Z-SCORE OR STANDARDIZED SCORE

What is Z-score and how to calculate them?

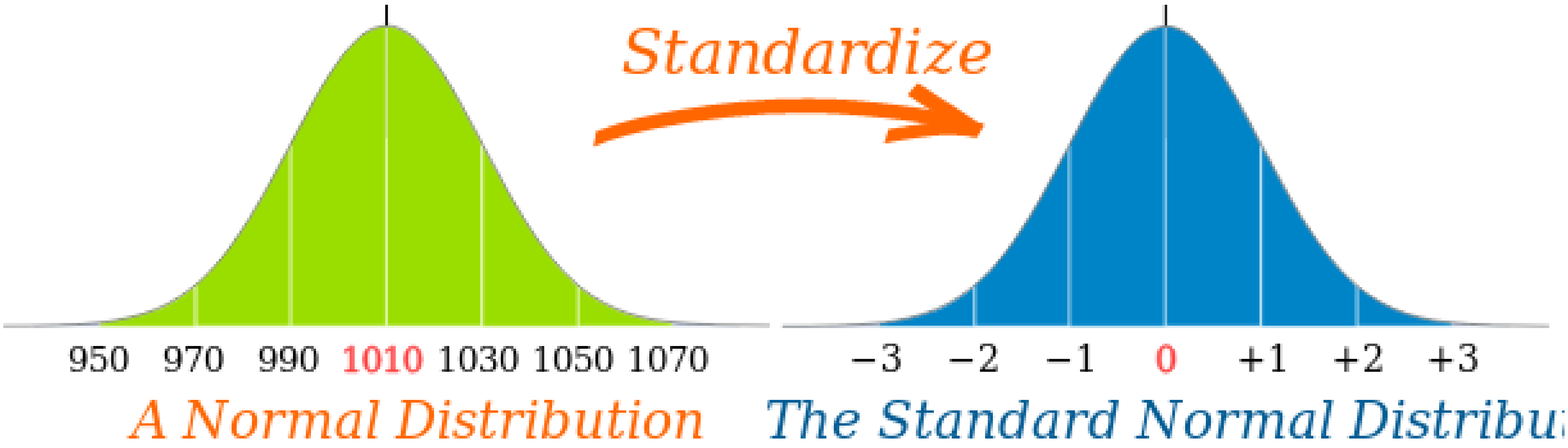
- A Standardized Score (Z-Score) is useful to know how many standard deviations an element falls from the mean.
- A z-score can be calculated from the following formula.

$$\text{Z Score} = \frac{X - \mu}{\sigma}$$

$$\text{Z Score} = \frac{\text{Raw score - Mean}}{\text{Standard deviation}}$$

- where z is the z-score, X is the value of the element, μ is the population mean, and σ is the standard deviation.

$$Z = \frac{\overset{\text{Score}}{x} - \overset{\text{Mean}}{\mu}}{\underset{\text{SD}}{\sigma}}$$


Standardize
950
970
990
1010
1030
1050
1070
A Normal Distribution
−3
−2
−1
0
+1
+2
+3
The Standard Normal Distribu

