HEIGHT AND DISTANCE

1.Question:A man is standing on the deck of a ship, which is 10m above water level. He observes the angle of elevation of the top of a light house as 60^0 and the angle of depression of the base of lighthouse as 30^0. Find the height of the light house.

Solution:

Let AB is the light house and the man is standing at C so, $\angle BCD = 60^0$ and $\angle ACD = 30^0$.
Let BD = h

In ΔADC, $\tan 30^0 = 10/CD$
$\Rightarrow 1/\sqrt{3} = 10/CD \Rightarrow CD = 10\sqrt{3}m$
In ΔBDC, $\tan 60^0 = h/CD$
$\Rightarrow \sqrt{3} = h/10\sqrt{3}$
$\Rightarrow h = 30m$
So the height of the light house is AB = AD + BD = 10 + 30 = 40m

2.Question:A person standing on the bank of a river observes that the angle of elevation of the top of a tree on the opposite bank is 45^0. When he moves 20m away from the bank, he finds the angle of elevation to be 30^0. Find the height of the tree.

Solution:
Let AB = x is the tree and AC = y is the river. Let the angle of elevation at point C is 45^0 and at point D is 30^0 s.t. CD = 20 m
In ΔACB

$\tan 45^0 = x/y \Rightarrow 1 = x/y \Rightarrow x = y$ ……(1)
In ΔADB, $\tan 30^0 = AB/AD$
$\Rightarrow 1/\sqrt{3} = x/ (20 + y)$
$\Rightarrow 1/\sqrt{3} = x/ (20 + x)$ [$\because$ of (1)]
$\Rightarrow 20 + x = \sqrt{3}x \Rightarrow (\sqrt{3}-1)x = 20$
$\Rightarrow x = 20/(\sqrt{3} - 1) = 20/(\sqrt{3} - 1) \times (\sqrt{3} + 1)/(\sqrt{3} + 1) = [20(\sqrt{3} + 1)]/3-1 \Rightarrow x = [20(\sqrt{3} + 1)]/2$
$\Rightarrow x = 10(\sqrt{3} + 1)m$

3.Question:From the top of a building 60m high, the angle of elevation and depression of the top and the foot of another building are α and β respectively. Find the height of the second building.

Solution:
Let AB is the building of height 60m and CD is the second building such that $\angle DBE = \alpha$ and $\angle CBE = \angle BCA = \beta$.

In ΔBAC, $\tan\beta = 60/AC$
$\Rightarrow BE = AC = 60/\tan\beta = 60\cot\beta$
In ΔBED, $\tan \alpha = DE/BE \Rightarrow \tan\alpha = DE/60\cot\beta$
$\Rightarrow DE = 60 \cot \beta \tan \alpha$
$\therefore$ The height of the building = CD = CE + ED

$= 60 + 60 \cot \beta \tan \alpha$
$= 60 (1 + \tan \alpha \cot \beta)$

4.Question:From the top of a tower 75m high, the angles of depression of the top and bottom of a pole standing on the same plane as the tower are observed to be 30^0 and 45^0 respectively. Find the height of the pole.

Solution:
Let AB is the tower of height 75 m and CD is the pole, such that $\angle BDE = 30^0$ and $\angle BCA = 45^0$
In ΔBAC, $\tan 45^0 = AB/AC$

$\Rightarrow 1 = AB/AC \Rightarrow AB = AC \Rightarrow AC = 75m$
Now $DE = AC = 75m$
In ΔBED,
$\tan 30^0 = BE/DE$
$\Rightarrow 1/\sqrt{3} = BE/75 \Rightarrow BE = 75/\sqrt{3}m$
$\Rightarrow BE = 25\sqrt{3}m = 43.3$ m
Hence the height of the pole
$= CD = AE = AB – BE = 75 – 43.3 = 31.7m$

5.Question:A 10 m long flagstaff is fixed on the top of a tower on the horizontal plane. From a point on the ground, the angles of elevation of the top and bottom of the flagstaff are 60^0 and 45^0 respectively. Find the height of the tower.

