Seat No. : _______________

AB-110

April-2019

BCA, Sem.-II

CC-111 : Mathematical Foundation of Computer Science (Old)

Time : 2:30 Hours] **[Max. Marks : 70**

Instructions : (1) **All** the questions are compulsory.

(2) Figures to the right indicate marks.

(3) Make suitable assumptions wherever necessary.

1. (A) Attempt the following :

(i) Prove that $(G,+_5)$ is a group where $G = \{0,1,2,3,4\}$. **7**

(ii) Show that the set of all positive rational numbers forms an abelian group under the composition defined by $a \times b = (ab)/2$ **7**

OR

Attempt the following :

(i) Find the composition fog and gof where

$$f = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 4 & 3 & 1 & 2 \end{pmatrix} \text{ and } g = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 1 & 4 & 3 \end{pmatrix}$$

(ii) Prove that every cyclic group is an abelian.

(B) Attempt any **Four** : **4**

(1) If '*' is any binary operation on any set S, then a * a = a for all $a \in S$. (True/False)

(2) Define transposition permutation in cyclic group.

(3) If a is generator of a group then ____ is also generator of group.

(a) a^{-1} (b) $a \oplus b$

(c) a * b (d) $a \bullet b$

(4) If * is binary operation on any set S, then a * a = a for all $a \in S$. (True/False)

(5) Every group is a subgroup of itself. (True/False)

(6) In a cyclic group, every element is generator. (True/False)

2. (A) Attempt the following :

(i) Draw Hasse diagram of (S_{100}, D). Determine the GLB and LUB of B, where B = {5, 10, 20, 25}. **7**

(ii) Determine whether the relation R on the set of all integers is reflexive, symmetric, anti-symmetric and or transitive, where $(x, y) \in R$ i.e. xRy if & only if

(a) $x \geq y^2$ (b) $x \leq y + 1$

(c) x is multiple of y. (d) $x \neq y$

(e) $x = y(\text{mod}7)$. **7**

OR

Attempt the following :

(i) Draw the Hasse diagram for the POSET (S_{150}, D) and (S_{30}, D).

(ii) Define the Partition of set. Determine whether or not each of the following is a partition of the set of natural numbers of positive integer with justification.

$P_1 = \{\{x/x>4\}, \{x/x<4\}\}$

$P_2 = \{(x/x>4\},\{0\},\{x/x<4\}\}$

$P_3 = \{\{x/x^2 \geq 4\}, \{x/x^2 < 4\}\}$

(B) Attempt any **four** : **4**

(1) If the domain and range of a relation are same then relation is ______.

(a) Reflexive (b) Symmetric

(c) Equivalence (d) None of these

(2) Give an example of relation R on set A = {1,2,3} which is reflexive, symmetric and transitive.

(3) Total number of distinct relation from set A to B is _____ where A = {1, 2, 3} and B = {a,b}.

(4) Find the maximal and minimal elements of the set P = {2, 3, 5, 7,11, 13} ordered by divisibility ?

(5) ______ element has no multiplicative inverse in group <Z, *>

(a) 0 (b) 1

(c) –1 (d) none

(6) Which relation is called Partial order relation ?

3. (A) Attempt the following :

(i) In any Boolean algebra prove that a = b $\Leftrightarrow$ ab' + a'b = 0. **7**

(ii) Define the following terms : **7**

Join Irreducible, Anti-atoms, Sub-Boolean Algebra, Complete Lattice, Complimented lattice, Distributive lattice, Bounded Lattice.

OR

Attempt the following :

(i) Write the following Boolean expression in equivalent product-of-sum canonical form. $x_1 * (x_2 \oplus x'_3)$

(ii) In a Boolean algebra prove that (a + b) (a' + c) = ac + a'b + bc = ac + a'b.

