Objectives

The student will be able to:

- interpret the **variance** of a data set.
- find the **standard deviation** of a data set.
- find the **z-scores** of a data set

SOL: A.9 2009

Standard Deviation and Variance

- *Deviation just means how far from the normal*

Standard Deviation

The Standard Deviation is a measure of how spread out numbers are.

Its symbol is σ (the greek letter sigma)

The formula is easy: it is the **square root** of the **Variance.** So now you ask, "What is the Variance?"

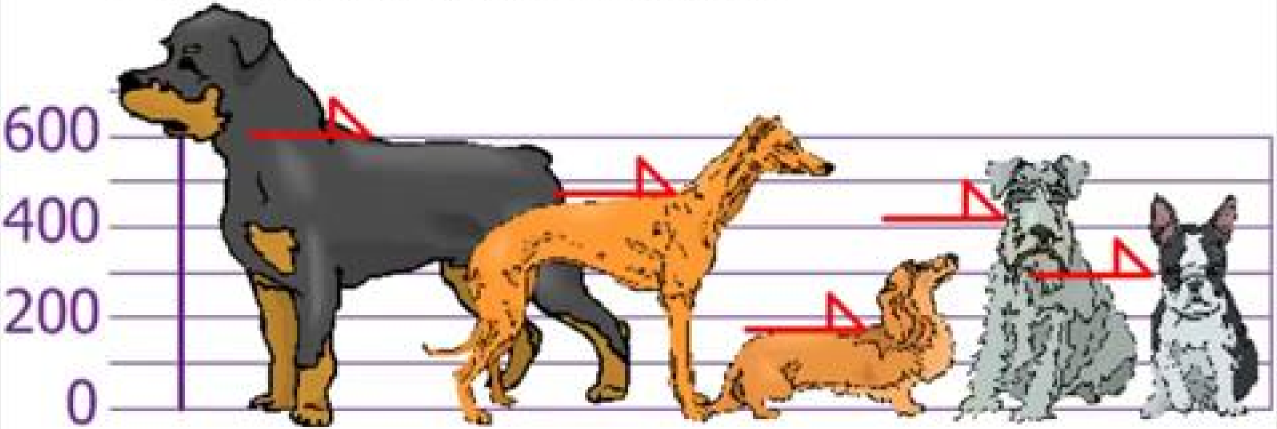
Variance

- The average of the **squared** differences from the Mean.
- To calculate the variance follow these steps:
  - Work out the Mean (the simple average of the numbers)
  - Then for each number: subtract the Mean and square the result (the *squared difference*).
  - Then work out the average of those squared differences.

Variance

Example

You and your friends have just measured the heights of your dogs (in millimeters):



The heights (at the shoulders) are: 600mm, 470mm, 170mm, 430mm and 300mm.

Find out the Mean, the Variance, and the Standard Deviation.

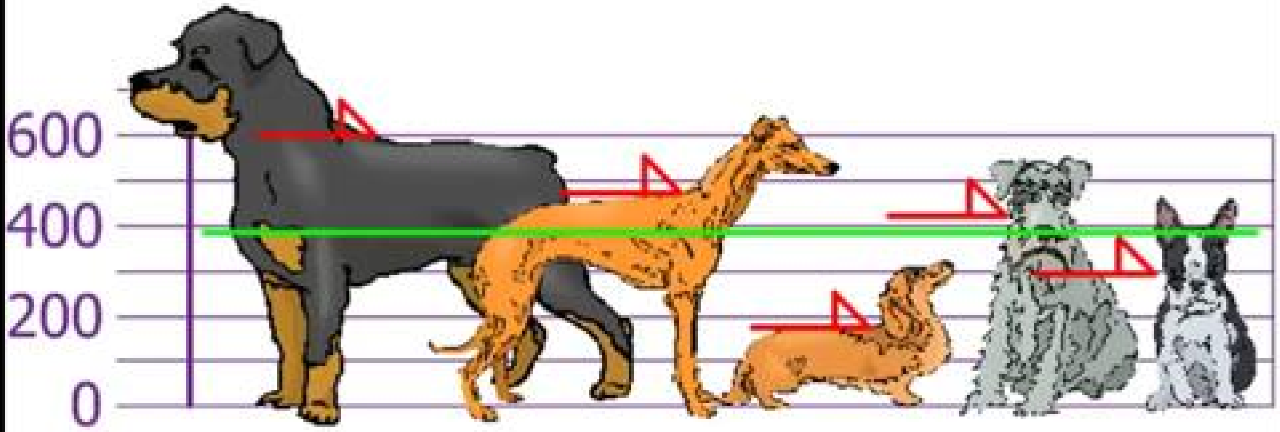
Find out the Mean, the Variance, and the Standard Deviation.