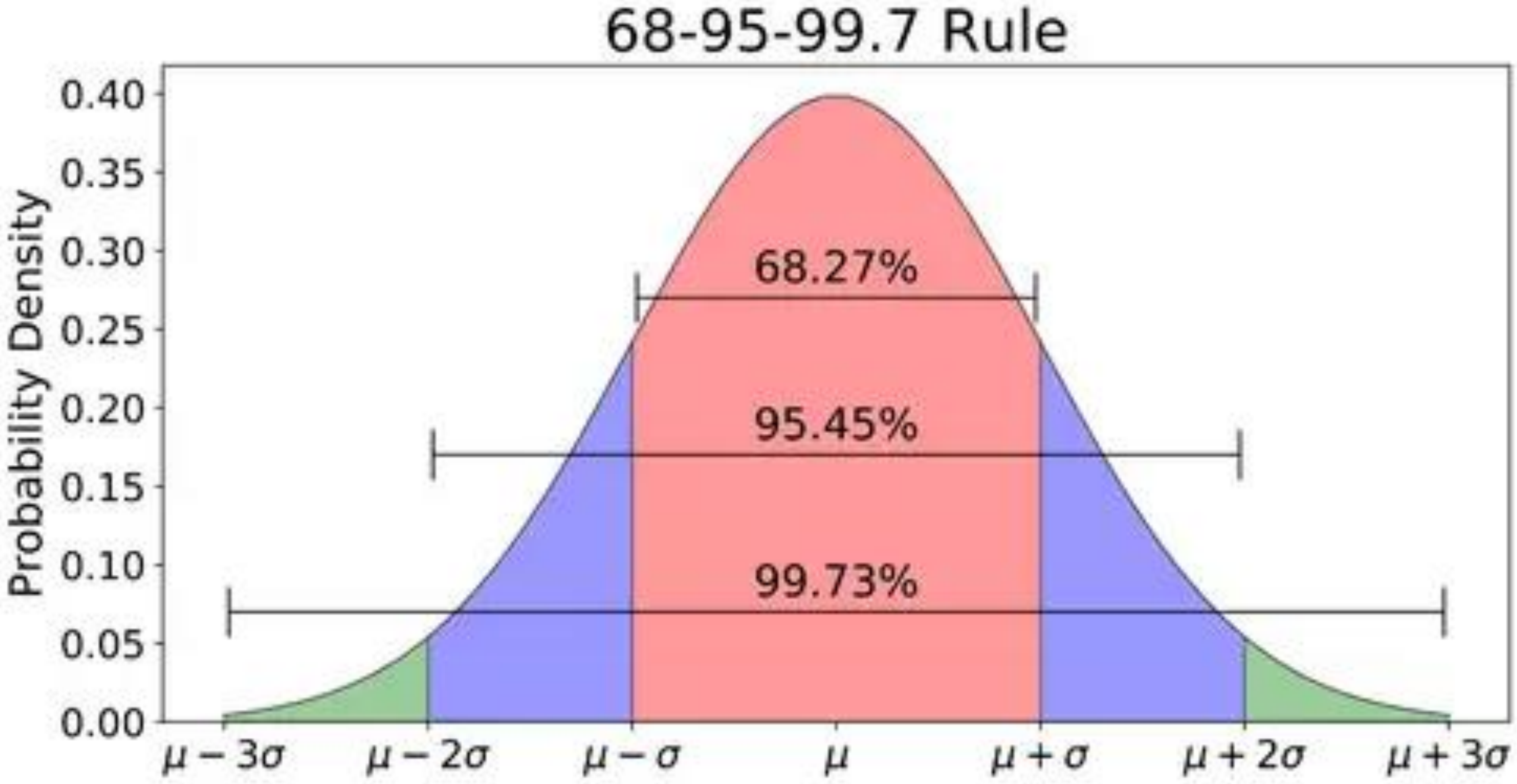
How do you interpret a z-score?

The value of the z-score tells you how many standard deviations you are away from the mean. If a z-score is equal to 0, it is on the mean.

A positive z-score indicates the raw score is higher than the mean average. For example, if a z-score is equal to +1, it is 1 standard deviation above the mean.

A negative z-score reveals the raw score is below the mean average. For example, if a z-score is equal to -2, it is 2 standard deviations below the mean.

Another way to interpret z-scores is by creating a standard normal distribution (also known as the z-score distribution or probability distribution).


68-95-99.7 Rule
Probability Density
0.40
0.35
0.30
0.25
0.20
0.15
0.10
0.05
0.00
68.27%
95.45%
99.73%
$\mu - 3\sigma$
$\mu - 2\sigma$
$\mu - \sigma$
μ
$\mu + \sigma$
$\mu + 2\sigma$
$\mu + 3\sigma$

- A z-score is also known as a **standard score** and it can be placed on a normal distribution curve.
- Z-scores range from -3 standard deviations (which would fall to the far left of the normal distribution curve) up to +3 standard deviations (which would fall to the far right of the normal distribution curve).
- In order to use a z-score, you need to know the mean μ and also the population standard deviation σ.

Z-score will help to understand a specific observation is common or exceptional in your study.

As mean is the middle point. So, negative z-score represent values below the mean. While positive z-score represent values above the mean

If you add all Z-Score you will get a value 0 because positive and negative z-score will cancel out each other.