(B) Attempt any **three** : **3**

(1) (S_6, Z)) is a sublattice of

(a) (S_{12}, D) (b) (S_{30}, D)

(c) (S_{45}, D) (d) None of these

(2) State the Associative law of Lattice.

(3) State Absorption law of lattice.

(4) In a Boolean Algebra (B, *, $\oplus$, ′, 0, l) , for any a, b $\in$ B, (a * b}’ = ________.

(5) Let (N, D) be lattice. Which of the following subset of N is linearly ordered ?

(a) {2, 3, 15} (b) {2, 4, 16}

4. (A) Attempt the following :

(i) Define a complete graph. Draw a complete graph on eight vertices. **7**

(ii) Draw the digraph G corresponding to the following matrix : **7**

$$A = \begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 2 & 0 & 0 \end{bmatrix}$$ And hence find Path matrix.

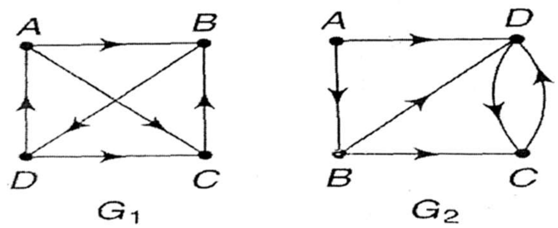
OR

Attempt the following :

(i) Give three other representation of tree expressed by

(A(B(C)(D)(E))(F(G)(H))(J(K)(L)(M(P)(Q))(N))).

(ii) Define Isomorphic graphs. Check whether the following graphs are isomorphic or not ? Why ?



(B) Attempt any **three** : **3**

(1) Draw a graph with four vertices of degree 1,1,3,3.

(2) A directed graph G(V, E) is said to be finite if its

(a) Set V of vertices is finite.

(b) Set V of vertices and set E of edges are finite.

(c) Set E of edges is finite.

(d) None of the above

(3) Tree is acyclic connected graph. (True/ False)

(4) Draw a graph with six vertex and four edges.

(5) Define loop in a graph theory.

Seat No. : ______________

AB-110

April-2019

BCA, Sem.-II

CC-111 : Discrete Mathematics
(New)

Time : 2:30 Hours] **[Max. Marks : 70**

Instructions : (1) **All** the questions are compulsory.
(2) Figures to the right indicate marks.
(3) Make suitable assumptions wherever necessary.

1. (A) Attempt the following :

(i) Define Sub-Group. Is the set S = {2, 4, 6, 8, 0} forms the subgroup of $<Z_{10},+_{10}>$. **7**

(ii) Let the permutation of elements of {1, 2, 3, 4, 5, 6} be given by **7**

$$\alpha = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 2 & 3 & 1 & 4 & 5 & 6 \end{pmatrix}; \beta = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 1 & 2 & 3 & 5 & 4 & 6 \end{pmatrix};$$

$$\lambda = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 5 & 4 & 3 & 1 & 2 & 6 \end{pmatrix} \text{ and } \delta = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 3 & 2 & 1 & 5 & 6 & 4 \end{pmatrix}$$

Calculate $\alpha\beta$, $\beta\alpha$, α^2, $\lambda\beta$, δ^{-1}, $\alpha\beta\lambda$ and $\alpha^{-1}\beta$.

OR

Attempt the following :

(i) Prepare the composition table for multiplication on the element in the set A = {1, –1, i, –i}, where i is the fourth root of unity. Show that multiplication satisfies the closure property, associative law, commutative law. Find identity element and inverse of each element.

(ii) Show that the group {(1, 2, 4, 5, 7, 8), x_9} is cyclic group. What are its generators ?

(B) Attempt any **four** : **4**

(1) Given that $\{Z^+,*\}$ where * is defined by a * b = a, for all a, b $\in Z^+$, is a monoid ?

(2) If S = {1,2,3,6} and * is defined by a* b = lcm(a, b), where a, b $\in$ S, given that {S, *} is a monoid. What is the identity element ?

