# Z- Transform

**Definitions**

1. The Z- transform of the sequence $\{f(k)\}$ is defined as $Z\{f(k)\} = F(z) = \sum_{k=-\infty}^{\infty} f(k)z^{-k}$

2. The Z- transform of the causal sequence $\{f(k)\}_{k=0}^{\infty}$ is $Z\{f(k)\} = F(z) = \sum_{k=0}^{\infty} f(k)z^{-k}$

**Note:**

**1.** $Z\{f(k)\} = \sum_{k=-\infty}^{\infty} f(k)z^{-k} = \cdots + f(-2)z^{-2} + f(-1)z^{1} + f(0)z^{-k} + f(1)z^{-1} + f(2)z^{-2} + \cdots$

**2.** $Z\{f(k)\} = \sum_{k=-\infty}^{\infty} f(k)z^{-k} = \underbrace{\sum_{k=-\infty}^{-1} f(k)z^{-k}}_{Non\ Causal} + \underbrace{\sum_{k=0}^{\infty} f(k)z^{-k}}_{Causal}$

**3.** $\frac{1}{1-x} = 1 + x + x^2 + ...,\ |x|<1$ , $\frac{1}{1-x} = 1 - x + x^2 \mp ...,\ |x|<1$

**4.** $\cos x = \frac{e^{ix} + e^{-ix}}{2}$, $\sin x = \frac{e^{ix} - e^{-ix}}{2i}$, $\cosh x = \frac{e^{x} + e^{-x}}{2}$, $\sinh x = \frac{e^{x} - e^{-x}}{2}$

**Multiple Choice Questions:**

$Z\{\delta(k)\} = \ldots$

A) $\frac{z}{z-1}$ B) $\frac{z}{1-z}$ **C)** 1 D) 0

1) $Z\{U(k)\} = \ldots$

**A)** $\frac{z}{z-1}$ B) $\frac{z}{1-z}$ C) 1 D) 0

2) $Z\{1\} = \ldots$

**A)** $\frac{z}{z-1}$ B) $\frac{z}{1-z}$ C) 1 D) 0

3) If $f(k) = a^k,\ k \geq 0$ then $Z\{f(k)\}$ in ROC $|z|>|a|$ is

A) $\frac{z}{z-1}$ **B)** $\frac{z}{z-a}$ C) $\frac{-z}{z-a}$ D) $\frac{az}{1-az} + \frac{z}{z-a}$

4) If $f(k) = a^k,\ k < 0$ then $Z\{f(k)\}$ in Roc $|z|<|a|$ is

A) $\frac{z}{z-1}$ B) $\frac{z}{z-a}$ **C)** $\frac{-z}{z-a}$ D) $\frac{az}{1-az} + \frac{z}{z-a}$

5) If $f(k) = a^{|k|},\ \forall k$ then $Z\{f(k)\}$ in Roc $|a|<|z|<\frac{1}{|a|}$ is

A) $\frac{z}{z-1}$ B) $\frac{z}{z-a}$ C) $\frac{-z}{z-a}$ **D)** $\frac{az}{1-az} + \frac{z}{z-a}$

6) If $f(k) = \cos \alpha k,\ k \geq 0$ then $Z\{f(k)\}$ in Roc $|z|>1$ is

**A)** $\frac{z(z-\cos\alpha)}{z^2 - 2z\cos\alpha + 1}$ B) $\frac{z\sin\alpha}{z^2 - 2z\cos\alpha + 1}$ C) $\frac{z(z-\cosh\alpha)}{z^2 - 2z\cosh\alpha + 1}$ D) $\frac{z\sinh\alpha}{z^2 - 2z\cosh\alpha + 1}$

7) If $f(k) = \sin \alpha k,\ k \geq 0$ then $Z\{f(k)\}$ in Roc $|z|>1$ is

A) $\frac{z(z-\cos\alpha)}{z^2 - 2z\cos\alpha + 1}$ **B)** $\frac{z\sin\alpha}{z^2 - 2z\cos\alpha + 1}$ C) $\frac{z(z-\cosh\alpha)}{z^2 - 2z\cosh\alpha + 1}$ D) $\frac{z\sinh\alpha}{z^2 - 2z\cosh\alpha + 1}$

8) If $f(k)=\cosh(\alpha k),\ k\geq 0$ then $Z\{f(k)\}$ in Roc $|z|>\max(|e^{\alpha}|,|e^{-\alpha}|)$ is

A) $\frac{z(z-\cos\alpha)}{z^2-2z\cos\alpha+1}$ B) $\frac{z\sin\alpha}{z^2-2z\cos\alpha+1}$ **C)** $\frac{z(z-\cosh\alpha)}{z^2-2z\cosh\alpha+1}$ D) $\frac{z\sinh\alpha}{z^2-2z\cosh\alpha+1}$