Solution:
Let AB is the tower of height x m and BC is the flagstaff of height 10 m. Let D be the point from where the angles of elevation are 45^0 and
60^0 such that $\angle BDA = 45^0$ and $\angle CDA = 60^0$

In ΔDAB, $\tan 45^0 = AB/AD$
$\Rightarrow 1 = x/AD \Rightarrow AD = x$
In ΔDAC, $\tan 60^0 = AC/AD$
$\Rightarrow \sqrt{3} = (10+x)/x$
$\Rightarrow \sqrt{3}x = 10+x$
$\Rightarrow (\sqrt{3}-1)x = 10$
$\Rightarrow x = 10/(\sqrt{3} - 1) = 10/(\sqrt{3} - 1) \times (\sqrt{3} + 1)/(\sqrt{3} + 1)$
$\Rightarrow x = 5(\sqrt{3} + 1)$

6.Question:The angles of elevation of the top of a tower from two points on the same side of the tower are α and β ($\alpha > \beta$). If the distance between the two points is 40m, find the height of the tower.

Solution:
Let AB is the tower of height x m and C, D are the points where the angles of elevation of the top of the tower are β and α respectively.
Also CD = 40 m

In ΔABD, $\tan \alpha = AB/AD \Rightarrow \tan\alpha = x/AD \Rightarrow AD = x/\tan\alpha$
In Δ ACB, $\tan \beta = AB/AC \Rightarrow \tan\beta = x/[40 + (x/\tan\alpha)]$

$\Rightarrow x = 40\tan\beta + x(\tan\beta/\tan\alpha)$
$\Rightarrow x(1-\tan\beta/\tan\alpha) = 40\tan\beta$
$\Rightarrow x[(\tan\alpha-\tan\beta)/\tan\alpha] = 40\tan\beta$
$\Rightarrow x = 40\tan\alpha\tan\beta/(\tan\alpha - \tan\beta)$

7.Question: The angle of elevation of the top of a tower from point A on the ground is 30^0. On moving a distance of 40m towards the foot of the tower, the angle of elevation increases to 45^0. Find the height of the tower.

Solution:
Let CD is the tower of height 'x' m and A, B are the points where the angles of elevation are 30^0 and 45^0 respect.
In Δ BCD, $\tan 45^0 = DC/BC$

$\Rightarrow 1 = x/BC \Rightarrow BC = x$ ....(1)
In ΔACD, $\tan 30^0 = CD/AC$
$\Rightarrow 1/\sqrt{3} = x/(40+x)$
$\Rightarrow 40+x = \sqrt{3}x$
$\Rightarrow (\sqrt{3} - 1)x = 40 \Rightarrow x = 40/(\sqrt{3}-1) = 40/(\sqrt{3}-1) \times (\sqrt{3}+1)/(\sqrt{3}+1) \Rightarrow x = 20 (\sqrt{} + 1) = 54.64$ m

8.Question: An aeroplane, when 4000m high from the ground, pass vertically above another aeroplane at an instance when the angles of elevation of the two aeroplanes from the same point on the ground are 60^0 and 30^0 respectively. Find the vertical distance between the two aero planes.

Solution:
Let A and B is the position of the aero planes such that AB = x. Also D is the point on the ground such that $\angle BDC = 30^0$ and $\angle ADC = 60^0$

In Δ ACD, $\tan 60^0 = AC/CD$
$\Rightarrow \sqrt{3} = 4000/CD$ m
$\Rightarrow CD = 4000/\sqrt{3}$ m
In Δ BCD, $\tan 30^0 = BC/CD$
$\Rightarrow 1/\sqrt{3} = BC/(4000/\sqrt{3}) \Rightarrow BC = 4000/\sqrt{3} \times 1/\sqrt{3} = 4000/3$m
$\therefore$ The distance between the planes = AB = AC – BC = 4000 - 4000/3 = 8000/3m

9.Question:A car is moving at uniform speed towards a tower. It takes 15 minutes for the angle of depression from the top of tower to the car to change from 30^0 to 60^0. What time after this, the car will reach the base of the tower?

Solution:
Let AB is the tower of height x m. Let C and D be the points on the ground where the angles of depression are 30^0 and 60^0 respectively. It took the car 15 minutes to go from C to D.

