

PROBABILITY

1. Basic Probability

1.Question: A box contains 3 red balls, 4 blue balls, and 5 green balls. If one ball is picked randomly, what is the probability that it is blue?

Solution:

Total balls = $3+4+5=12$

Favorable outcomes = 4 (blue balls)

Probability = $\frac{\text{Favorable outcomes}}{\text{Total outcomes}} = \frac{4}{12} = \frac{1}{3}$

2.Question: A bag contains 5 red balls, 7 blue balls, and 3 green balls. If one ball is drawn randomly, what is the probability that it is?

Solution:

Total number of balls = $5+7+3=15$

a) Probability of drawing a red ball = $\frac{\text{Number of red balls}}{\text{Total balls}} = \frac{5}{15} = \frac{1}{3}$

b) Probability of drawing a blue ball = $\frac{\text{Number of blue balls}}{\text{Total balls}} = \frac{7}{15}$

c) Probability of not a green ball = $1 - \text{Probability of green ball} = 1 - \frac{\text{Number of green balls}}{\text{Total balls}} = 1 - \frac{3}{15} = \frac{12}{15} = \frac{4}{5}$

3.Question: A card is drawn randomly from a standard deck of 52 cards. What is the probability that it is?

Solution:

Total cards in a deck = 52

a) Probability of drawing a heart = $\frac{\text{Number of hearts}}{\text{Total cards}} = \frac{13}{52} = \frac{1}{4}$

b) Probability of drawing a queen = $\frac{\text{Number of queens}}{\text{Total cards}} = \frac{4}{52} = \frac{1}{13}$

c) Probability of drawing a red card = $\frac{\text{Number of red cards}}{\text{Total cards}} = \frac{26}{52} = \frac{1}{2}$

4.Question: Two dice are rolled. What is the probability that the sum of the numbers on the dice is?

Solution:

Total outcomes when two dice are rolled = $6 \times 6 = 36$

a) Possible outcomes for a sum of 8: (2,6),(3,5),(4,4),(5,3),(6,2) = 5 outcomes
Probability = $\frac{5}{36}$

b) Possible outcomes for a sum less than 5: (1,1),(1,2),(1,3),(2,1),(2,2),(3,1) = 6 outcomes
Probability = $\frac{6}{36} = \frac{1}{6}$

5.Question: A box contains 10 bulbs, 6 of which are defective. If one bulb is chosen randomly, what is the probability that it is?

Solution:

- Total bulbs = 10
- Defective bulbs = 6

a) Probability of defective = $\frac{\text{Number of defective bulbs}}{\text{Total bulbs}} = \frac{6}{10} = \frac{3}{5}$

b) Probability of non-defective = $1 - \text{Probability of defective} = 1 - \frac{6}{10} = \frac{4}{10} = \frac{2}{5}$

2. Complementary Probability

1.Question: If the probability of raining today is 0.70.70.7, what is the probability that it will not rain?

Solution:

Probability of not raining = $1 - \text{Probability of raining} = 1 - 0.7 = 0.3$

2.Question :A die is rolled. What is the probability of not rolling a number greater than 4?

Solution:

Total outcomes = 6 (1, 2, 3, 4, 5, 6)

Outcomes where the number is greater than 4 = 2 (5 and 6)

Probability of rolling a number greater than 4 = $\frac{2}{6} = \frac{1}{3}$

Probability of not rolling a number greater than 4 = $1 - \frac{1}{3} = \frac{2}{3}$

3.Question :A bag contains 6 red balls, 4 green balls, and 5 blue balls. What is the probability of NOT picking a green ball when a ball is drawn randomly?

Solution:

Total balls = $6 + 4 + 5 = 15$

Green balls = 4

Probability of picking a green ball = $\frac{4}{15}$

Probability of NOT picking a green ball = $1 - \frac{4}{15} = \frac{11}{15}$

4.Question :A person randomly guesses the answer to a multiple-choice question with 4 options. What is the probability that the person does NOT guess the correct answer?