9) If $f(k)=\sinh(\alpha k),\ k\geq 0$ then $Z\{f(k)\}$ in Roc $|z|>\max(|e^{\alpha}|,|e^{-\alpha}|)$ is

A) $\frac{z(z-\cos\alpha)}{z^2-2z\cos\alpha+1}$ B) $\frac{z\sin\alpha}{z^2-2z\cos\alpha+1}$ C) $\frac{z(z-\cosh\alpha)}{z^2-2z\cosh\alpha+1}$ **D)** $\frac{z\sinh\alpha}{z^2-2z\cosh\alpha+1}$

10) If $f(k)=\frac{a^k}{k!},\ k\geq 0$ then $Z\{f(k)\}$ is

A) $e^{z/a}$ B) $e^{-a/z}$ C) $e^{-z/a}$ **D)** $e^{a/z}$

**Properties:**

11) If $Z\{af(k)+bg(k)\}=....$

**A)** $aZ\{f(k)\}+bZ\{g(k)\}$ B) $aZ\{f(k)\}-bZ\{g(k)\}$ C) $aZ^{-1}\{f(k)\}+bZ^{-1}\{g(k)\}$ D) $aZ^{-1}\{f(k)\}-bZ^{-1}\{g(k)\}$

12) If $Z^{-1}\{af(k)+bg(k)\}=....$

A) $aZ\{f(k)\}+bZ\{g(k)\}$ B) $aZ\{f(k)\}-bZ\{g(k)\}$ **C)** $aZ^{-1}\{f(k)\}+bZ^{-1}\{g(k)\}$ D) $aZ^{-1}\{f(k)\}-bZ^{-1}\{g(k)\}$

13) If $Z\{f(k)\}=F(z)$, then $Z\{a^k f(k)\}=....$

**A)** $F\left(\frac{z}{a}\right)$ B) $F\left(\frac{a}{z}\right)$ C) $F\left(e^{a}z\right)$ D) $F\left(e^{-a}z\right)$

14) If $Z\{f(k)\}=F(z)$, then $Z\{e^{-ak} f(k)\}=....$

A) $F\left(\frac{z}{a}\right)$ B) $F\left(\frac{a}{z}\right)$ **C)** $F\left(e^{a}z\right)$ D) $F\left(e^{-a}z\right)$

15) If $Z\{f(k)\}=F(z)$, then $Z\{kf(k)\}=....$

A) $F\left(\frac{z}{a}\right)$ **B)** $-z\frac{d}{dz}F(z)$ C) $\left(-z\frac{d}{dz}\right)^n F(z)$ D) $F\left(e^{-a}z\right)$

16) If $Z\{f(k)\}=F(z)$, then $Z\{kf(k)\}=....$

A) $F\left(\frac{z}{a}\right)$ B) $-z\frac{d}{dz}F(z)$ **C)** $\left(-z\frac{d}{dz}\right)^n F(z)$ D) $-\int^{z} z^{-1}F(z)dz$

17) If $Z\{f(k)\}=F(z),$ , then $Z\left\{\frac{f(k)}{k}\right\}=....$

**A)** $F\left(\frac{z}{a}\right)$ B) $-z\frac{d}{dz}F(z)$ C) $\left(-z\frac{d}{dz}\right)^n F(z)$ D) $-\int^{z} z^{-1}F(z)dz$

18) If $Z\{f(k)\}=F(z),\ k\geq 0$, then $Z\{f(k+1)\}=....$

A) $zF(z)+zf(0)$ **B)** $zF(z)-zf(0)$ C) $zF(z)-f(0)$ D) $z^2F(z)-f(0)$

19) If $Z\{f(k)\}=F(z),\ k\geq 0$, then $Z\{f(k+2)\}=....$

A) $z^2F(z)-zf(0)-f(1)$ B) $z^2F(z)+z^2f(0)+zf(1)$ C) $z^2F(z)+zf(0)+f(1)$ **D)** $z^2F(z)-z^2f(0)-zf(1)$

20) If $Z\{f(k)\}=F(z),\ k\geq 0$, then $Z\{f(k-1)\}=....$

**A)** $z^{-1}F(z)$ B) $z^{-1}F(z)-f(0)$ C) $z^{-2}F(z)-z^{-1}f(0)$ D) $zF(z)$

21) If $Z\{f(k)\}=F(z),\ k\geq 0$, then $Z\{f(k-2)\}=....$

**A)** $z^{-2}F(z)$ B) $z^{-2}F(z)-f(0)$ C) $z^{-2}F(z)-z^{-1}f(0)$ D) $zF(z)$

22) Convolution of two sequences $\{f(k)\}$ and $\{g(k)\}$ is $\{h(k)\}=\{f(k)\}*\{g(k)\}$. Then $Z\{h(k)\}$ is