If your data is extremely right skewed then probably you will get large positive Z-Score. On the other way, if distribution is left skewed then you will get large negative Z-Score

Z-Score of Mean is 0 as it is the middle value

If value of $|Z|$ is greater than 2 then we can tell a distribution is **unusual or exceptional.**

Why Z-Score is needed?

- Sometimes, in your statistical analysis you want to figure out a specific observation is common or exceptional case.
- Then Z-score will help to understand the standard deviation it falls below or above the mean.

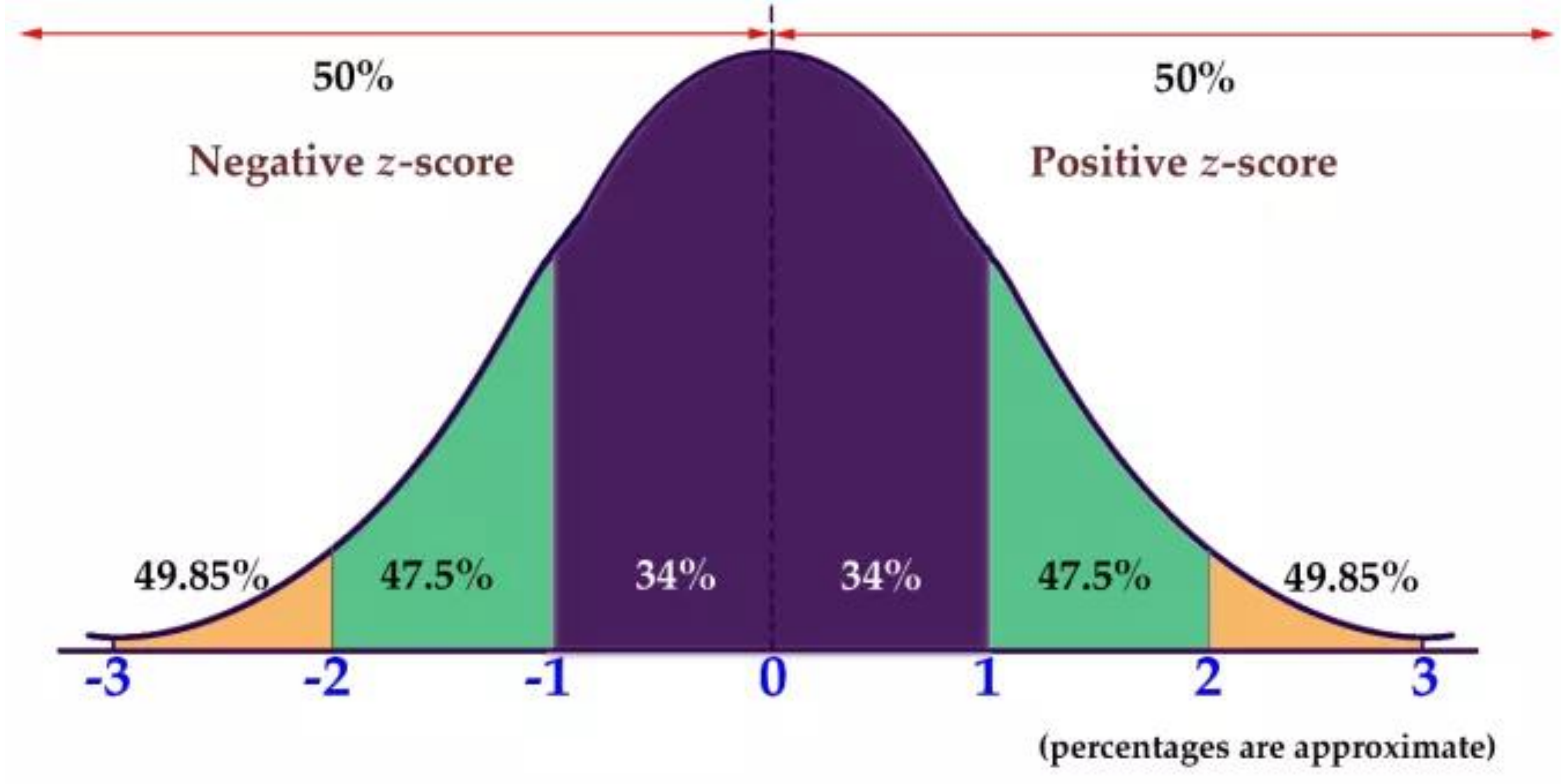
Bell Shaped Distribution and Empirical Rule:

If distribution is bell shape then it is assumed that :

-about 68% of the elements have a z-score between -1 and 1;

-about 95% have a z-score between -2 and 2;

-about 99% have a z-score between -3 and 3.