(3) Given that {N, *} where * is defined by a * b = gcd (a, b), for all a, b $\in$ N. Is this algebraic structure follows associative law ?

(4) If a set A has 2 elements, then how many binary operations are possible in A ?

(a) 8 (b) 12
(c) 16 (d) 20

(5) If * is a binary operation defined as a* b = 3a–b, then (2*3) *4= ?

(a) 2 (b) 3
(c) 4 (d) 5

(6) Every abelian group is a cyclic group. [True/ False]

2. (A) Attempt the following :

(i) Draw the Hasse diagram of (S_{24}, D) and $(P(S), \subseteq)$, where $S = \{a,b,c\}$ **7**

(ii) Let a set $X = \{1,2,3,4\}$. The relation matrix $M(R)$ on a set X is given below : **7**

$$M(R) = \begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix}$$

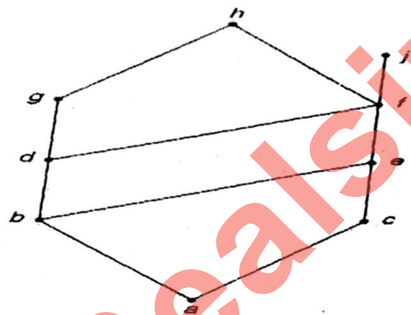
Answer the following questions :

(a) Give R as set of ordered pairs.
(b) Give domain of relation.
(c) Give range of relation.
(d) Is the relation reflexive ?
(e) Is the relation symmetric ?
(f) Is the relation transitive ?
(g) Draw the digraph representing this relation.

OR

Attempt the following :

(i) Define LUB and GLB. Also find LUB and GLB of the subset A={b,d,g} and B={j, h}, if they exist.



(ii) Let A= {a,b,c,d} and
R = {(a, b), (a, a), (b, a),(b, b), (c, c),(d, d),(d, e),(e, d),(e, e)} and
S = {(a, a), (b, b), (c, c), (d, d), (e, e), (a, c), (c, d), (d, e), (e, d)} be the equivalence relation on A. Determine the partitions corresponding to $R \cap S$.

(B) Attempt any **four** : **4**

(1) For a relation R on set A, let $M_R = [m_{ij}]$, $m_{ij} = 1$ if a_j R a_j and 0 otherwise, be the matrix of relation R. If $(M_R)^2 = M_R$ then R is _____.

(a) Symmetric
(b) transitive
(c) Antisymmetric
(d) Reflexive

(2) Let L be a set with a relation R which is transitive, antisymmetric and reflexive and for any two elements a, b $\in$ L. Let least upper bound lub (a, b) and the greatest lower bound glb (a, b) exist. Which of the following is/are TRUE ?

(a) L is a Poset.
(b) L is Boolean algebra.
(c) L is a lattice.
(d) None of these

(3) The less than relation, <, on reals is

(a) a partial ordering since it is asymmetric and reflexive.
(b) a partial ordering since it is anti-symmetric and reflexive.
(c) not a partial ordering because it is not asymmetric and not reflexive
(d) not a partial ordering because it is not anti-symmetric and not reflexive.

(4) Which of the following pair is not congruent modulo7 ?
(a) 10,24 (b) 25,56
(c) –31,11 (d) –64,15

(5) A relation on set A is subset of ________.
(a) $A \times A$ (b) $\varnothing$
(c) A (d) None of the above

(6) Let R be a relation on N defined by X+2Y=8. The domain of R is
(a) {2, 4, 8} (b) {2, 4, 6, 8}
(c) {2, 4, 6} (d) {1, 2, 3, 4}

3. (A) Attempt the following :

(i) Define Distributive lattice and Complimented lattice. Check the $<S_{12}, D>$ is complemented or not ? Distributive or not ? **7**

(ii) Show that in a complimented and distributive lattice **7**
$a \le b \Rightarrow a * b' = 0 \Rightarrow a' \oplus b = 1 \Rightarrow b' \le a'$

OR

Attempt the following :

(i) Write the Boolean expression, $ab' + c$, in an equivalent sum-of-product and product-of-sum canonical form in three variables a, b and c.

