3 CO-ORDINATE GEOMETRY

University Syllabus

- *Introduction to co–ordinates*
- *Quadrants and lines*
- *Distance between two points in R^2 (without proof)*
- *Section formula (without proof)*
- *Area of triangle (without proof)*
- *Typical examples.*

An Outline of The Chapter

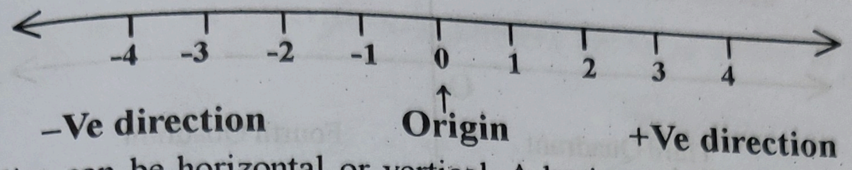
[1] Introduction

Co-ordinate Geometry, the branch of geometry in which two real numbers, called co- ordinates, are used to give the position of a point in a plane. This branch of geometry was developed by the French Mathematician Renatus Cartesius (1956–1650), so it is sometimes called as Cartesian Geometry.

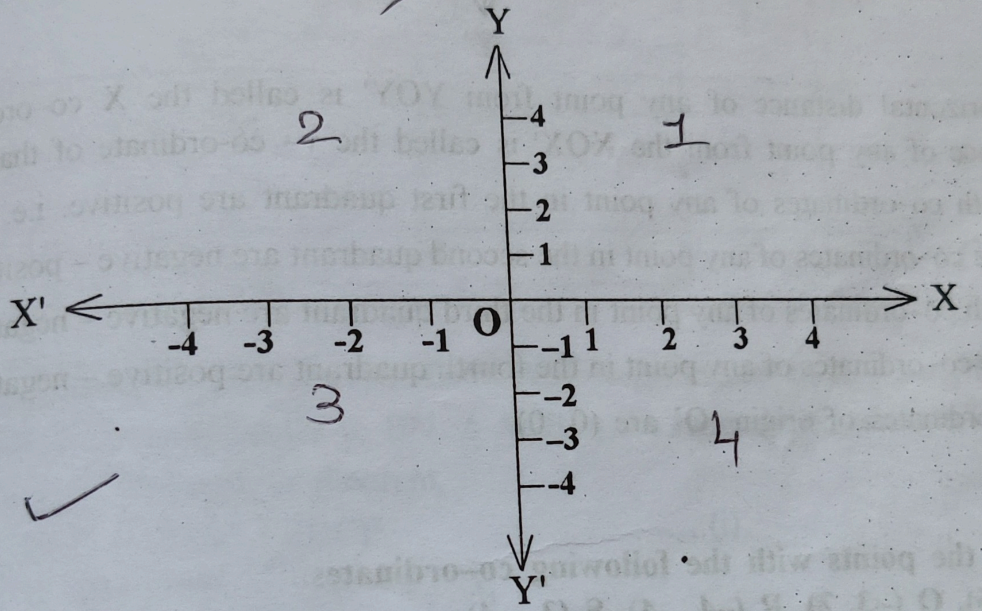
The main contribution of co-ordinate geometry is that it has enabled intergration of algebra and geometry. This is evident from the fact that algebraic methods are employed to represent and prove the fundamental properties of geometrical theorems. Equations are also employed to represent the different geometric figures.

[2] Directed Line

A directed line is a straight line with number units positive, negative and zero. The point of origin is 0. The arrow indicates its direction. On the right side of the arrow the units are positive numbers and on the left side of the arrow the units are negative numbers.



A directed line can be horizontal or vertical. A horizontal directed line indicated by X'OX axis and vertical directed line indicated by Y'OY axis. The point where these two lines intersect each other is called the point of origin O. The following figure illustrates the above concept.



The two lines together are called rectangular axis and are at right angle to each other.

Remember :

1. The coordinates of the origin are (0, 0).
2. The co-ordinates of any point on X axis is $(x, 0)$ e.g. (3, 0) which denotes the point is + 3 units on X axis from the origin on the right hand side towards the direction of the arrow.
3. The co-ordinate of any point on Y axis is $(0, y)$ e.g. (0, –3) which denotes the points is –3 units on Y axis from the origin downwards on the vertical axis.

[3] Quadrants

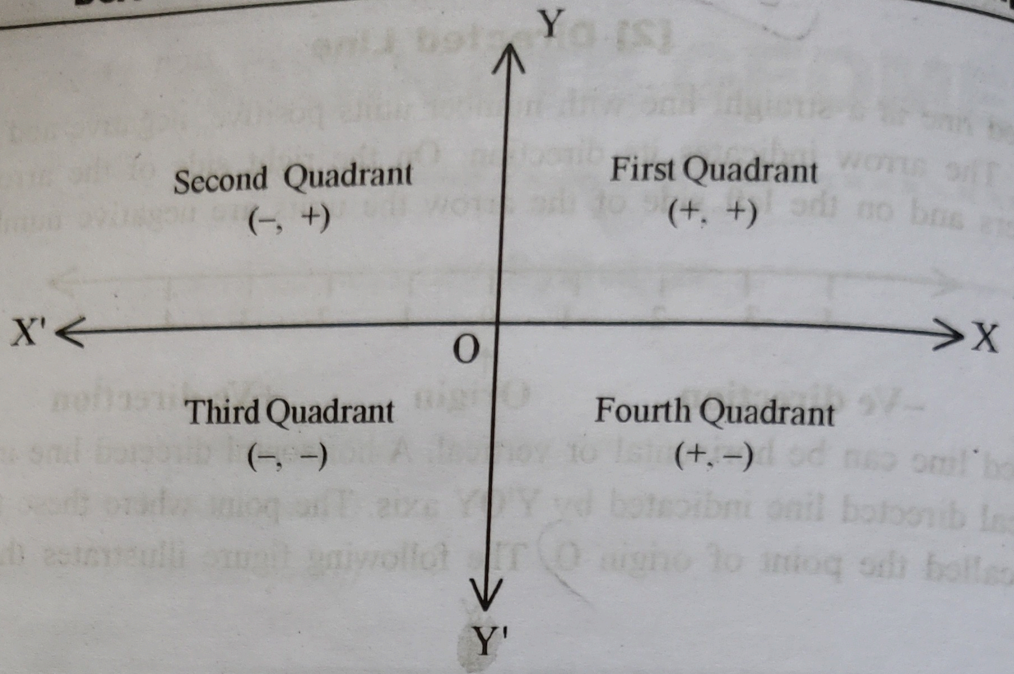
When two directed lines intersect at the right angles at the point of origin, they divide their plane into four parts or regions namely XOY, X'OY, X'OY', and XOY.'