In Δ ABD, $\tan 60^0 = AB/AD \Rightarrow \sqrt{3} = x/AD \Rightarrow AD = x/\sqrt{3}$ m
Again in Δ BAC,
$\tan 30^0 = x/AC \Rightarrow 1/\sqrt{3} = x/AC \Rightarrow AC = \sqrt{3}x$ m
Now CD = AC – AD
$CD = \sqrt{3}x - x/\sqrt{3} = 2x/\sqrt{3}$ m.

Now the car covered $2x/\sqrt{3}$ m in 15 minutes
So it will cover AD = $x/\sqrt{}$m in $15 \times \sqrt{3}/2x \times x/\sqrt{3} = 7.5$ minutes.

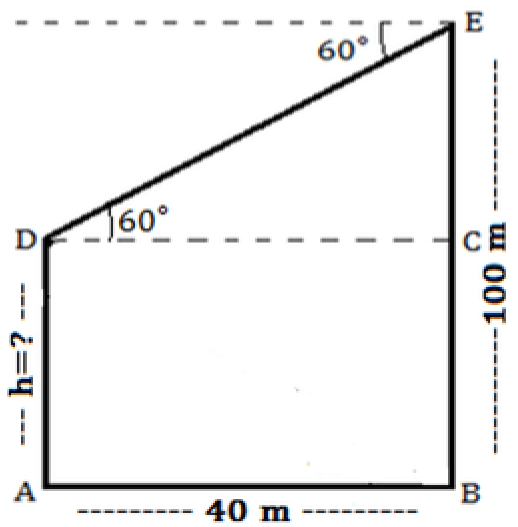
10.Question:A man is watching from the top of a tower, a boat speeding away from the tower. The angle of depression from the top of the tower to the boat is 60^0 when the boat is 80m from the tower. After 10 seconds, the angle of depression becomes 30^0. What is the speed of the boat? (Assume that the boat is running in still water).

Solution:
Let AB is the tower and boat is at points C and D when the angles of depression are 60^0 and 30^0 respectively.

In ΔABC,
$\tan 60^0 = AB/AC \Rightarrow \sqrt{3} = AB/80 \Rightarrow AB = 80\sqrt{3}$m
Again in ΔBAD,
$\tan 30^0 = AB/AD \Rightarrow 1/\sqrt{3} = 80\sqrt{3}/AD$
$\Rightarrow AD = 80\sqrt{3} \times \sqrt{3} = 240$m
$\therefore CD = 240 - 80 = 160$m
The boat took 10 seconds to cover 160m
$\therefore$ The speed of the boat = 160/10 = 16m/s

11.Question: Two buildings are 40 m apart. The angle of depression of the top of one building of height 100 m with the top of second building of unknown height is 60°. Find the height of second building?

Solution:



Le t the height of the second building AD be h.

EC = 100 - h

DC = AB = 40

$\frac{EC}{DC}\left(\frac{Perpendicular}{\text{Base}}\right) = \tan 60° \left(= \sqrt{3}\right)$

$\frac{100 - h}{40} = \sqrt{3}$

100 -h = 40 * √3

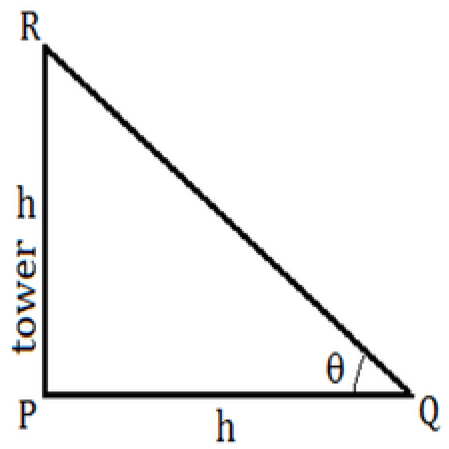
- h = -100 + 40 * 1.73

- h = -100 + 69.2

h = 30.8 m

12.Question: The angle of elevation of the top of a tower of height h meter at point Q is θ. If the distance between point Q and base of the tower is equal to the height of the tower, find the value of θ?