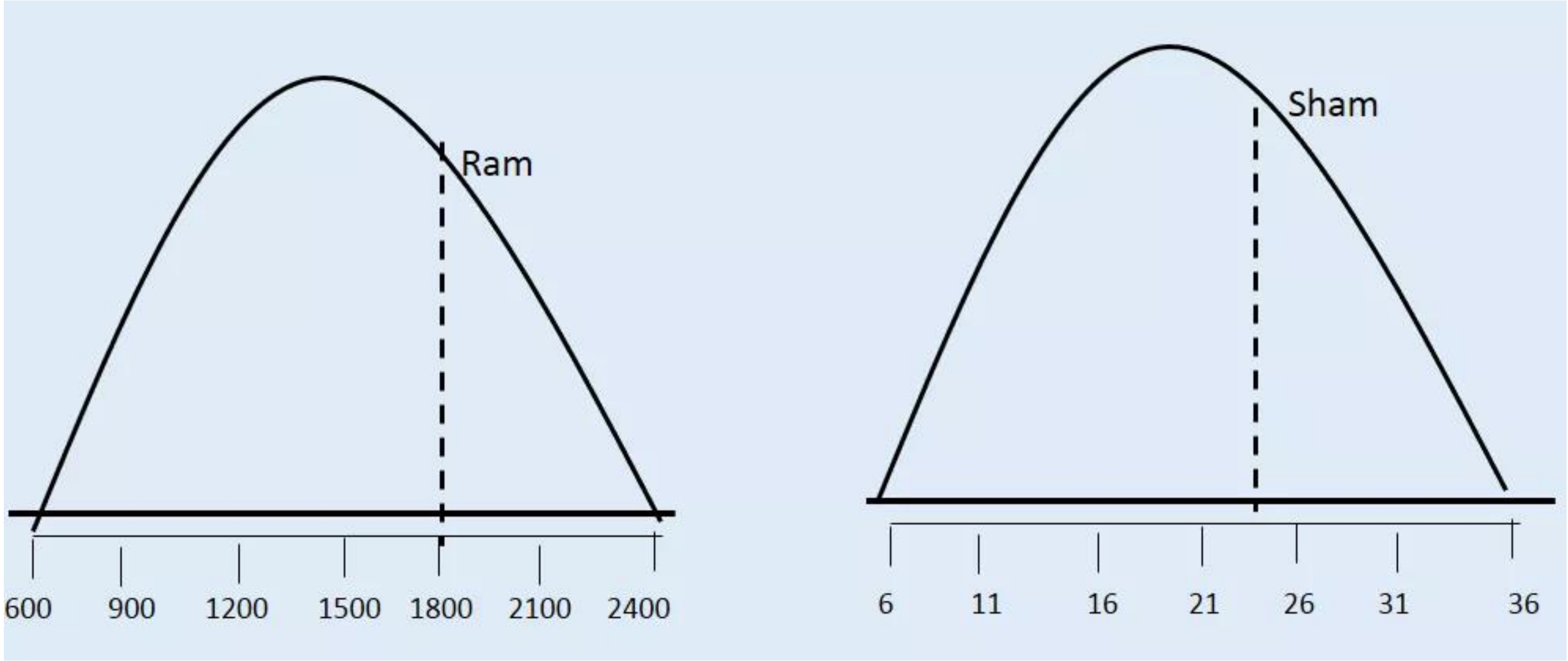
Let’s take an example why Z-Score are useful?

- A person is having two sons. He wants to know who scored better on their standardized test with respect to the other test takers.
- Ram who earned an 1800 on his SAT or Sham who scored a 24 on his ACT Exam ?