XOY is called first quadrant

X'OY is called second quadrant

X'OY' is called third quadrant

XOY' is called fourth quadrant



The horizontal distance of any point from YOY' is called the X co–ordinate and the vertical distance of any point from the XOX' is called the y– co-ordinate of that point.

(i) Both co–ordinates of any point in the first quadrant are positive. i.e. (+, +)

(ii) The co–ordinates of any point in the second quadrant are negative – positive i.e. (–, +)

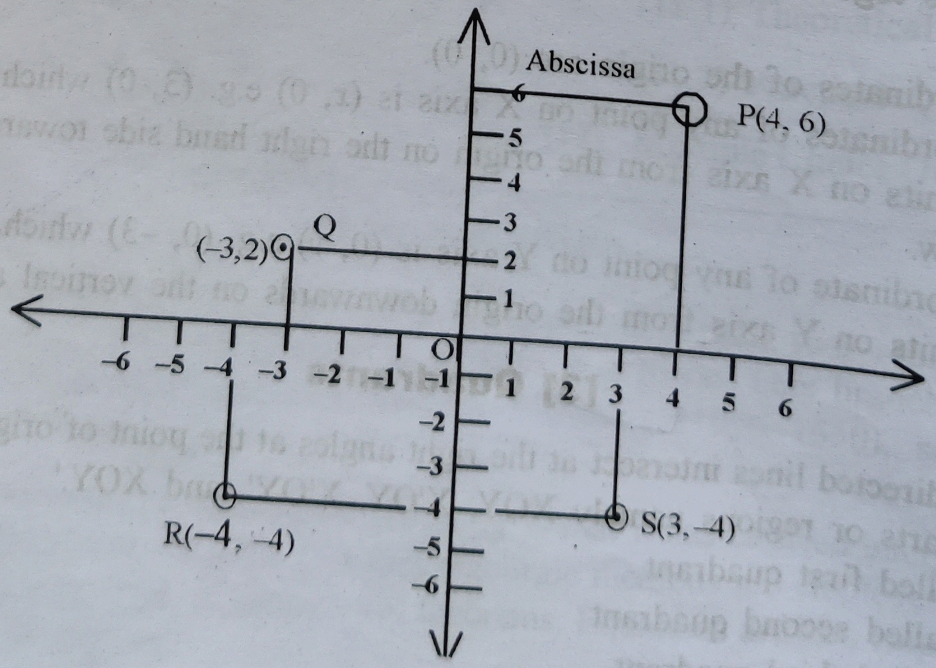
(iii) Both co–ordinates of any point in the third quadrant are negative – negative i.e. (–, -)

(iv) The co–ordinates of any point in the fourth quadrant are positive – negative (+, –)

The co–ordinates of origin 'O' are (0, 0)

Illu. 1. Plot the points with the following co–ordinates.
P(4,6), Q (–3, 2), R (–4, –4), S (3, –4)

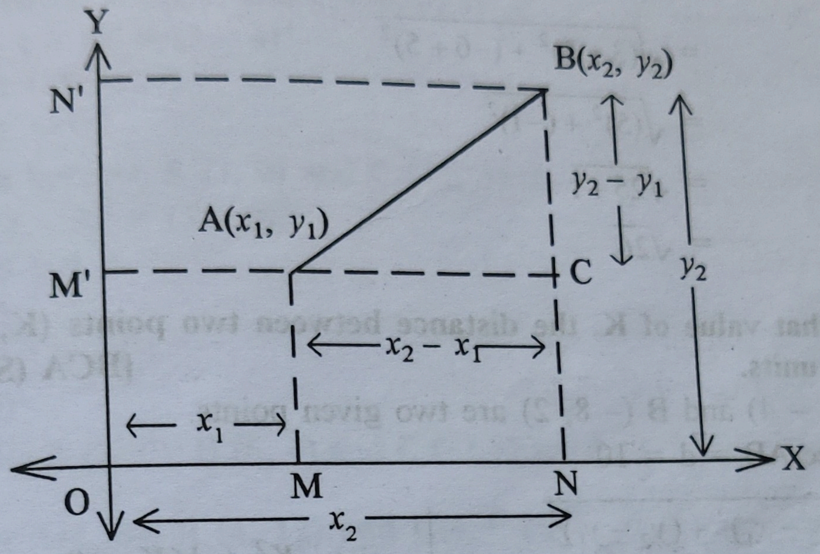
Sol.



It should be noted that the horizontal distance of the point from the Y'OY is called the x - co-ordinate or abscissa and the vertical distance of the point from the X'OX is called the y - co-ordinate.

[4] Distance Between Two Points

Let A (x_1, y_1) and B(x_2, y_2) be any two different points as shown in the following figure.



We want to find the distance AB. Draw AM perpendicular to X – axis, BN perpendicular to X – axis and AC perpendicular to BN. Δ ACB is the right angled triangle ($\because m\angle C = 90°$). Hence, according to Pythagoras theorem,

$$(AB)^2 = (AC)^2 + (BC)^2 \qquad .....(i)$$

But, AC = MN = ON – OM = $x_2 - x_1$

Similarly BC = N'M' = N'O M'O = $y_2 - y_1$

Substituting these values in (i)

$$(AB)^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$$

$$\therefore \ AB = d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Illu. 2. Find the distance between the points (4, 3) and (2, 6)

Sol. Distance between two points (x_1, y_1) and (x_2, y_2) is,

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Here, $x_1 = 4, y_1 = 3, x_2 = 2, y_2 = 6$

$$d = \sqrt{(2-4)^2 + (6-3)^2}$$

$$= \sqrt{(-2)^2 + (3)^2}$$

$$= \sqrt{4+9} = \sqrt{13}$$

Illu. 3. **Find the distance between the points A (–2, –5) and B (3, – 6)**

Sol. Here, $x_1 = -2,\ y_1 = -5,\ x_2 = 3,\ y_2 = -6$

The distance
$$\begin{aligned} d &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(3-(-2))^2 + (-6-(-5))^2} \\ &= \sqrt{(3+2)^2 + (-6+5)^2} \\ &= \sqrt{(5)^2 + (-1)^2} \\ &= \sqrt{25+1} \\ &= \sqrt{26} \end{aligned}$$

Illu. 4. **For what value of K, the distance between two points (K, – 4) and (–8, 2) will be 10 units.** **[BCA (Sem – II) Oct. – 2009]**

Sol. Here, A (K, – 4) and B (– 8, 2) are two given points.