Solution:



Let the height of tower PR be h.

PQ = h as point Q is at a distance of h meter from the base of tower.

Then, $\frac{PR}{PQ}\left(\frac{Perpendicular}{\text{Base}}\right) = \tan\ \theta$

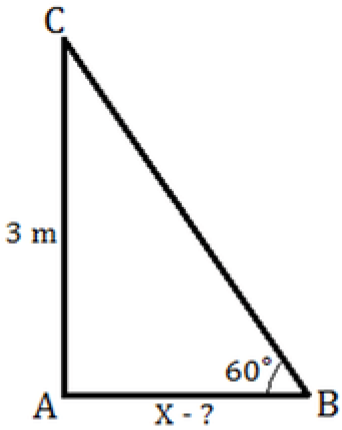
$\frac{h}{\text{h}} = \tan\ \theta$

tan θ=1

tanθ= 45°

13.Question: The shadow of a pole of height 3 meter when the angle of elevation of the sun is 60°, is

Solution:



Let the length of the shadow be AB = X meter.

Height of tower, AC = 3 meter

Then $\frac{AC}{AB}\left(\frac{Perpendicular}{Base}\right) = \tan 60° (\sqrt{3})$

$\frac{3}{X} = \sqrt{3}$

$X = \frac{3}{\sqrt{3}}$

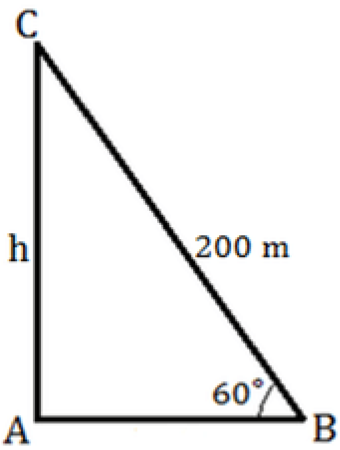
$X = \frac{3}{\sqrt{3}} * \frac{\sqrt{3}}{\sqrt{3}}$

$X = \frac{3\sqrt{3}}{3}$

X = √3 m

14.Question: A kite is flown with a thread of length 200 meter. The thread is fully stretched and makes an angle of 60° with the horizontal, find the height of the kite above the ground.

Solution:



Let height of the kite above the ground be AC = h.

Length of thread, BC = 200 m

$$\text{Then } \frac{AC}{BC}\left(\frac{Perpendicular}{\text{hypotenuse}}\right) = \sin 60° \quad \left(= \frac{\sqrt{3}}{2}\right)$$

$$\frac{h}{200} = \frac{\sqrt{3}}{2}$$

$$h * 2 = 200 * \sqrt{3}$$

$$h = \frac{200 * \sqrt{3}}{2}$$

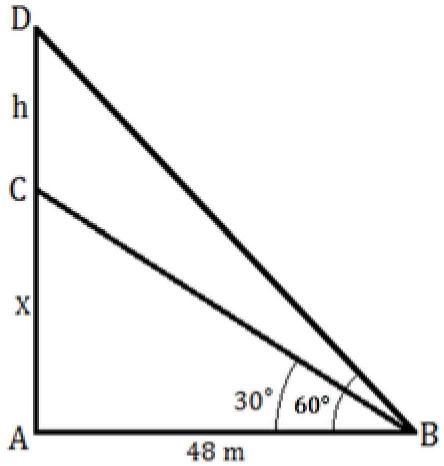
= 100 * √l3

= 100 * 1.73

= 173 m

15.Question: The top and bottom of a flag on a building subtend angles of 60° and 30° respectively at a point B which is 48 meter away from the building. Find the height of the flag?

SOLUTION:



Let height of building be AC = X and height of flag be CD = h.