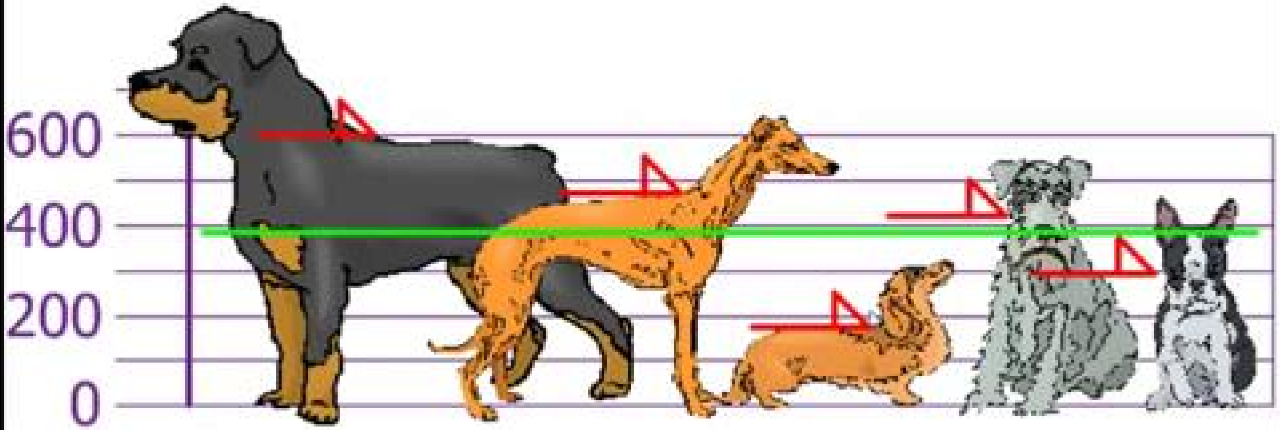
Your first step is to find the Mean:

Answer:

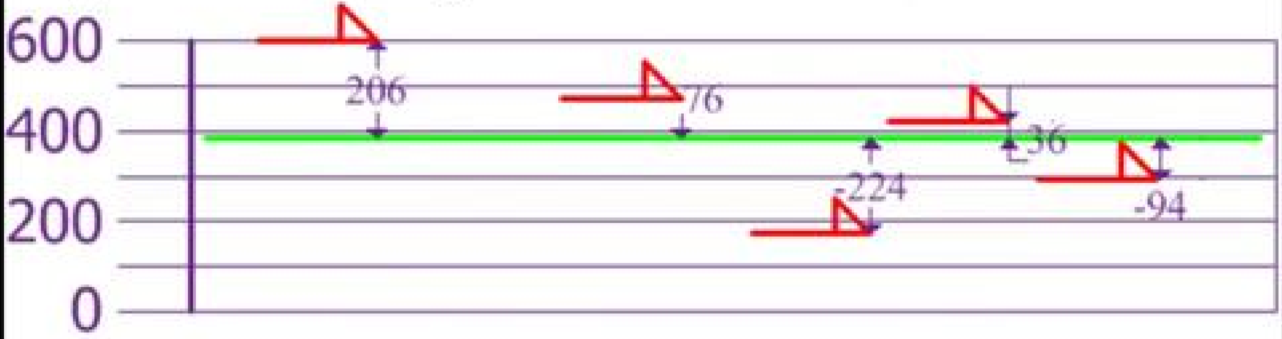
$$\text{Mean} = \frac{600 + 470 + 170 + 430 + 300}{5} = \frac{1970}{5} = 394$$

Students will find the variance, standard deviation, and z-score.


600
400
200
0



Now, we calculate each dogs difference from the Mean:



To calculate the Variance, take each difference, square it, and then average the result:

$$\text{Variance: } \sigma^2 = \frac{206^2 + 76^2 + (-224)^2 + 36^2 + (-94)^2}{5}$$

$=$

$=$

So, the Variance is

And the Standard Deviation is just the square root of Variance, so:

Students will find the variance, standard deviation, and z-score.

$$\text{Variance: } \sigma^2 = \frac{206^2 + 76^2 + (-224)^2 + 36^2 + (-94)^2}{5}$$

$$= \frac{42,436 + 5,776 + 50,176 + 1,296 + 8,836}{5}$$

$$=$$

So, the Variance is

And the Standard Deviation is just the square root of Variance, so:

Students will find the variance, standard deviation, and z-score.

$$\text{Variance: } \sigma^2 = \frac{206^2 + 76^2 + (-224)^2 + 36^2 + (-94)^2}{5}$$

$$= \frac{42,436 + 5,776 + 50,176 + 1,296 + 8,836}{5}$$

$$= \frac{108,520}{5} = 21,704$$

So, the Variance is

And the Standard Deviation is just the square root of Variance, so:

Students will find the variance, standard deviation, and z-score.

$$\text{Variance: } \sigma^2 = \frac{206^2 + 76^2 + (-224)^2 + 36^2 + (-94)^2}{5}$$

$$= \frac{42,436 + 5,776 + 50,176 + 1,296 + 8,836}{5}$$

$$= \frac{108,520}{5} = 21,704$$

So, the Variance is **21,704**.

