

Assignment 1

AI1110: Probability and Random Variables
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1 12.13.3.5: QUESTION

A laboratory blood test is 99% effective in detecting a certain disease when it is in fact, present. However, the test also yields a false positive result for 0.5% of the healthy person tested (i.e. if a healthy person is tested, then, with probability 0.005, the test will imply he has the disease). If 0.1 percent of the population actually has the disease, what is the probability that a person has the disease given that his test result is positive ?

Answer:

$$\frac{22}{133}$$

Solution: It is given that A is the event that blood test is positive. Let E_1 be the event that person has a disease and E_2 be the event that person don't have a disease.

According to the question $\Pr(A|E_1)$ is chances of positive blood test when person is having a disease is 99%. $\Pr(A|E_2)$ is chances of positive blood test when person is not having a disease is 0.5%

Here E_1 and E_2 are the events which are complementary to each other. So by boolean logic,

$$\Pr(E_1) + \Pr(E_2) = 1 \quad (1)$$

$$\Pr(E_2) = 1 - \Pr(E_1) \quad (2)$$

$$\Pr(E_2) = 1 - 0.001 = 0.999 \quad (3)$$

Here, we are supposed to find the probability of a person has a disease given that his test result is positive. i.e. $\Pr(E_1|A)$

$$\Pr(E_1|A) = \frac{\Pr(E_1) \Pr(A|E_1)}{\sum_{i=1}^2 \Pr(E_i) \Pr(A|E_i)} \quad (4)$$

$$\quad (5)$$

$$= \frac{0.001 \times 0.99}{0.001 \times 0.99 + 0.999 \times 0.005} \quad (6)$$

$$= \frac{0.00099}{0.00099 + 0.004995} \quad (7)$$

$$= \frac{0.00099}{0.005985} \quad (8)$$

$$= \frac{22}{133} \quad (9)$$

So, 22/133 is the probability that a person has the disease given that his test result is positive.

A:	Person with positive blood test	$\Pr(A)$
E_1 :	Person suffering from a disease	$\Pr(E_1)=0.001$
E_2 :	Person not suffering from a disease	$\Pr(E_2)=0.999$
$A E_1$:	Event of positive blood test when person suffers from disease	$\Pr(A E_1)=0.99$
$A E_2$:	Event of positive blood test when person not suffers from disease	$\Pr(A E_2)=0.005$

TABLE 0: Given Information