Assignment 1

AI1110: Probability and Random Variables
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1 12.13.3.5: Question

A laboratory blood test is 99% effective in detecting a certain disease when it is in fact, present. However, the test also yields a false positive result for 0.5% of the healthy person tested (i.e. if a healthy person is tested, then, with probability 0.005, the test will imply he has the disease). If 0.1 percent of the population actually has the disease, what is the probability that a person has the disease given that his test result is positive ?

Answer:

$$\frac{22}{133}$$

Solution: It is given that A is the event that blood test is positive. Let E1 be the event that person has a disease and E2 be the event that person don't have a disease.
According to the question $\Pr(A|E_1)$ is chances of positive blood test when person is having a disease is 99%. $\Pr(A|E_2)$ is chances of positive blood test when person is not having a disease is 0.5%

Here E1 and E2 are the events which are complimentary to each other. So by boolean logic,

$\Pr(E_1)$+ $\Pr(E_2)$ = 1
$\Pr(E_2)$ = 1 - $\Pr(E_1)$)
Then, $\Pr(E_2)$ = 1 -0.001 = 0.999

A:	Person with positive blood test	$\Pr(A)$
E_1:	Person suffering from a disease	$\Pr(E_1)$=0.001
E_2:	Person not suffering from a disease	$\Pr(E_2)$=0.999

TABLE 0: Given Information

Here, we are supposed to find the probability of a person has a disease given that his test result is positive. i.e. $\Pr(E_1|A)$

$$\Pr(E_1|A) = \frac{\Pr(E_1)\Pr(A|E_1)}{\sum_{i=1}^{2}\Pr(E_i)\Pr(A|E_i)} \tag{1}$$

$$\tag{2}$$

$$= \frac{0.001 \times 0.99}{0.001 \times 0.99 + 0.999 \times 0.005} \tag{3}$$

$$= \frac{0.00099}{0.00099 + 0.004995} \tag{4}$$

$$= \frac{0.00099}{0.005985} \tag{5}$$

$$= \frac{22}{133} \tag{6}$$

So,22/133 is the probability that a person has the disease given that his test result is positive.