Also distance AB = d = 10

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$10 = \sqrt{(-8-K)^2 + (2+4)^2}$$

$\therefore\ 100 = (-8 - K)^2 + (2 + 4)^2$

$\therefore\ 100 = 64 + 16K + K^2 + 36$

$\therefore\ K^2 + 16K + 100 - 100 = 0$

$\therefore\ K^2 + 16K = 0$

$\therefore\ K^2 = -16K$

$\therefore\ \dfrac{K^2}{K} = -16$

$\therefore\ K = -16$

Illu. 5. **The distance between two points (K, – 5) and (2, K) is 13. Find the value of K.** **[BCA (Sem (II) – April – 2009]**

Sol. Here, $x_1 = K,\ y_1 = -5,\ x_2 = 2,\ y_2 = K$

Also distance d = 13

Now,
$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$13 = \sqrt{(2-K)^2 + (K+5)^2}$$

$\therefore\ 169 = (2 - K)^2 + (K + 5)^2$

$\therefore\ 169 = 4 - 4K + K^2 + K^2 + 10K + 25$

$\therefore\ 169 = 2K^2 + 6K + 29$

$\therefore\ 2K^2 + 6K - 140 = 0$

$\therefore\ K^2 + 3K - 70 = 0$ ($\because$ Dividing by 2)

$\therefore\ (K - 7)(K + 10) = 0$

$\therefore\ K - 7 = 0$ or $K + 10 = 0$

$\therefore\ K = 7$ or $K = -10$

Illu. 6. **Prove that (3,0), (6, 4) and (– 1, 3) are the vertices of a right angled triangle.**

Sol. Here, A (3, 0), B (6, 4) and C(–1, 3) are the given points.

First we shall find the distances AB, BC and AC.

Now, distance between A (3, 0) and B (6, 4) is,

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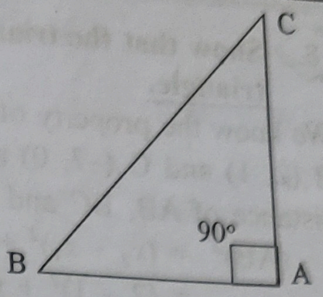


$$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$\therefore (AB)^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$$
$$= (6 - 3)^2 + (4 - 0)^2$$
$$= 9 + 16$$
$$= 25$$

The distance between B (6, 4) and C (–1, 3) is,

$$(BC)^2 = (-1 - 6)^2 + (3 - 4)^2$$
$$= 49 + 1$$
$$= 50$$

and the distance between A (3, 0) and C (–1, 3) is,

$$(AC)^2 = (-1 - 3)^2 + (3 - 0)^2$$
$$= 16 + 9$$
$$= 25$$

Here $BC^2 = AB^2 + AC^2$

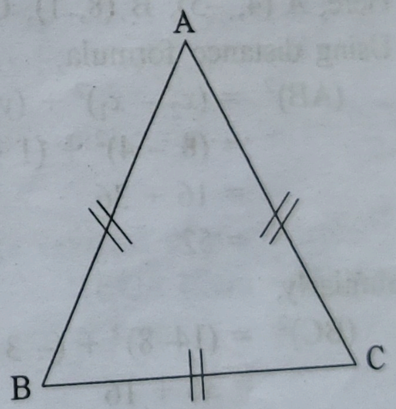
$\therefore m\angle A = 90°$

Hence, the points A (3, 0), B (6, 4) and C (–1,3) are the vertices of a right angle triangle.

Illu. 7. Show that A(4, 8), B (4, 12) and C($4 + 2\sqrt{3}$, 10) from an equilateral triangle.

Sol. We know that the property of an equilateral triangle is that all of its sides are equal. A (4, 8), B (4, 12) and C ($4 + 2\sqrt{3}$, 10) are the three given points.

Using distance formula,



$$(AB)^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$$
$$= (4 - 4)^2 + (12 - 8)^2$$
$$= 0 + 16 = 16$$

Similarly,

$$(BC)^2 = (4 + 2\sqrt{3} - 4)^2 + (10 - 12)^2$$
$$= (2\sqrt{3})^2 + (-2)^2$$
$$= 12 + 4 = 16$$

and $(AC)^2 = (4 + 2\sqrt{3} - 4)^2 + (10 - 8)^2$

$$= (2\sqrt{3})^2 + (2)^2$$
$$= 12 + 4$$
$$= 16$$

Here, $AB^2 = BC^2 = AC^2 = 16$

$\therefore AB = BC = AC = 4$

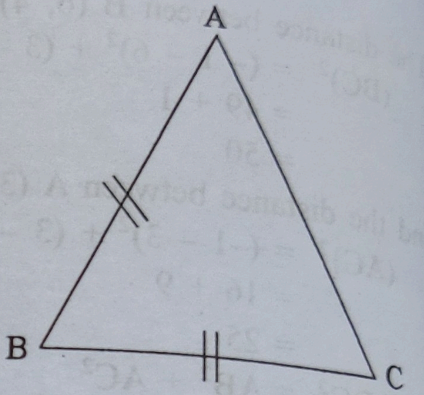
Since all of the three sides are equal i.e. AB = BC = AC.

The triangle ABC is an equilateral triangle.

Hence, ABC are the vertices of an equilateral triangle.

Illu. 8. **Show that the triangle whose vertices are (1, 10), (2, 1) and (–7, 0) is an isosceles triangle.** সমদ্বিভুজ

Sol. We know the property of an isosceles triangle is that two of its sides are equal. A (1, 10), B (2, 1) and C (–7, 0) are the three given points. Using distance formula, we shall find distance of AB, BC and AC.



$$(AB)^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$$
$$= (2 - 1)^2 + (1 - 10)^2$$
$$= 1 + 81 = 82$$

Similarly,

$$(BC) = (-7 - 2)^2 + (0 - 1)^2$$
$$= 81 + 1$$
$$= 82$$

$$\text{and } (AC)^2 = (-7 - 1)^2 + (0 - 10)^2$$
$$= 64 + 100$$
$$= 164$$

Here $AB^2 = BC^2 = 82$

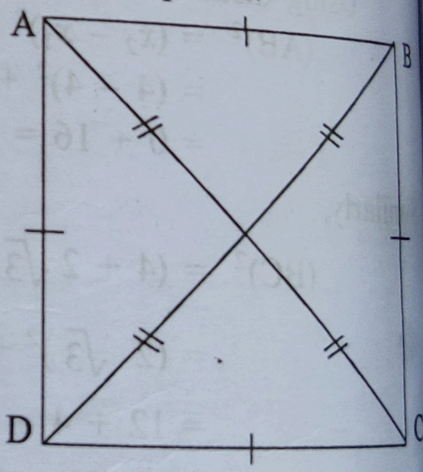
$\therefore \quad AB = BC = \sqrt{82}$

Since two of the sides are equal i.e. AB = BC, the triangle ABC is an isosceles triangle.