A) $F(z)+G(z)$ **B)** $F(z)G(z)$ C) $F(z)-G(z)$ D) $\frac{F(z)}{G(z)}$

23)If $f(k)=\cos\pi k,\ k\geq 0$ then $Z\{f(k)\}$ in Roc $|z|>1$ is

A) $\frac{z(z-1)}{(z+1)^2}$ B) 0 C) $\frac{(z-1)}{(z+1)^2}$ D) $\frac{z}{z+1}$

24) If $f(k)=\sin\pi k,\ k\geq 0$ then $Z\{f(k)\}$ in Roc $|z|>1$ is

A) $\frac{z(z-1)}{(z+1)^2}$ B) 0 C) $\frac{(z-1)}{(z+1)^2}$ D) $\frac{z}{z+1}$

25)If $f(k)=\cos\left(\frac{\pi}{2}\right)k,\ k\geq 0$ then $Z\{f(k)\}$ in Roc $|z|>1$ is

**A)** $\frac{z^2}{z^2+1}$ B) $\frac{z^2}{z^2-1}$ C) $\frac{z}{z-1}$ D) $\frac{z}{z+1}$

26) If $f(k)=\sin\left(\frac{\pi}{2}\right)k,\ k\geq 0$ then $Z\{f(k)\}$ in Roc $|z|>1$ is

A) $\frac{z}{z^2+1}$ B) $\frac{z}{z^2-1}$ C) $\frac{z}{z-1}$ **D)** $\frac{z}{z^2+1}$

27) If $f(k)=2^k\sin\left(\frac{\pi}{2}\right)k,\ k\geq 0$ then $Z\{f(k)\}$ is

A) $\frac{2z}{z^2-4},|z|>2$ B) $\frac{2z}{z^2-4},|z|<2$ C) $\frac{2z}{z^2+4},|z|<2$ **D)** $\frac{2z}{z^2+4},|z|>2$

28) If $f(k)=2^k\cosh 3k,\ k\geq 0$ then $Z\{f(k)\}$ is

A) $\frac{z(z-2\cosh 3)}{z^2-4z\cosh 3+4},|z|>max(|e^2|,|e^{-2}|)$ **B)** $\frac{z(z-2\cosh 3)}{z^2-4z\cosh 3+4},|z|>max(|e^3|,|e^{-3}|)$

C) $\frac{z(z+2\cosh 3)}{z^2+4z\cosh 3+4},|z|<max(|e^3|,|e^{-3}|)$ D) $\frac{z(z+2\sinh 3)}{z^2+4z\sinh 3+4},|z|>max(|e^3|,|e^{-3}|)$

29) If $f(k)=2^k\sinh 3k,\ k\geq 0$ then $Z\{f(k)\}$ is

A) $\frac{z(z-2\cosh 3)}{z^2-4z\cosh 3+4},|z|>max(|e^2|,|e^{-2}|)$ B) $\frac{3z\sinh 3}{z^2+6z\cosh 3+9},|z|>max(|e^3|,|e^{-3}|)$

C) $\frac{3z\sinh 3}{z^2-6z\cosh 3+9},|z|>max(|e^3|,|e^{-3}|)$ D) $\frac{z(z+2\sinh 3)}{z^2+4z\sinh 3+9},|z|>max(|e^3|,|e^{-3}|)$

30)If $f(k)=2^k,\ k\geq 0$ then $Z\{f(k)\}$ is

**A)** $\frac{z}{z-4},\ |z|>2$ B) $\frac{z}{z-4},\ |z|<2$ C) $\frac{z}{z+4},\ |z|>2$ D) $\frac{z}{z+4},\ |z|<2$

31) If $f(k)=k,\ k\geq 0$ then $Z\{f(k)\}$ is

**A)** $\frac{z}{(z-1)^2},\ |z|>1$ B) $\frac{z}{(z-1)^2},\ |z|<1$ C) $\frac{z}{(z-1)},\ |z|>1$ D) $\frac{z}{(z-1)},\ |z|<1$

32)If $f(k)=k5^k,\ k\geq 0$ then $Z\{f(k)\}$ is

A) $\frac{z}{(z-1)^2},\ |z|>1$ B) $\frac{z}{(z-1)^2},\ |z|<1$ **C)** $\frac{5z}{(z-5)^2},\ |z|>5$ D) $\frac{5z}{(z-5)^2},\ |z|<5$