From Δ DAB, $\frac{DA}{AB}\left(\frac{Perpendicular}{\text{Base}}\right) = \tan 60° (\sqrt{3})$

$\frac{X+h}{48} = \sqrt{3}$

$X + h = 48 * \sqrt{3}$

$h = 48\sqrt{3} - X$ ....(1)

From Δ CAB, $\frac{CA}{AB}\left(\frac{Perpendicular}{\text{Base}}\right) = \tan 30° \left(= \frac{1}{\sqrt{3}}\right)$

$\frac{X}{48} = \frac{1}{\sqrt{3}}$

$X = \frac{48}{\sqrt{3}}$ ....(2)

Put value of X in equation (1) from equation (2)

$h = 48\sqrt{3} - \frac{48}{\sqrt{3}}$

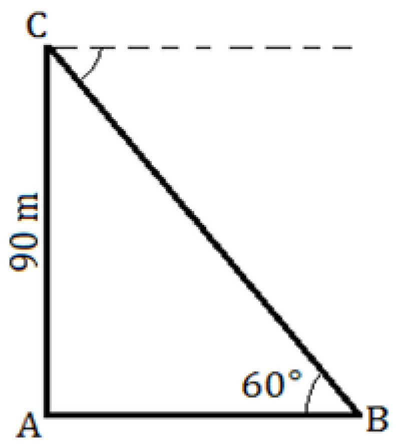
$= \frac{48*3-48}{\sqrt{3}}$

$= \frac{144-48}{\sqrt{3}} = \frac{96}{\sqrt{3}} \rightarrow \frac{96}{\sqrt{3}} * \frac{\sqrt{3}}{\sqrt{3}} \rightarrow \frac{96\sqrt{3}}{3} \rightarrow 32\sqrt{3}$ m

16.Question: From the top of a lighthouse which is 90 m above the sea, the angle of depression of a ship is 60°. How far is the ship from the lighthouse?

Solution:

Answer with explanation:



Let the height of the lighthouse above sea be AC and it is given 90 m.

Ship is at point B so the distance between the base of lighthouse A and ship is AB.

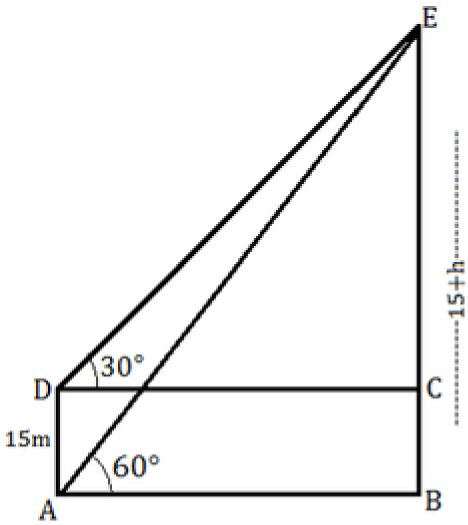
$$\text{Then, From } \Delta\, ABC, \frac{AC}{AB}\left(\frac{Perpendicular}{\text{Base}}\right) = \tan 60^\circ\,(\sqrt{3})$$

$$\frac{90}{AB} = \sqrt{3}$$

$$AB = \frac{90}{\sqrt{3}} * \frac{\sqrt{3}}{\sqrt{3}} = 30\sqrt{3}\text{ m}$$

17.Question: The angles of elevation of the top of a tower from the top and bottom of a tree of height 15 m are 30° and 60° respectively. Find the height of the tower?

Solution:



Let the CE be h meter.

Height of tree be AD = 15m

BE is the height of tower = BC + CE = 15 + h

AB = CD, let it is = X m

From Δ DCE, $\frac{EC}{CD}\left(\frac{Perpendicular}{Base}\right) = \tan 30° \left(\frac{1}{\sqrt{3}}\right)$

$\frac{h}{X} = \frac{1}{\sqrt{3}}$

$X = \sqrt{3}\,h$ ……………………(1)

From Δ ABE, $\frac{BE}{AB}\left(\frac{Perpendicular}{Base}\right) = \tan 60° (\sqrt{3}\,)$

$\frac{15+h}{X} = \sqrt{3}$

$X = \frac{15+h}{\sqrt{3}}$ ……………………(2)

From equation (1) and (2):