Illu. 9. **Show that the points (4, –5), (8, 1), (14, –3) and (10, –9) are the vertices of a square.**

Sol. Here, A (4, –5), B (8, 1), C (14, –3) and D (10, –9) are the given points.

Using distance formula,



$$(AB)^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$$
$$= (8 - 4)^2 + (1 + 5)^2$$
$$= 16 + 36$$
$$= 52$$

Similarly,

$$(BC)^2 = (14-8)^2 + (-3 - 1)^2$$
$$= 36 + 16$$
$$= 52$$

$$(CD)^2 = (10 - 14)^2 + (-9 + 3)^2 = 16 + 36 = 52$$

$$\text{and } (DA)^2 = (4 - 10)^2 + (-5 + 9)^2 = 36 + 16 = 52$$

Since $(AB)^2 = (BC)^2 = (CD)^2 = (DA)^2 = 52$

$\therefore AB = BC = CD = DA = \sqrt{52}$

Therefore, ABCD is either square of a rhombus. But in a square two diagonals are also equal (while in a rhombus, they are not equal).

Now, diagonal

$$(AC)^2 = (14 - 4)^2 + (-3 + 5)^2$$
$$= 100 + 4$$
$$= 104$$

and, $(BD)^2 = (10-8)^2 + (-9-1)^2$
$= 4 + 100$
$= 104$

Thus, $(AC)^2 = (BD)^2 = 104$

$\therefore \quad AC = BD = \sqrt{104}$

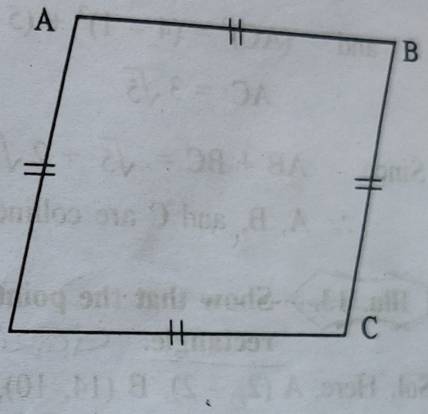
Hence, ABCD is a square.

Illu. 10. Show that the quadrilateral with vertices (2, –1), (3, 4), (–2, 3) and (–3, –2) is a rhombus. ~~समचतुष्कोण चतु~~ સમચતુર્ભુજ

Sol. We know that the quadrilateral is a rhombus, if its all the four sides are equal. Suppose A(2, –1), B(3, 4), C(–2, 3) and D(–3, –2) are the four vertices of the quadrilateral.

Using distance formula,

$(AB)^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$
$= (3-2)^2 + (4+1)^2$
$= 1 + 25$
$= 26$



Similarly, $(BC)^2 = (-2-3)^2 + (3-4)^2 = 25 + 1 = 26$
$(CD)^2 = (-3+2)^2 + (-2-3)^2 = 1 + 25 = 26$
$(DA)^2 = (2+3)^2 + (-1+2)^2 = 25 + 1 = 26$

Since $(AB)^2 = (BC)^2 = (CD)^2 = (DA)^2 = 26$

$\therefore$ AB = BC = CD = DA = $\sqrt{26}$

In rhombus two diagonals are not same.

Here, $(AC)^2 = (-2-2)^2 + (3+1)^2 = 16 + 16 = 32$
$(BD)^2 = (-3-3)^2 + (-2-4)^2 = 36 + 36 = 72$

$\therefore \quad (AC)^2 \neq (BD)^2$

$\therefore$ ABCD is a rhombus.

Illu. 11. Prove that (3, 2), (5, 4), (3, 6), (1, 4) are the vertices of a square. ચોરસ
[BCA (Sem – II) April-2009]

Sol. Here, A(3, 2), B (5, 4), C (3, 6), D (1, 4) are the given points.

Using distance formula

$(AB)^2 = (x_2 - x_y)^2 + (y_2 - y_1)^2$
$= (5-3)^2 + (4-2)^2$
$= 4 + 4 = 8$

Similarly

$(BC)^2 = (3-5)^2 + (6-4)^2 = 4 + 4 = 8$
$(CD)^2 = (1-3)^2 + (4-6)^2 = 4 + 4 = 8$
$(DA)^2 = (3-1)^2 + (2-4)^2 = 4 + 4 = 8$

Therefore, ABCD is either square or a rhombus. But in a square two diagonals are also equal.

Now diagonal $(AC)^2 = (3-3)^2 + (6-2)^2$
$= 0 + 16 = 16$

and $(BD)^2 = (1-5)^2 + (4-4)^2$
$= 16 + 0$
$= 16$

Thus, $(AC)^2 = (BD)^2 = 16$

$\therefore$ AC = BD = 4

Hence ABCD is a square.

Illu. 12. **Show that (1, –1), (2, 1) and (4, 5) are collinear points.**

Sol. Here, A (1, –1) , B (2, 1) and C (4, 5) are the given points.

Using distance formula if A, B, C are collinear, then

$$AB + BC = AC$$

$$(AB)^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$$
$$= (2 - 1)^2 + (1 + 1)^2 = 1 + 4 = 5$$

$$\therefore \ AB = \sqrt{5}$$

Similarly,

$$(BC)^2 = (4 - 2)^2 + (5 + 1)^2 = 4 + 16 = 20$$

$$\therefore \quad BC = 2\sqrt{5}$$

and $(AC)^2 = (4 - 1)^2 + (5 - 1)^2 = 9 + 36 = 45$

$$\therefore \quad AC = 3\sqrt{5}$$

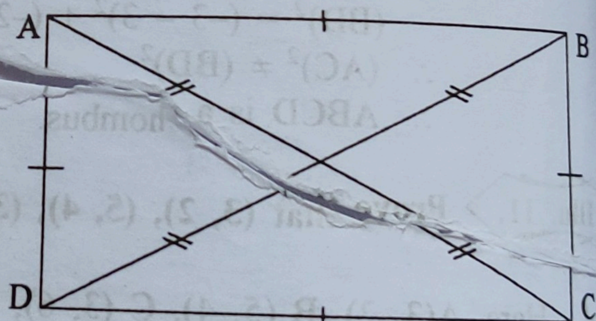
Since $AB + BC = \sqrt{5} + 2\sqrt{5} = 3\sqrt{5} = AC$

$\therefore$ A, B, and C are collinear points.