Solution:

Total options = 4

Correct answer = 1

Probability of guessing the correct answer = $\frac{1}{4}$

Probability of NOT guessing the correct answer = $1 - \frac{1}{4} = \frac{3}{4}$

5.Question :A card is drawn from a standard deck of 52 cards. What is the probability that the card drawn is NOT a heart?

Solution:

Total cards = 52

Hearts = 13

Probability of drawing a heart = $\frac{13}{52} = \frac{1}{4}$

Probability of NOT drawing a heart = $1 - \frac{1}{4} = \frac{3}{4}$

6.Question : The probability of a machine failing on a given day is 0.1. What is the probability that the machine does NOT fail on the same day?

Solution:

Probability of machine failing = 0.1

Probability of machine NOT failing = $1 - 0.1 = 0.9$

3. Conditional Probability

1.Question: A card is drawn from a standard deck of 52 cards. What is the probability that it is a face card, given that it is a spade?

Solution:

Face cards in spades = 3 (Jack, Queen, King)

Total spades = 13

Conditional probability = $\frac{3}{13}$

2.Question: A card is drawn at random from a standard deck of 52 cards. What is the probability that the card is a King, given that it is a face card?

Solution:

Total face cards in a deck = 12 (3 face cards for each of the 4 suits: Jack, Queen, King).

Favorable outcomes (Kings) = 4 (one King for each suit).

Conditional Probability = $\frac{\text{Favorable outcomes (Kings)}}{\text{Total outcomes (Face cards)}} = \frac{4}{12} = \frac{1}{3}$

3.Question: A factory produces 60% of its products from Machine A and 40% from Machine B. The probability that a product from Machine A is defective is 3%, and the probability that a product from Machine B is defective is 5%. If a randomly chosen product is defective, what is the probability it came from Machine A?

Solution:

This is a **Bayes' Theorem** problem.

Let $P(A)$ = Probability that the product came from Machine A = 0.6

Let $P(B)$ = Probability that the product came from Machine B = 0.4

Let $P(D|A)$ = Probability that a product is defective given it came from Machine A = 0.03

Let $P(D|B)$ = Probability that a product is defective given it came from Machine B = 0.05

The total probability that a product is defective:

$P(D) = P(A)P(D|A) + P(B)P(D|B) = (0.6)(0.03) + (0.4)(0.05) = 0.018 + 0.02 = 0.038$

Probability that the defective product came from Machine A:

$P(A|D) = \frac{P(A)P(D|A)}{P(D)} = \frac{(0.6)(0.03)}{0.038} = \frac{0.018}{0.038} = 0.4737$

Answer: $\approx 47.37\%$

4.Question: A box contains 5 red marbles and 3 blue marbles. Two marbles are drawn one after the other without replacement. What is the probability that the second marble is blue, given that the first marble was red?

Solution:

After the first marble is red, there are 4 red marbles and 3 blue marbles left.

Total marbles left = 7

Probability that the second marble is blue = $\frac{\text{Number of blue marbles}}{\text{Remaining total marbles}} = \frac{3}{7}$

Answer: $\frac{3}{7}$

5.Question: A coin is tossed twice. What is the probability of getting heads on the second toss, given that the first toss was a head?

Solution:

The two tosses are independent events.

The outcome of the second toss does not depend on the first toss.

Probability of heads on the second toss = $\frac{1}{2}$

4. Independent Events

1.Question: A coin is tossed twice. What is the probability of getting heads on both tosses?

Solution:

Probability of heads in one toss = $\frac{1}{2}$

Since the tosses are independent,

Probability of heads on both tosses = $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$

2.Question : A coin is tossed and a die is rolled. What is the probability of getting a "heads" on the coin and a "6" on the die?

Solution:

Probability of getting heads on the coin = $\frac{1}{2}$

Probability of getting a 6 on the die = $\frac{1}{6}$

Since these are independent events:

Probability of both events = $\frac{1}{2} \times \frac{1}{6} = \frac{1}{12}$

Answer: $\frac{1}{12}$

3.Question : Two cards are drawn from two different decks of cards (one card from each deck). What is the probability that both cards are aces?