SAT Score ~ N(mean = 1500, Sd= 300)

ACT Score ~ N(mean = 21, Sd= 5)

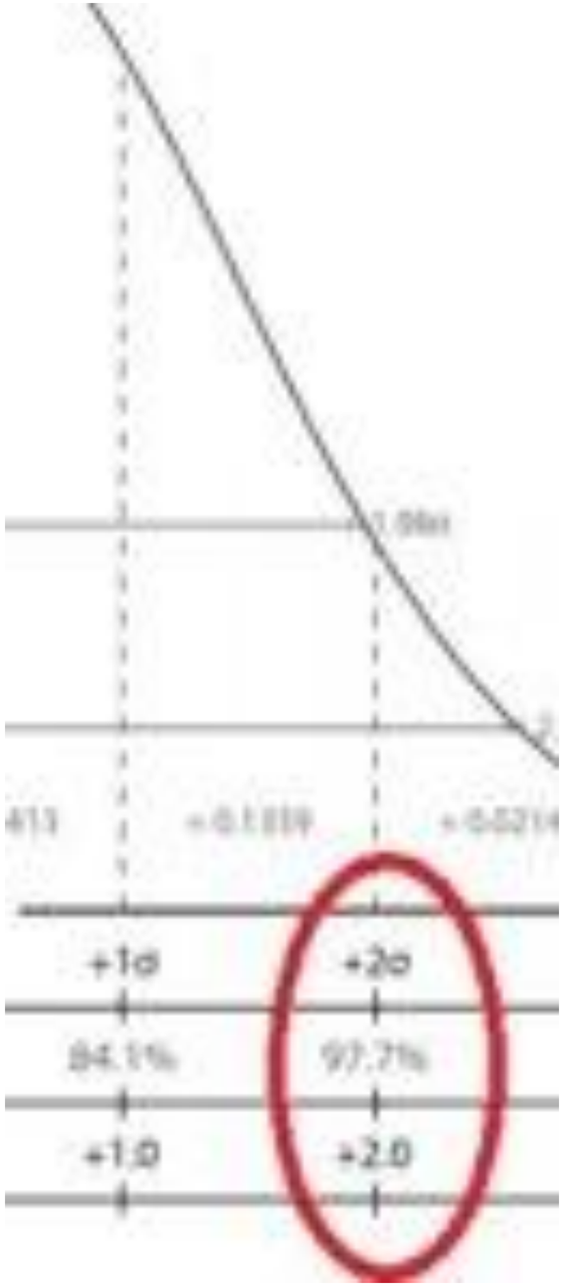
- Here we cannot simply compare and tell who has done better as they are measured in different scale.



So, his father will be interested to observe how many standard deviation of their respective mean of their distribution Ram and Sham score.

- Ram = (1800- 1500) / 300 =1 standard deviation above the mean
- Sham = (24 – 21) / 5= 0.6 standard deviation above the mean
- Now his father can conclude Ram indeed did a better score than Sham.
- If it is still not clear to you and want to explore more then can go to the below sites and have a look.

- The z-score in the center of the curve is zero.
- The z-scores to the **right of the mean are positive** and the z-scores to the **left of the mean are negative**.
- If you look up the score in the [z-table](), you can tell what *percentage* of the population is above or below your score.
- The table below shows a z-score of 2.0 highlighted, showing .9772 (which converts to 97.72%).
- If you look at the same score (2.0) of the normal distribution curve above, you'll see it corresponds with 97.72%.



Sample question:

You take the SAT and score 1100.

The mean score for the SAT is 1026 and the standard deviation is 209.

How well did you score on the test compared to the average test taker?

- Step 1: **Write your X-value into the z-score equation**. For this sample question the X-value is your SAT score, 1100.

$$Z = \frac{1100 - \mu}{\sigma}$$

- Step 2: **Put the mean, μ, into the z-score equation**.

$$Z = \frac{1100 - 1026}{\sigma}$$

Step 3: **Write the standard deviation, σ into the z-score equation**.

$$Z = \frac{1100 - 1026}{209}$$

- Step 4: **Calculate the answer using a calculator**: (1100 – 1026) / 209 = .354. This means that your score was .354 std devs above the mean.
- Step 5: (**Optional**) Look up your z-value in the z-table to see what percentage of test-takers scored below you.
- A z-score of .354 is .1368 + .5000* = .6368 or 63.68%.