Illu. 13. **Show that the points (2, –2), (14, 10), (11, 13) and (1, 1) are the vertices of a rectangle.**

Sol. Here, A (2, – 2), B (14, 10), C (11, 13) and D (1, 1) are the given points.

We know the property of rectangle is that its opposite sides are equal and also its diagonal are equal.



Using distance formula,

$$(AB)^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$$
$$= (14 - 2)^2 + (10 + 2)^2$$
$$= 144 + 144$$
$$= 288$$

Similarly,

$$(BC)^2 = (11 - 14)^2 + (13 - 10)^2 = 9 + 9 = 18$$
$$(CD)^2 = (-1 - 11)^2 + (1 - 13)^2 = 144 + 144 = 288$$
$$(DA)^2 = (-1 - 2)^2 + (1 + 2)^2 = 9 + 9 = 18$$

Also diagonal $(AC)^2 = (11 - 2)^2 + (13 + 2)^2$
$$= 81 + 225 = 306$$

and diagonal $(BD)^2 = (-1 - 14)^2 + (1 - 10)^2$
$$= 225 + 81 = 306$$

Since the opposite sides of the quadrilateral ABCD are equal i.e. AB = CD and BC = DA, and The diagonals are also equal i.e. AC = BD

$\therefore$ ABCD is a rectangle.

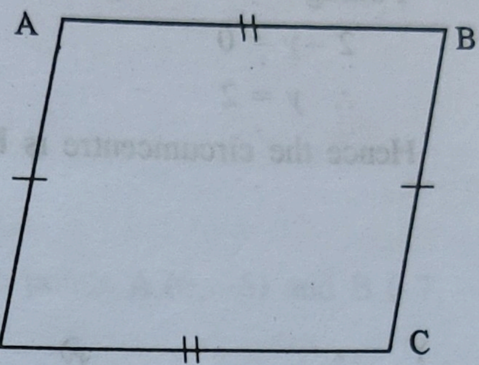
Illu. 14. **Prove that (– 2, – 1), (1, 0), (4, 3) and (1, 2) are the vertices of a parallelogram.**

Sol. Here, A (– 2, – 1), B (1, 0), C (4, 3) and D (1, 2) are the given points.

We know the property of Parallelogram is that its opposite sides are equal.

Using distance formula.

$$(AB)^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$$
$$= (1 + 2)^2 + (0 + 1)^2$$
$$= 9 + 1 = 10$$



Similarly,

$$(BC)^2 = (4 - 1)^2 + (3 - 0)^2 = 9 + 9 = 18$$
$$(CD)^2 = (1 - 4)^2 + (2 - 3)^2 = 9 + 1 = 10$$
$$(DA)^2 = (-2 - 1)^2 + (-1 - 2)^2 = 9 + 9 = 18$$

Since $(AB)^2 = (CD)^2$ and $(BC)^2 = (DA)^2$

$\therefore$ AB = CD and BC = DA

Hence ABCD is a parallelogram

$\therefore\ x = \frac{4}{2}$

$\therefore\ x = 2$

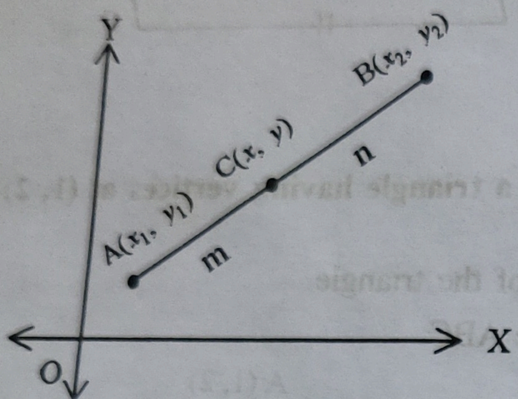
Putting $x = 2$ in equ. (ii) we get,

$2 - y = 0$

$\therefore\ y = 2$

Hence the circumcentre is P (2, 2)

[5] Section Formula



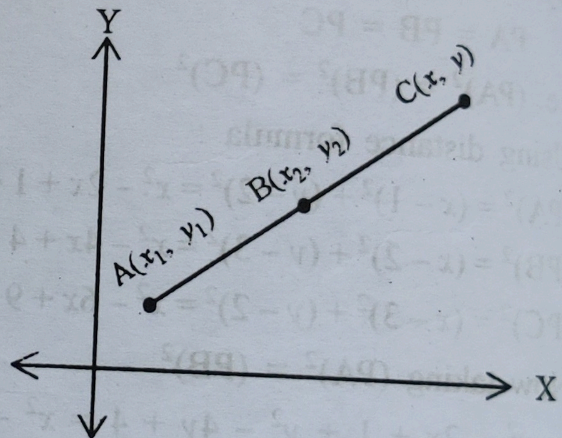
As shown in the adjoining figure, the co-ordinate of a point C (x, y) dividing a line internally in the ratio m : n connecting the points A (x_1, y_1) and B(x_2, y_2) is given by

$$x = \frac{mx_2 + nx_1}{m+n} \text{ and } y = \frac{my_2 + ny_1}{m+n}$$

* **External Division**

In the above figure it was assumed that the point C divides AB externally in the ratio m : n. If AB is divided externally by C i.e. C lies outside AB, then the co-ordinate of C will be,

$$x = \frac{mx_2 - nx_1}{m-n} \text{ and } y = \frac{my_2 - ny_1}{m-n}$$



Illu. 16. **Find the co-ordinates of the point which divides the points (6, –5) and (–7, – 15) internally in the ratio of 4 : 7.** **[Sau. Uni. B.C.A. Sem-II]**

Sol. Here, A (6, –5) and B (– 7, – 15) are the given points.