Solution:

Probability of drawing an ace from the first deck = $\frac{4}{52} = \frac{1}{13}$

Probability of drawing an ace from the second deck = $\frac{4}{52} = \frac{1}{13}$

Since these are independent events:

Probability of both events = $\frac{1}{13} \times \frac{1}{13} = \frac{1}{169}$

Answer: $\frac{1}{169}$

4.Question : A bag contains 5 red balls and 3 green balls. Another bag contains 4 blue balls and 6 yellow balls. If one ball is drawn from each bag, what is the probability of getting a red ball from the first bag and a blue ball from the second bag?

Solution:

Probability of getting a red ball from the first bag = $\frac{5}{8}$

Probability of getting a blue ball from the second bag = $\frac{4}{10} = \frac{2}{5}$

Since these are independent events:

Probability of both events = $\frac{5}{8} \times \frac{2}{5} = \frac{10}{40} = \frac{1}{4}$

Answer: $\frac{1}{4}$

5.Question : A factory has two independent machines. Machine A produces a defective item with a probability of 0.02. Machine B produces a defective item with a probability of 0.03. What is the probability that both machines produce a defective item on the same day?

Solution:

Probability of defective item from Machine A = 0.02

Probability of defective item from Machine B = 0.03

Since these are independent events:

Probability of both events = $0.02 \times 0.03 = 0.0006$

Answer: 0.0006 or 0.06%

6.Question: A coin is tossed 3 times. What is the probability of getting heads on the first toss, tails on the second toss, and heads on the third toss?

Solution:

Probability of heads on a single toss = $\frac{1}{2}$

Probability of tails on a single toss = $\frac{1}{2}$

Since these are independent events:

Probability = $\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{8}$

Answer: $\frac{1}{8}$

5. Dependent Events

1.Question: A bag contains 5 white balls and 3 black balls. If two balls are drawn one after the other without replacement, what is the probability that both are white?

Solution:

Probability of first white ball = $\frac{5}{8}$

Probability of second white ball (after one white is removed) = $\frac{4}{7}$

Probability of both white = $\frac{5}{8} \times \frac{4}{7} = \frac{20}{56} = \frac{5}{14}$

2.Question: A bag contains 5 red balls and 4 green balls. Two balls are drawn one after the other **without replacement**. What is the probability that both balls are red?

Solution:

Probability of drawing the first red ball = $\frac{5}{9}$

Probability of drawing the second red ball (after one red ball is removed) = $\frac{4}{8} = \frac{1}{2}$

Probability of both balls being red = $\frac{5}{9} \times \frac{1}{2} = \frac{5}{18} = \frac{5}{18}$

Answer: $\frac{5}{18}$

3.Question: A card is drawn from a standard deck of 52 cards. Without replacing the first card, a second card is drawn. What is the probability that both cards are aces?

Solution:

Total aces in the deck = 4

Probability of drawing the first ace = $\frac{4}{52} = \frac{1}{13}$

After the first ace is drawn, remaining aces = 3 and total cards = 51

Probability of drawing the second ace = $\frac{3}{51} = \frac{1}{17}$

Probability of both cards being aces = $\frac{1}{13} \times \frac{1}{17} = \frac{1}{221}$

Answer: $\frac{1}{221}$

4.Question: A class has 10 boys and 8 girls. Two students are selected one after the other **without replacement**. What is the probability that the first student is a girl and the second is a boy?

Solution:

Probability of selecting a girl first = $\frac{8}{18} = \frac{4}{9}$

After selecting a girl, remaining students = 17, and boys = 10

Probability of selecting a boy second = $\frac{10}{17}$

Probability of the first being a girl and the second being a boy = $\frac{4}{9} \times \frac{10}{17} = \frac{40}{153}$

Answer: $\frac{40}{153}$

5.Question : A jar contains 3 blue marbles, 4 red marbles, and 5 white marbles. Two marbles are picked randomly one after the other **without replacement**. What is the probability that the first marble is blue and the second marble is red?