And the Standard Deviation is just the square root of Variance, so:

$$\text{Variance: } \sigma^2 = \frac{206^2 + 76^2 + (-224)^2 + 36^2 + (-94)^2}{5}$$

$$= \frac{42,436 + 5,776 + 50,176 + 1,296 + 8,836}{5}$$

$$= \frac{108,520}{5} = 21,704$$

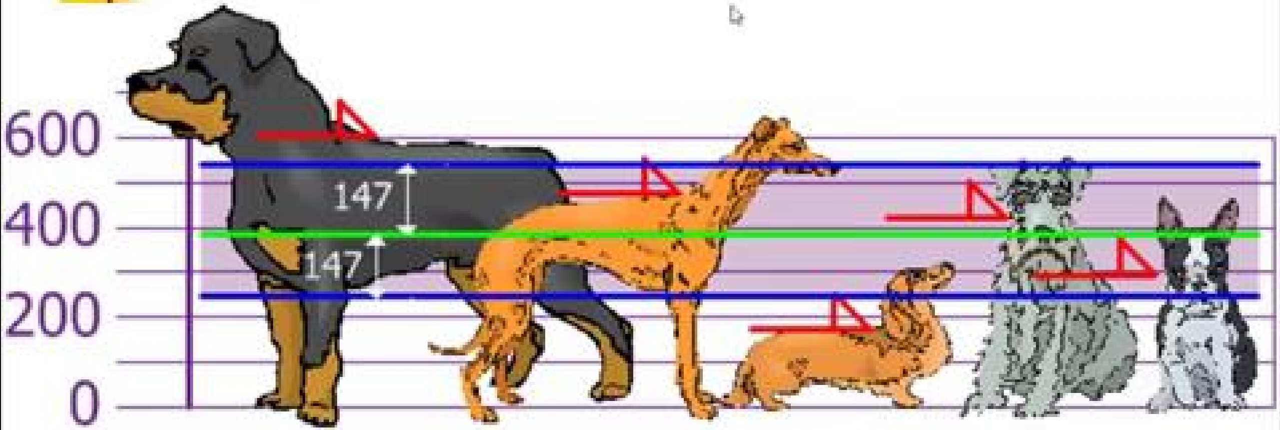
So, the Variance is **21,704**.

And the Standard Deviation is just the square root of Variance, so:

Standard Deviation: σ = $\sqrt{21704}$

≈147.32... ≈ 147 (to the nearest mm)

And the good thing about the Standard Deviation is that it is useful. Now we can show which heights are within one Standard Deviation (147mm) of the Mean:



So, using the Standard Deviation we have a "standard" way of knowing what is normal, and what is extra large or extra small. Rottweilers **are** tall dogs. And Dachshunds **are** a bit short ... but don't tell them!

Students will find the variance, standard deviation, and z-score.

Variance Formula

The variance formula includes the Sigma Notation, $\sum$, which represents the sum of all the items to the right of Sigma.

$$\frac{\sum(x-\mu)^2}{n}$$

$$(\sigma^2) = \frac{\sum_{i=1}^{n}(x_i-\mu)^2}{n}$$

Mean is represented by μ and n is the number of items.

Standard Deviation

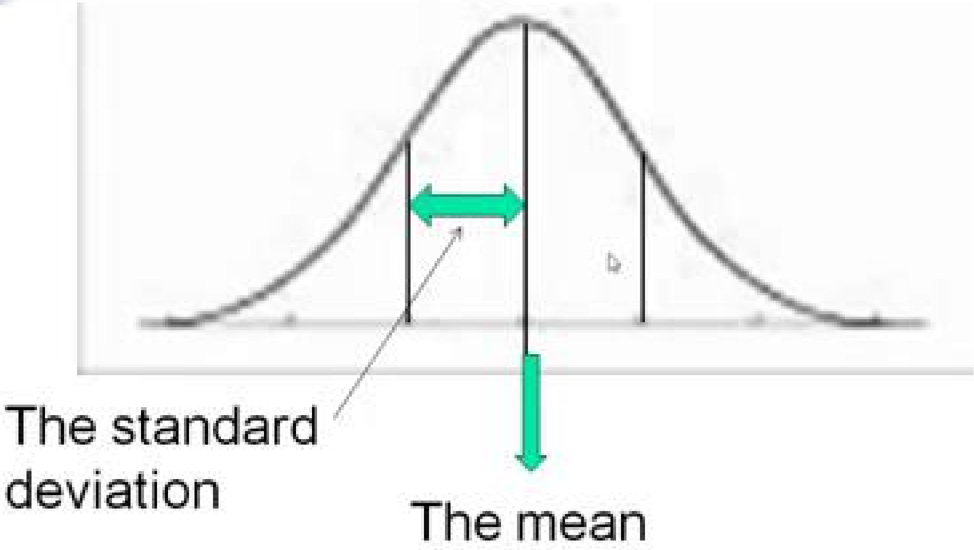
Standard Deviation shows the variation in data. If the data is close together, the standard deviation will be small. If the data is spread out, the standard deviation will be large.