$\therefore$ The coordinates are

$x_1 = 6,\ y_1 = -5,\ x_2 = -7,\ y_2 = -15$

Internal division in the ratio of 4 : 7,

$\therefore$ m = 4, n = 7

By the section formula,

$$x = \frac{mx_2 + nx_1}{m+n} \quad \text{and} \quad y = \frac{my_2 + ny_1}{m+n}$$

$$= \frac{4(-7)+7(6)}{4+7} \qquad = \frac{4(-15)+7(-5)}{4+7}$$

$$= \frac{-28+42}{11} \qquad = \frac{-60-35}{11}$$

$$= \frac{14}{11} \qquad = \frac{-95}{11}$$

Hence, the co-ordinates of the points which divides the points A (6, –5) and B (–7, –15) internally in the ratio 4 : 7 is $\left(\frac{14}{11}, \frac{-95}{11}\right)$

Illu. 17. **Find the co-ordinate of the point which divides internally the join of the pair of points (5, 2) and (7, 9) in the ratio of 2 : 7**

Sol. Here the given co-ordinates are $x_1 = 5$, $y_1 = 2$, $x_2 = 7$, $y_2 = 9$

Internal division in the ratio 2 : 7

$\therefore$ m = 2 and n = 7

By section formula,

$$x = \frac{mx_2 + nx_1}{m+n} \quad \text{and} \quad y = \frac{my_2 + ny_1}{m+n}$$

$$= \frac{2(7)+7(5)}{2+7} \qquad = \frac{2(9)+7(2)}{2+7}$$

$$= \frac{14+35}{9} \qquad = \frac{18+14}{9}$$

$$= \frac{49}{9} \qquad = \frac{32}{9}$$

Hence, the co-ordinate of the point which divides the join of the pair of points (5, 2) and (7, 9) in the ratio of 2 : 7 is $\left(\frac{49}{9}, \frac{32}{9}\right)$

Illu. 18. **Find the co-ordinate of the point which divides internally the join of the pair of points (a + b, a – b) and (a – b, a + b) in the ratio of a : b**

Sol. Here the given co-ordinates are $x_1 = a + b$, $y_1 = a - b$, $x_2 = a - b$, $y_2 = a + b$

internal division in the ratio of a : b

$\therefore$ m = a and n = b

By section formula,

$$x = \frac{mx_2 + nx_1}{m+n} = \frac{a(a-b)+b(a+b)}{a+b} = \frac{a^2 - ab + ab + b^2}{a+b} = \frac{a^2+b^2}{a+b}$$

$$y = \frac{my_2 + ny_1}{m+n} = \frac{a(a+b)+b(a-b)}{a+b} = \frac{a^2 + ab + ab - b^2}{a+b} = \frac{a^2 + 2ab - b^2}{a+b}$$

Hence, the co-ordinate of the point which divides internally the join of the pair of points $(a + b, a - b)$ and $(a - b, a + b)$ in the ratio of $a : b$ is $\left(\frac{a^2+b^2}{a+b}, \frac{a^2+2ab-b^2}{a+b}\right)$

Illu. 19. **Find the co-ordinate of the point which divides the points (4, 7) and (1, – 2) externally in the ratio of 3 : 2.**

Sol. Here the given co–ordinates are $x_1 = 4$, $y_1 = 7$, $x_2 = 1$, $y_2 = -2$,

For external division in the ratio of 3 : 2

$\therefore$ m = 3 and n = 2

By section formula.

$$x = \frac{mx_2 - nx_1}{m-n} = \frac{3(1)-2(4)}{3-2} = \frac{3-8}{1} = -5$$

$$y = \frac{my_2 - ny_1}{m-n} = \frac{3(-2)-2(7)}{3-2} = -\frac{6-14}{1} = -20$$

Hence, the co-ordinate of the point which divides the points (4, 7) and (1, –2) externally in the ratio of 3 : 2 is (– 5, – 20)

Illu. 20. **Find the co-ordinates of the point which divides externally the join of the pair of points (p, q) and (q, p) in the ratio of p – q : p + q**

Sol. Here the given co-ordinates are $x_1 = p$, $y_1 = q$, $x_2 = q$, $y_2 = p$

For external division in the ratio of p – q : p + q

$\therefore$ m = p – q, n = p + q

By Section formula,

$$x = \frac{mx_2 - nx_1}{m-n} = \frac{(p-q)(q)-(p+q)(p)}{p-q-(p+q)}$$

$$y = \frac{my_2 - ny_1}{m-n}$$

$$y = \frac{(p-q)(p)-(p+q)(q)}{p-q-(p+q)}$$

$$= \frac{pq - q^2 - p^2 - qp}{p - q - p - q}$$

$$= \frac{-(p^2 + q^2)}{-2q}$$

$$= \frac{p^2 + q^2}{2q}$$

$$= \frac{p^2 - pq - pq - q^2}{p - q - p - q}$$

$$= \frac{p^2 - 2pq - q^2}{-2q}$$

$$= \frac{-(p^2 + 2pq + q^2)}{-2q}$$

$$= \frac{-(p^2 + 2pq + q^2)}{2q}$$

Hence, the co-ordinates of the point which divides externally the join of the pair of points (p, q) and (q, p) in the ratio of p – q : p + q is

$$\left(\frac{p^2 + q^2}{2q}, \frac{-(p^2 + 2pq + q^2)}{2q}\right)$$

Illu. 21. Find the ratio in which the join of the points A (4, 4) and B (7, 7) divided by C(– 1, – 1).

Sol. Let the points A (4, 4) and B (7, 7) be divided by C (– 1, – 1) in the ratio of m : 1

Here given co-ordinates are

$x_1 = 4,\ y_1 = 4,\ x_2 = 7,\ y_2 = 7$

m = m and n = 1

By Section formula,

$$x = \frac{mx_2 + nx_1}{m + n}$$

$$\therefore\ -1 = \frac{m(7) + 1(4)}{m + 1}$$

$\therefore\ -m - 1 = 7m + 4$

$\therefore\ -m - 7m = 4 + 1$

$\therefore\ -8m = 5$

$\therefore\ m = \frac{-5}{8}$

Here the value of m is negative therefore point C divides the line externally.

$\therefore$ C divides AB externally in the ratio of $\frac{5}{8} : 1$ i.e. 5 : 8

Illu. 22. Find the ratio in which the point (2, 14) divides the line segment joining (5, 4) and (11, – 16) externally.