(ii) Define the following terminology :
Complete Lattice, Complemented lattice, Boolean Algebra, Atoms in Boolean Algebra, Sub-Boolean Algebra, Bounded Lattice, Meet-Irreducible

(B) Attempt any **three** : **3**

(1) If B is Boolean Algebra then which of the following is true ?
(a) B is finite but not complemented lattice.
(b) B is finite, complemented and distributive lattice.
(c) B is finite, distributive but not complemented lattice.
(d) B is not distributive lattice.

(2) The absorption law is defined as
(a) $a * (a * b) = b$ (b) $a * (a \oplus b) = b$
(c) $a * (a * b) = a \oplus b$ (d) $a * (a \oplus b) = a$

(3) Complement of the expression A'B + CD' is
(a) $(A' + B)(C' + D)$ (b) $(A + B')(C' + D)$
(c) $(A' + B')(C + D)$ (d) $(A + B')(C + D')$

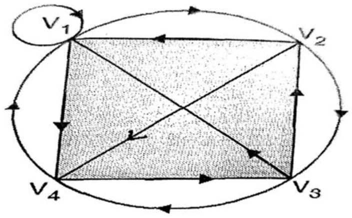
(4) Every Chain is a Lattice. [True/False]

(5) A product term containing all K variables of the function in either complemented or uncomplemented form is called a
(a) Minterm (b) Maxterm
(c) Midterm (d) None of the above

4. (A) Attempt the following :

(i) Define Path, Simple Path and Elementary path. For the graph given in the figure find **7**
(a) Find an elementary path of length 2 from V_1 to V_3.
(b) Find a Simple path from V_1 to V_3, which is not elementary.

(c) Find all possible paths from V_2 to V_4 and how many of them are simple and elementary ?



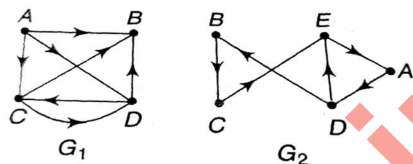
(ii) Define the following terminology : **7**

Complete Graph, Compliment of a Graph, Isolated Vertex, Pendent vertex, Null Graph, Forest, Degree of a vertex in a tree.

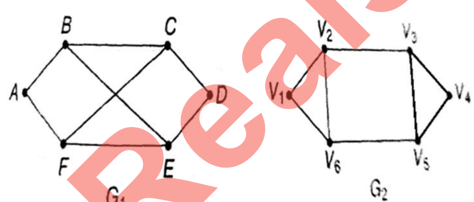
OR

Attempt the following.

(i) Define Strongly, Weakly and Unilaterally connected graphs. Find which of the following graphs are strongly or unilaterally connected. Give the reason.



(ii) Define Isomorphic Graphs. Check whether the following graphs are isomorphic or not ?



(B) Attempt any **three** : **3**

(1) Which of the following statements is/are TRUE for an undirected graph ?

P : Number of odd degree vertices is even.

Q : Sum of degrees of all vertices is even.

(a) P Only (b) Q Only (c) Both P and Q (d) Neither P nor Q

(2) Adjacency matrix of all graphs are symmetric.

(a) True (b) False

(3) Which of the following statement is true ?

(a) Every Graph is not its own subgraph.

(b) The terminal vertex of a graph are of degree two.

(c) A tree with n vertices has n edges.

(d) A single vertex in graph G is subgraph of G.

(4) A tree with n vertices has _______ edges.

(a) n (b) n + 1 (c) n – 2 (d) n – 1

(5) The complete graph with four vertices has k edges where k is

(a) 3 (b) 4 (c) 5 (d) 6