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## Calculating ANOVA

Let's explore calculating ANOVA for the scenario:

Compare plant growth under 3 fertilizers (A, B, C):

- Fertilizer A: [10, 11, 12]
- Fertilizer B: [7, 8, 9]
- Fertilizer C: [4, 5, 6]

### 1. State Hypothesis

- **Null Hypothesis ( $H_0$ ):**  $\mu_A = \mu_B = \mu_C$
- **Alternative Hypothesis ( $H_a$ ):** At least one  $\mu$  differs.

### 2. Calculate Group means and Grand mean.

- **Group Means:**  $\bar{X}_A, \bar{X}_B, \text{ and } \bar{X}_C$
- **Grand Mean:**  $\bar{X}_{\text{grand}}$

$$\bar{X}_A = \frac{10+11+12}{3} = 11 \quad \bar{X}_B = \frac{7+8+9}{3} = 8 \quad \bar{X}_C = \frac{4+5+6}{3} = 5$$

$$\bar{X}_{\text{grand}} = \frac{10+11+12+7+8+9+4+5+6}{9} = 8$$

### 3. Compute Sum of Squares (SS):

**SSB (Sum of Squares Between Groups):** Accounts for variation due to the treatment or independent variable.

$$SSB = \sum n_i (\bar{X}_i - \bar{X}_{\text{grand}})^2$$

**SSE (Sum of Squares Error or Within Groups):** Accounts for variation within groups (random error or residuals).

$$SSE = \sum (x_i - \bar{X})^2$$

**SST (Total Sum of Squares):** Accounts for total variation from overall mean.

$$SST = SSB + SSE$$

$$SSB = 3(11 - 8)^2 + 3(8 - 8)^2 + 3(5 - 8)^2 = 3(9) + 3(0) + 3(9) = 54$$

SSE:

- Fertilizer A:  $(10 - 11)^2 + (11 - 11)^2 + (12 - 11)^2 = 1 + 0 + 1 = 2$
- Fertilizer B:  $(7 - 8)^2 + (8 - 8)^2 + (9 - 8)^2 = 1 + 0 + 1 = 2$

- Fertilizer C:  $(4 - 5)^2 + (5 - 5)^2 + (6 - 5)^2 = 1 + 0 + 1 = 2$

$$SSW = 2 + 2 + 2 = 6$$

$$SST = 54 + 6 = 60$$

#### 4. Calculate Degrees of Freedom (df):

*df1 (Between Groups) = k - 1, where k is number of groups.*

*df2 (Within Groups) = N - k, where N is the total observations.*

*df3 (Total) = N - 1.*

- $df1 = 3 - 1 = 2$
- $df2 = 9 - 3 = 6$
- $df3 = 9 - 1 = 8$

#### 5. Calculate Mean Squares (MS):

*MSB (Mean Square Between Groups) = SSB / df1.*

*MSE (Mean Square Error) = SSE / df2.*

- $MSB = SSB / df1 = 54 / 2 = 27$   $MSB = df1 / SSB = 254 / 27$
- $MSW = SSW / df2 = 6 / 6 = 1$   $MSW = df2 / SSW = 66 / 1$

#### 6. F-statistic:

*The F-statistic is calculated as the ratio of MSB to MSE:*

$$F = MSB / MSE$$

- $F = 27 / 1 = 27$   $F = 127 / 27$

#### 7. P-value:

*The p-value is used to decide whether differences among groups are statistically significant. When the p-value is smaller than the significance level ( $\alpha$ ), the null hypothesis is rejected.*

*If  $F > F_{critical} \rightarrow p < 0.05$  : Null Hypothesis Rejected*

**Use the F-distribution table** or software with: Numerator  $df1 = 2$  ,  
Denominator  $df2 = 6$ ,  $\alpha = 0.05$

**Critical F-value,  $F_{critical}$ : 5.14 (From F-distribution table)**

$F > F_{critical}$  :  $27 > 5.14 \rightarrow p < 0.05$ ; Reject null hypothesis