Sol. Let the point C (2, 14) divides the line segment joining A (5, 4) and B (11, – 16) externally in the ratio of m : 1

Here the given co-ordinates are

$x_1 = 5,\ y_1 = 4,\ x_2 = 11,\ y_2 = -16$

For external division in the ratio of m : 1 by section formula.

$$x = \frac{mx_2 - nx_1}{m - n} \quad \& \quad y = \frac{my_2 - ny_1}{m - n}$$

$$2 = \frac{m(11) - 1(5)}{m - 1} \qquad 14 = \frac{m(-16) - 1(4)}{m - 1}$$

$\therefore$ 2m – 2 = 11 m – 5

$\therefore$ 2m – 11m = – 5 + 2

$\therefore$ –9m = – 3

$$\therefore m = \frac{-3}{-9}$$

$$\therefore m = \frac{1}{3}$$

$\therefore$ C divides AB externally in the ratio of $\frac{1}{3}$: 1 i.e. 1 : 3

Illu. 23. **In what ratio is the line segment joining (2, 3) and (5, –4) is divided by (i) X – axis and (ii) Y – axis ?**

Sol. (i) Let the point C(x, 0) be on the X-axis divides the join of points A(2, 3) and B(5, –4) in the ratio of m : 1

Here given co-ordinates are $x_1 = 2$, $y_1 = 3$, $x_2 = 5$, $y_2 = -4$, $x = x$, $y = 0$

By section formula.

$$y = \frac{my_2 + ny_1}{m + n}$$

$$\therefore 0 = \frac{m(-4) + 1(3)}{m + 1}$$

$\therefore$ 0 = – 4m + 3

$\therefore$ 4m = 3

$$\therefore m = \frac{3}{4}$$

$\therefore$ C divides AB internally in the ratio of $\frac{3}{4}$: 1 i.e. 3 : 4

(ii) Let the point C (0, y) be on the Y–axis divide the join of points A (2, 3) and B(5, – 4) in the ratio of m : 1.

Here the given co-ordinates are $x_1 = 2$, $y_1 = 3$, $x_2 = 5$, $y_2 = -4$, $x = 0$, $y = y$

By section formula,

$$x = \frac{mx_2 + nx_1}{m + n}$$

$$\therefore 0 = \frac{m(5) + 1(2)}{m + n}$$

$\therefore$ 0 = 5m + 2

$\therefore$ 5m = – 2

$$\therefore m = -\frac{2}{5}$$

$\therefore$ C divides AB externally (as the value of m is negative) in the ratio of $\frac{2}{5}$: 1 i.e. 2 : 5

Illu. 24. **Find the ratio in which the points C (–1, –1) divides the join of points A (3, 3) and B (7, 7) in the ratio of m : 1**

[BCA Sem. (II) Oct.-2009]

Sol. Let the point (–1, 1) divides the join of points A(3, 3) and B(7, 7) in the ratio of m : 1

Here the given co ordinates are $x_1 = 3$, $y_1 = 3$, $x_2 = 7$, $y_2 = 7$, $x = -1$, $y = -1$

By section formula.

$$x = \frac{mx_2 + nx_1}{m + n}$$

$$-1 = \frac{m(7) + 1(3)}{m + 1}$$

$$\therefore -m - 1 = 7m + 3$$

$$\therefore -m - 7m = 3 + 1$$

$$\therefore -8m = 4$$

$$\therefore m = -\frac{4}{8}$$

$$\therefore m = -\frac{1}{2}$$

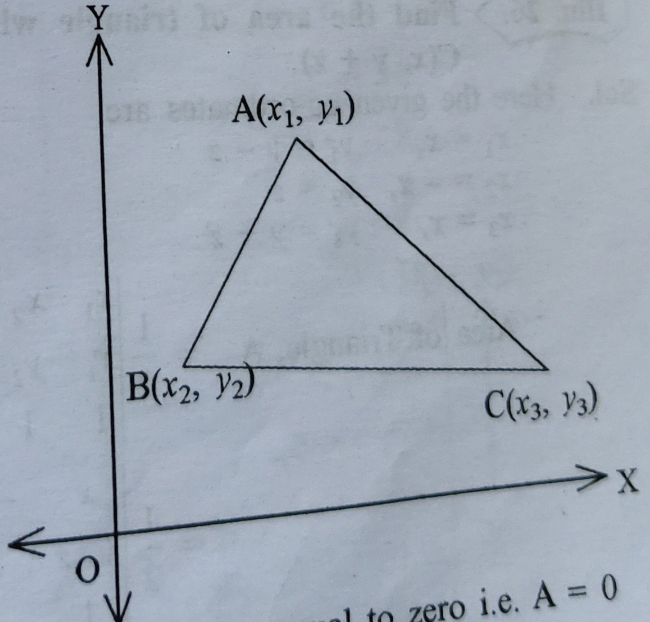
Hence C divides AB externally (as the value of m is found to be negative) in the ratio of

$\frac{1}{2} : 1$

i.e. 1 : 2

[6] Area of Triangle (without proof)

From the above figure, by connecting the points $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$ form a triangle.



The area of the triangle can be obtained by using the following formula.

Area of Triangle

$$A = \frac{1}{2}\begin{vmatrix} x_1 & x_2 & x_3 \\ y_1 & y_2 & y_3 \\ 1 & 1 & 1 \end{vmatrix}$$

$$= \frac{1}{2}\{x_1(y_2 - y_3) - x_2(y_1 - y_3) + x_3(y_1 - y_2)\}$$

If three points of a triangle are in a straight line, they are called collinear. The area of such a triangle is equal to zero i.e. A = 0

$$\therefore A = \frac{1}{2}\{x_1(y_2 - y_3) - x_2(y_1 - y_3) + x_3(y_1 - y_2)\} = 0$$

Illu. 25. **Find the area of the triangle whose vertices are (2, 3), (5, 7) and (– 3, 4)** **[Sau. Uni. B.C.A.]**

Sol. Here the given co ordinates are

$x_1 = 2, \quad y_1 = 3, \quad x_2 = 5, \quad y_2 = 7, \quad x_3 = -3, \quad y_3 = 4$

$$\text{Area of Triangle, } A = \frac{1}{2}\begin{vmatrix} x_1 & x_2 & x_3 \\ y_1 & y_2 & y_3 \\ 1 & 1 & 1 \end{vmatrix}$$

$$= \frac{1}{2}\{x_1(y_2 - y_3) - x_2(y_1 - y_3) + x_3(y_1 - y_2)\}$$

$$\therefore A = \frac{1}{2}\begin{vmatrix} 2 & 5 & -3 \\ 3 & 7 & 4 \\ 1 & 1 & 1 \end{vmatrix}$$

$$= \frac{1}{2}\{2(7-4) - 5(3-4) + (-3)(3-7)\}$$

$$= \frac{1}{2}\{2(-3) - 5(-1) - 3(-4)\}$$

$$= \frac{1}{2}\{-6 + 5 + 12\}$$

$$= \frac{1}{2}(11)$$

$$\therefore A = \frac{11}{2}$$

Illu. 26. **Find the area of triangle whose vertices are A (x, y – z), B (–x, z) and C(x, y + z)** **[BCA Sem. (II)) Oct. – 2009]**