$\sqrt{3}\,h = \frac{15+h}{\sqrt{3}}$

3 h = 15 + h

2 h = 15

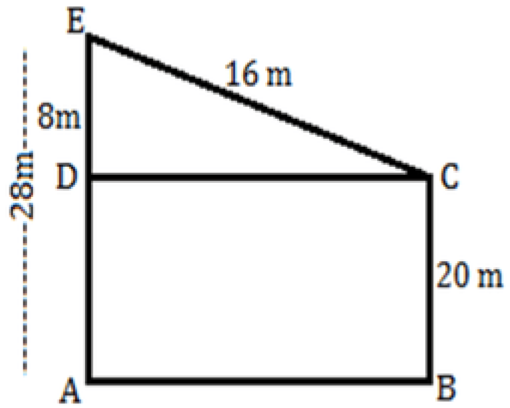
h = 7.5 m

Height of tower = 15 + 7.5 = 22.5 m

18.Question: The distance between the tops of two trees is 16 m. If the heights of the trees are 20 m and 28 m respectively, find the horizontal distance between the two trees?

Solution:

Answer with explanation:



Let AE and BC be the heights of trees.

AE = 28 m

BC = 20 m

Horizontal distance between trees AB = DC

In Δ EDC, $EC^2 = ED^2 + DC^2$ (Pythagoras theorem**)**

$DC^2 = EC^2 - ED^2$

$= 16^2 - 8^2$

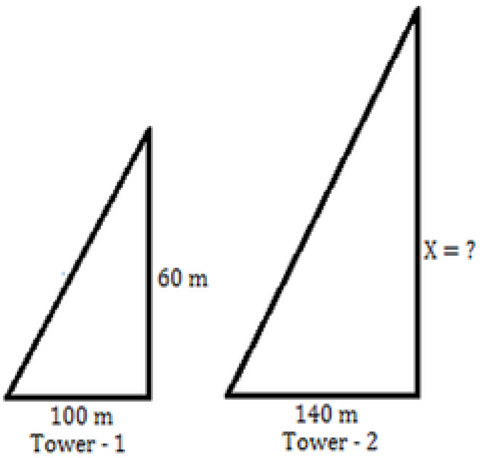
$= 256 - 64$

$DC^2 = 192$

DC =√192 m

19.Questions: There are two towers. The first tower of height 60 m casts a shadow of length 100m. At the same time if the second tower casts a shadow of length 140 m, find its height?

Solution:



Let the height of the second tower = X

We know that the length of the shadow is directly proportional to the height of the tower.

Therefore; $\frac{X}{140} = \frac{60}{100}$

X = 84 m

20.Question: 40 % of 280 =?

Solution:

x % of a given number 'n' $= \frac{x}{100} * n$

x = 40 and n = 280

∴ 40 % of 280 $= \frac{40}{100} * 280 = 112$

21.Question: Whose 35% is 280?

Solution:

Let the required value is x.

As per question; 35% of x = 280

$\therefore \frac{35}{100} * x = 280$

$X = \frac{280 * 100}{35} = 800$

22.Question:A ladder is leaning against a wall, forming an angle of 60∘60^\circ60∘ with the ground. If the foot of the ladder is 4 m away from the wall, find the length of the ladder.
Solution:
Using cosθ=adjacent\hypotenuse :

cos60∘=4\L

1\2=4\L ⟹ L=8 m.

23.Question

The angle of depression from the top of a building 30 m high to a point on the ground is 45∘45^\circ45∘. Find the distance of the point from the base of the building.
Solution:
Using tanθ=opposite\adjacent=

tan45∘=30\x

1=30\x ⟹ x=30 m.

24. Question: Two buildings are 60 m apart. The height of one building is 30 m, and the height of the other is 20 m. Find the angle of elevation of the taller building from the top of the shorter building.
Solution:
The height difference is 30−20=10m. Using tanθ=opposite\adjacent :

tanθ=10\60=1\6.

θ=tan−1(1\6)≈9.46∘.

25.Question

The angle of elevation of the top of a lighthouse from a point 50 m away is 45∘45^\circ45∘. Find the height of the lighthouse.
Solution:
Using tanθ=opposite\adjacent= :

tan45∘=h\50\

1=h\50 ⟹ h=50 m.