[CO5]	[8]  [8]
5.	Using convolution theorem, find $L^{-1}\left[\frac{s^2}{(s^2+a^2)(s^2+b^2)}\right]$	[BL5]	[CO5]	[16]
6.	(i) Verify Initial and Final Value theorem for $f(t) = 1 + e^{-t}(\sin t + \cos t)$ .	[BL4]	[CO5]	[8]

	(ii) Find $L\left(\frac{e^{-at} - e^{-bt}}{t}\right)$			[8]
7.	Using Laplace transform, solve $y'' - 3y' + 2y = 1$ given that $y(0) = 1, y'(0) = 1$ .	[BL5]	[CO5]	[16]
8.	Using Laplace transform solve $y'' - 3y' + 2y = e^{3t}$ given that $y(0) = 1, y'(0) = 0$ .	[BL5]	[CO5]	[16]
9.	Using Laplace transform, solve $(D^2 + 9)y = \cos 2t$ , given that if $y(0) = 1, y\left(\frac{\pi}{2}\right) = -1$	[BL5]	[CO5]	[16]
10.	Using Laplace transform, solve $\frac{d^2y}{dt^2} + \frac{dy}{dt} = t^2 + 2t$ given that $y = 4$ and $y' = -2$ when $t = 0$	[BL5]	[CO5]	[16]