Sol. Here the given co-ordinates are

$x_1 = x, \quad y_1 = y - z$

$x_2 = -x, \quad y_2 = z$

$x_3 = x, \quad y_3 = y + z$

$$\text{Area of Triangle, } A = \frac{1}{2}\begin{vmatrix} x_1 & x_2 & x_3 \\ y_1 & y_2 & y_3 \\ 1 & 1 & 1 \end{vmatrix}$$

$$= \frac{1}{2}\begin{vmatrix} x & -x & x \\ y-z & z & y+z \\ 1 & 1 & 1 \end{vmatrix}$$

$$= \frac{1}{2}\{x(z - (y+z)) - (-x)((y-z) - (y+z)) + x(y-z) - z)\}$$

$$= \frac{1}{2}\{x(z - y - z) + x(y - z - y - z) + x(y - z - z)\}$$

$$= \frac{1}{2}\{x(-y) + x(-2z) + x(y-2z)\}$$
$$= \frac{1}{2}\{-xy - 2xz + xy - 2xz\}$$
$$= \frac{1}{2}(-4xz)$$
$$= -2xz$$

$\therefore$ Area of Triangle $= |A|$
$= |-2xz|$
$= 2xz$

Illu. 27. For what value of K, the points (– 3, 8), (K, 5) and (–5, 2) will be collinear ?
[Sau. Un. B.C.A. (Sem.-II) April-2013]

Sol. If three points are collinear then Area of Triangle is equal to zero.
i.e. A = 0
Here the given co-ordinates are $x_1 = -3$, $y_1 = 8$, $x_2 = K$, $y_2 = 5$, $x_3 = -5$, $y_3 = 2$
Now,
Area of Triangle A = 0

$$\therefore \frac{1}{2}\begin{vmatrix} x_1 & x_2 & x_3 \\ y_1 & y_2 & y_3 \\ 1 & 1 & 1 \end{vmatrix} = 0$$
$$\therefore \frac{1}{2}\begin{vmatrix} -3 & K & -5 \\ 8 & 5 & 2 \\ 1 & 1 & 1 \end{vmatrix} = 0$$
$$\therefore \frac{1}{2}\{-3(5-2) - K(8-2) + (-5)(8-5)\} = 0$$
$$\therefore \frac{1}{2}\{-3(3) - K(6) - 5(3)\} = 0$$
$$\therefore \frac{1}{2}\{-9 - 6K - 15\} = 0$$
$$\therefore \frac{1}{2}\{-24 - 6K\} = 0$$
$$\therefore -24 - 6K = 0$$
$$\therefore -6K = 24$$
$$\therefore K = \frac{24}{-6}$$
$$\therefore K = -4$$

[7] Equations of Lines (without proof)

(7.1) Slope of A Line :

Straight line is defined as the shortest distance between two distinct points.

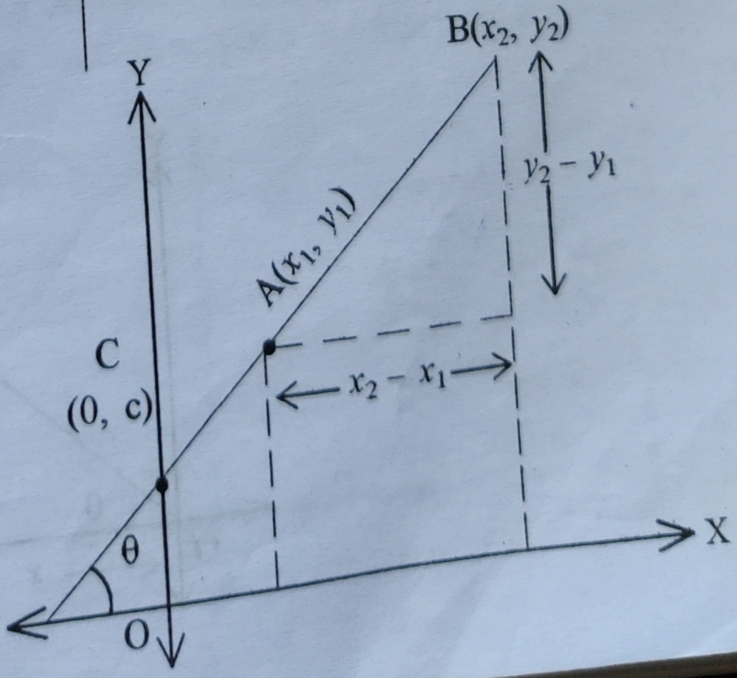
Equation of straight line is,

$y = mx + c$

where m = slope of line,

c = intercept on Y axis.

In the adjacent figure, a straight line is

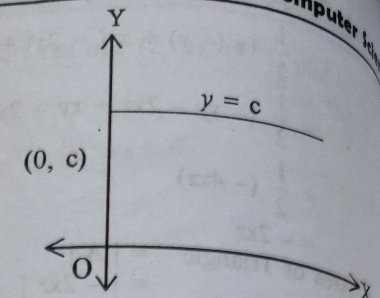


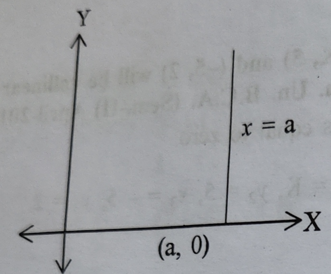
passing through two point A and B, intercept Y axis at C(0, c)

The slope of the line is given by,

$$\tan\theta = m = \frac{y_2 - y_1}{x_2 - x_1}$$

If the line is parallel to X-axis then the slope of line is zero i.e. m = 0 and equation of the line is $y = c$ (As shown in above figure)





If the line is parallel to Y-axis then the slope is not defined and the equation of the line is $x = a$ (As shown in side figure)

Illu. 28. **Find the slope of the line passing through the points A (1, 2) and B (3, 6)**

Sol. Here the given co-ordinates are $x_1 = 1$, $y_1 = 2$, $x_2 = 3$, $y_2 = 6$

$$\text{Slope of line, } m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{6-2}{3-1} = \frac{4}{2} = 2$$

(7.2) Origin Slope Form :

(The equation of a line passing through the origin and having slope m)