33) If $f(k)=\left(\frac{1}{3}\right)^k$, $k<0$ then $Z\{f(k)\}$ is

A) $\frac{3z}{3z-1}, |z|>\frac{1}{3}$ B) $\frac{3z}{3z-1}, |z|<\frac{1}{3}$ **C)** $\frac{3z}{1-3z}, |z|<\frac{1}{3}$ D) $\frac{3z}{1-3z}, |z|>\frac{1}{3}$

34) If $\{f(k)\}=\left\{\frac{1}{1^k}\right\}*\left\{\frac{1}{2^k}\right\}$ then $Z\{f(k)\}$ is

**A)** $\left(\frac{z}{z-1}\right)\left(\frac{2z}{2z-1}\right), |z|>1$ B) $\left(\frac{z}{z-1}\right)\left(\frac{2z}{2z-1}\right), |z|<1$ C) $\left(\frac{z}{z-1}\right)+\left(\frac{2z}{2z-1}\right), |z|>1$ D) $\left(\frac{z}{z-1}\right)\left(\frac{2z}{2z+1}\right), |z|>1$

**Inverse Z-Transform:**

35) If $F(z)=\frac{z}{z-a}$, then $Z^{-1}\{F(z)\}$ for $|z|>|a|$ is

**A)** $a^k,\ k\geq 0$ B) $-a^k,\ k<0$ C) $-a^{k-1},\ k<0$ D) $a^{k-1},\ k\geq 0$

36) If $F(z)=\frac{z}{z-a}$, then $Z^{-1}\{F(z)\}$ for $|z|<|a|$ is

A) $a^k,\ k\geq 0$ **B)** $-a^k,\ k<0$ C) $-a^{k-1},\ k<0$ D) $a^{k-1},\ k\geq 0$

37) If $f(z)=\frac{1}{z-a}$, then $Z^{-1}\{F(z)\}$ for $|z|>|a|$ is

A) $a^k,\ k\geq 0$ B) $-a^k,\ k<0$ **C)** $-a^{k-1},\ k\leq 0$ D) $a^{k-1},\ k\geq 1$

38) If $Z^{-1}\left\{\frac{1}{z-a}\right\}$, then $Z^{-1}\{F(z)\}$ for $|z|<|a|$ is

A) $a^k,\ k\geq 0$ B) $-a^k,\ k<0$ C) $-a^{k-1},\ k\leq 0$ **D)** $a^{k-1},\ k\geq 1$

39) If $F(z)=\frac{z^2}{(z-a)^2}$, then $Z^{-1}\{F(z)\}$ for $|z|>|a|$ is

**A)** $(k+1)a^k,\ k\geq 0$ B) $-(k+1)a^k,\ k<0$ C) $-a^{k-1},\ k\leq 0$ D) $a^{k-1},\ k\geq 1$

40) If $F(z)=\frac{z^2}{(z-a)^2}$, then $Z^{-1}\{F(z)\}$ for $|z|<|a|$ is

A) $(k+1)a^k,\ k\geq 0$ **B)** $-(k+1)a^k,\ k<0$ C) $-a^{k-1},\ k\leq 0$ D) $a^{k-1},\ k\geq 1$

41) If $F(z)=\frac{z}{(z-a)^2}$, then $Z^{-1}\{F(z)\}$ for $|z|>|a|$ is

A) $(k+1)a^k,\ k\geq 0$ B) $-(k+1)a^k,\ k<0$ **C)** $ka^{k-1},\ k\geq 0$ D) $-ka^{k-1},\ k<0$

42) If $F(z)=\frac{z}{(z-a)^2}$, then $Z^{-1}\{F(z)\}$ for $|z|<|a|$ is

A) $(k+1)a^k,\ k\geq 0$ B) $-(k+1)a^k,\ k<0$ C) $ka^{k-1},\ k\geq 0$ **D)** $-ka^{k-1},\ k<0$

43) By Inversion integral formula $Z^{-1}\{F(z)\}$ is

**A)** $\int_c F(z)z^{-k}dz$ B) $\frac{1}{2\pi}\int_c F(z)z^{-k}dz$ C) $\frac{2\pi}{1}\int_c F(z)z^{-k}dz$ D) $\int_c F(z)z^{k}dz$

44) By Inversion integral formula $Z^{-1}\{F(z)\}$ is

A) $2\pi i$ [sum of residues at $F(z)$] B) $2\pi i$ [sum of residues at $F(z)z^k$]

**C)** $2\pi i$ [sum of residues at $F(z)z^{-k}$] D) $2\pi i$