Solution:

Probability of picking a blue marble first = $\frac{3}{12} = \frac{1}{4}$

After the blue marble is removed, remaining marbles = 11, and red marbles = 4

Probability of picking a red marble second = $\frac{4}{11}$

Probability of first blue and second red = $\frac{1}{4} \times \frac{4}{11} = \frac{1}{11}$

Answer: $\frac{1}{11}$

6.Question: On a shelf, there are 5 math books and 3 science books. If two books are picked one after the other **without replacement**, what is the probability that the first book is a math book and the second is a science book?

Solution:

Probability of picking a math book first = $\frac{5}{8}$

After one math book is removed, remaining books = 7, and science books = 3

Probability of picking a science book second = $\frac{3}{7}$

Probability of first math and second science = $\frac{5}{8} \times \frac{3}{7} = \frac{15}{56}$

Answer: $\frac{15}{56}$

6. Probability with Dice

1.Question: Two dice are rolled. What is the probability of getting a sum of 7?

Solution:

Possible outcomes for a sum of 7 = (1,6),(2,5),(3,4),(4,3),(5,2),(6,1) = 6 outcomes

Total outcomes when two dice are rolled = $6 \times 6 = 36$

Probability = $\frac{6}{36} = \frac{1}{6}$

2.Question: A single six-sided die is rolled. What is the probability of getting a number greater than 4?

Solution:

Possible outcomes = {1, 2, 3, 4, 5, 6}

Numbers greater than 4 = {5, 6} (2 outcomes)

Total outcomes = 6

Probability = $\frac{\text{Favorable outcomes}}{\text{Total outcomes}} = \frac{2}{6} = \frac{1}{3}$

3.Question: Two six-sided dice are rolled. What is the probability of getting a sum of 8?

Solution:

Possible combinations for a sum of 8:

(2,6), (3,5), (4,4), (5,3), (6,2) = 5 outcomes

Total outcomes when two dice are rolled = $6 \times 6 = 36$

Probability = $\frac{5}{36}$

4.Question: Two dice are rolled. What is the probability of getting doubles (same number on both dice)?

Solution:

Possible doubles: (1,1), (2,2), (3,3), (4,4), (5,5), (6,6) = 6 outcomes

Total outcomes = $6 \times 6 = 36$

Probability = $\frac{6}{36} = \frac{1}{6}$

5.Question: Two dice are rolled. What is the probability of getting at least one six?

Solution:

Probability of not getting a six on one die = $\frac{5}{6}$

Probability of not getting a six on both dice = $\frac{5}{6} \times \frac{5}{6} = \frac{25}{36}$

Probability of getting at least one six = $1 - \frac{25}{36} = \frac{11}{36}$

6.Question: Two dice are rolled. What is the probability that both dice show even numbers?

Solution:

Even numbers on a die = {2, 4, 6} = 3 outcomes

Probability of an even number on one die = $\frac{3}{6} = \frac{1}{2}$

Probability of even numbers on both dice = $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$

7.Question: Two dice are rolled. What is the probability that the product of the numbers on the dice is greater than 15?

Solution:

List all pairs of outcomes (i, j) where $i \times j > 15$:

- (3,6), (4,5), (4,6), (5,4), (5,5), (5,6), (6,3), (6,4), (6,5), (6,6)

Count = 10 outcomes

Total outcomes = $6 \times 6 = 36$

Probability = $\frac{10}{36} = \frac{5}{18}$

8.Question: Two dice are rolled. What is the probability that neither die shows a 1?

Solution:

Probability that one die does not show a 1 = $\frac{5}{6}$

Probability that neither die shows a 1 = $\frac{5}{6} \times \frac{5}{6} = \frac{25}{36}$

9.Question: Two dice are rolled. What is the probability of rolling a (4,5) or (5,4)?

