

20-11-2025

## Calculating ANOVA

Let's explore calculating ANOVA for the scenario:

Compare plant growth under 3 fertilizers (A, B, C):

- Fertilizer A: [10, 11, 12]
- Fertilizer B: [7, 8, 9]
- Fertilizer C: [4, 5, 6]

### 1. State Hypothesis

- Null Hypothesis ( $H_0$ ):  $\mu_A = \mu_B = \mu_C$
- Alternative Hypothesis ( $H_a$ ): At least one  $\mu$  differs.

### 2. Calculate Group means and Grand mean.

- Group Means:  $\bar{X}_A, \bar{X}_B, \text{ and } \bar{X}_C$
- Grand Mean:  $\bar{X}_{\text{grand}}$

$$\begin{aligned}\bar{X}_A &= 10 + 11 + 12 = 11 \\ \bar{X}_B &= 7 + 8 + 9 = 8 \\ \bar{X}_C &= 4 + 5 + 6 = 5\end{aligned}$$

$$\begin{aligned}\bar{X}_{\text{grand}} &= 10 + 11 + 12 + 7 + 8 + 9 + 4 + 5 + 6 = 72 \\ &= 910 + 11 + 12 + 7 + 8 + 9 + 4 + 5 + 6 = 972 = 8\end{aligned}$$

### 3. Compute Sum of Squares (SS):

**SSB (Sum of Squares Between Groups):** Accounts for variation due to the treatment or independent variable.

$$SSB = \sum ni(\bar{X}_i - \bar{X}_{\text{grand}})^2$$

**SSE (Sum of Squares Error or Within Groups):** Accounts for variation within groups (random error or residuals).

$$SSE = \sum (x_i - \bar{X})^2$$

**SST (Total Sum of Squares):** Accounts for total variation from overall mean.

$$SST = SSB + SSE$$

$$SSB = 3(11 - 8)^2 + 3(8 - 8)^2 + 3(5 - 8)^2 = 3(9) + 3(0) + 3(9) = 54$$

SSE:

- Fertilizer A:  $(10 - 11)^2 + (11 - 11)^2 + (12 - 11)^2 = 1 + 0 + 1 = 2$
- Fertilizer B:  $(7 - 8)^2 + (8 - 8)^2 + (9 - 8)^2 = 1 + 0 + 1 = 2$

- Fertilizer C:  $(4 - 5)^2 + (5 - 5)^2 + (6 - 5)^2 = 1 + 0 + 1 = 2$
- $SSW = 2 + 2 + 2 = 6$
- $SST = 54 + 6 = 60$

#### 4. Calculate Degrees of Freedom (df):

$df_1$  (Between Groups) =  $k - 1$ , where  $k$  is number of groups.

$df_2$  (Within Groups) =  $N - k$ , where  $N$  is the total observations.

$df_3$  (Total) =  $N - 1$ .

- $df_1 = 3 - 1 = 2$
- $df_2 = 9 - 3 = 6$
- $df_3 = 9 - 1 = 8$

#### 5. Calculate Mean Squares (MS):

$MSB$  (Mean Square Between Groups) =  $SSB / df_1$ .

$MSE$  (Mean Square Error) =  $SSE / df_2$ .

- $MSB = SSB / df_1 = 54 / 2 = 27$
- $MSW = SSE / df_2 = 66 / 6 = 11$

#### 6. F-statistic:

The F-statistic is calculated as the ratio of MSB to MSE:

$$F = \frac{MSB}{MSW}$$

- $F = 27 / 11 = 2.45$

#### 7. P-value:

The p-value is used to decide whether differences among groups are statistically significant. When the p-value is smaller than the significance level ( $\alpha$ ), the null hypothesis is rejected.

If  $F > F_{critical} \rightarrow p < 0.05$  : Null Hypothesis Rejected

Use the F-distribution table or software with: Numerator  $df_1 = 2$ , Denominator  $df_2 = 6$ ,  $\alpha=0.05$

Critical F-value,  $F_{critical}$ : 5.14 (From F-distribution table)

$F > F_{critical} : 2.45 > 5.14 \rightarrow p < 0.05$ ; Reject null hypothesis