Solution:

Outcomes of interest = (4,5), (5,4) = 2 outcomes

Total outcomes = $6 \times 6 = 36$

Probability = $\frac{2}{36} = \frac{1}{18}$

7. Permutations and Combinations in Probability

1.Question: A committee of 3 members is to be formed from 5 men and 4 women. What is the probability that the committee has exactly 2 women?

Solution:

Number of ways to choose 2 women out of 4 = ${}^4C_2 = 6$

Number of ways to choose 1 man out of 5 = ${}^5C_1 = 5$

Total favorable outcomes = $6 \times 5 = 30$

Total ways to form any 3-member committee = $(9\ 3) = 84$

Probability = $30/84 = 5/14$

2.Question: A committee of 3 members is to be formed from 5 men and 4 women. What is the probability that the committee has exactly 2 women?

Solution:

Step 1: Find the total ways to form a 3-member committee from 9 people:

Total ways = $(9\ 3) = 9 \times 8 \times 7 / (3 \times 2 \times 1) = 84$

Step 2: Find the favorable outcomes (exactly 2 women and 1 man):

Ways to choose 2 women from 4 = $(4\ 2) = 4 \times 3 / (2 \times 1) = 6$

Ways to choose 1 man from 5 = $(5\ 1) = 5$

Favorable outcomes = $6 \times 5 = 30$

Step 3: Probability:

$P = \text{Favorable outcomes} / \text{Total outcomes} = 30/84 = 5/14$

Answer: $5/14$

3.Question: What is the probability of forming the word "CAT" when arranging the letters C, A, T, R randomly?

Solution:

Step 1: Total number of arrangements of the 4 letters:

$4! = 4 \times 3 \times 2 \times 1 = 24$

Step 2: Favorable outcomes (only 1 "CAT"):

1

Step 3: Probability:

$P = \text{Favorable outcomes} / \text{Total outcomes} = 1/24$

Answer: $1/24$

4.Question: A bag contains 6 red balls, 4 blue balls, and 5 green balls. If 3 balls are drawn at random, what is the probability that all are red?

Solution:

Step 1: Total balls = $6 + 4 + 5 = 15$

Total ways to choose 3 balls =

$(15\ 3) = 15 \times 14 \times 13 / (3 \times 2 \times 1) = 455$

Step 2: Favorable outcomes (choosing 3 red balls):

$(6\ 3) = 6 \times 5 \times 4 / (3 \times 2 \times 1) = 20$

Step 3: Probability:

$$P = \frac{\text{Favorable outcomes}}{\text{Total outcomes}} = \frac{4}{455} = \frac{4}{91}$$

Answer: $\frac{4}{91}$

5.Question: How many 4-letter words can be formed using the letters A, B, C, and D if repetition of letters is allowed? What is the probability of forming the word "ABCD"?

Solution:

Step 1: Total number of 4-letter words (with repetition):

$$4 \times 4 \times 4 \times 4 = 256$$

Step 2: Favorable outcomes (forming "ABCD"):

1

Step 3: Probability:

$$P = \frac{\text{Favorable outcomes}}{\text{Total outcomes}} = \frac{1}{256}$$

Answer: $\frac{1}{256}$

6.Question: A bag contains 3 white, 4 black, and 2 green marbles. If 3 marbles are drawn at random, what is the probability that at least one of them is green?

Solution:

Step 1: Total marbles = $3 + 4 + 2 = 9$

Total ways to choose 3 marbles =

$${}^9C_3 = \frac{9 \times 8 \times 7}{3 \times 2 \times 1} = 84$$

Step 2: Find the complement (no green marble):

Ways to choose 3 marbles from 7 (only white and black):

$${}^7C_3 = \frac{7 \times 6 \times 5}{3 \times 2 \times 1} = 35$$

Step 3: Probability of at least one green marble:

$$P(\text{at least one green}) = 1 - P(\text{no green}) = 1 - \frac{35}{84} = \frac{49}{84} = \frac{7}{12}$$

Answer: $\frac{7}{12}$