Standard Deviation

Standard Deviation shows the variation in data. If the data is close together, the standard deviation will be small. If the data is spread out, the standard deviation will be large.

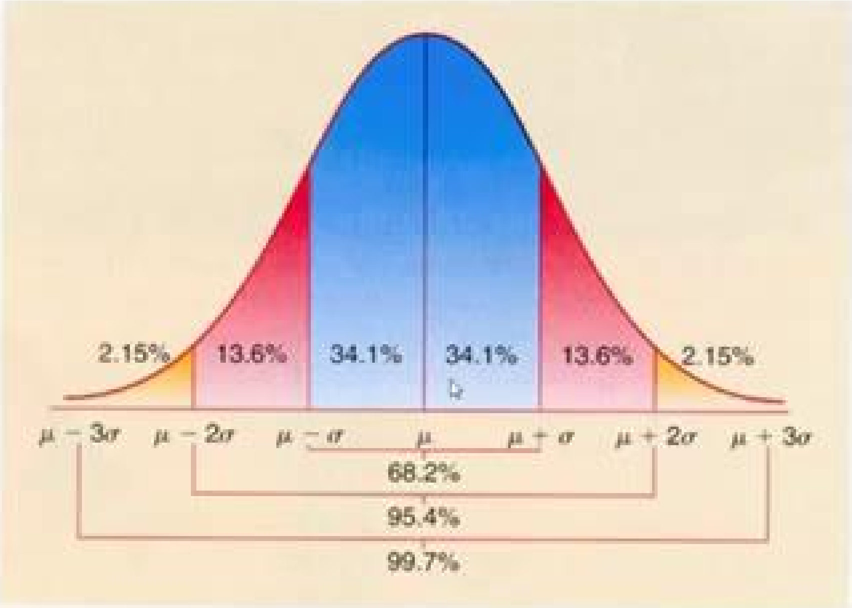
Standard Deviation is often denoted by the lowercase Greek letter sigma, σ.

Bell Curve



Students will find the variance, standard deviation, and z-score.

The Normal Curve (Bell Curve)



Students will find the variance, standard deviation, and z-score.

Standard Deviation Formula

The standard deviation formula can be represented using Sigma Notation:

$$\sigma = \sqrt{\frac{\sum (x-\mu)^2}{n}}$$

$$(\sigma) = \sqrt{\frac{\sum_{i=1}^{n} (x_i - \mu)^2}{n}}$$

Standard Deviation Formula

The standard deviation formula can be represented using Sigma Notation:

$$\sigma = \sqrt{\frac{\sum (x-\mu)^2}{n}}$$

$$(\sigma) = \sqrt{\frac{\sum_{i=1}^{n} (x_i-\mu)^2}{n}}$$

Standard Deviation Formula

The standard deviation formula can be represented using Sigma Notation:

$$\sigma = \sqrt{\frac{\sum (x-\mu)^2}{n}}$$

$$(\sigma) = \sqrt{\frac{\sum_{i=1}^{n} (x_i - \mu)^2}{n}}$$

Notice the standard deviation formula is the square root of the variance.

Find the variance and standard deviation

The math test scores of five students are: 92,88,80,68 and 52.

1) Find the **mean**: (92+88+80+68+52)/5 = 76.

2) Find the **deviation from the mean**:

92-76=16

88-76=12

80-76=4

68-76= -8

52-76= -24

Find the variance and standard deviation

The math test scores of five students are: 92,88,80,68 and 52.

3) Square the deviation from the mean:

$$(16)^2 = 256$$
$$(12)^2 = 144$$
$$(4)^2 = 16$$
$$(-8)^2 = 64$$
$$(-24)^2 = 576$$

Find the variance and standard deviation

The math test scores of five students are: 92,88,80,68 and 52.

4) Find the sum of the squares of the deviation from the mean:

   $256+144+16+64+576= 1056$

5) Divide by the number of data items to find the **variance**:

   $1056/5 = 211.2$

Find the variance and standard deviation

The math test scores of five students are: 92,88,80,68 and 52.

6) Find the square root of the variance: $\sqrt{211.2} \approx 14.53$

Thus the standard deviation of the test scores is about 14.53.

z-scores

A z-**score** reflects how many standard deviations above or below the mean a raw score is.

copy

z-scores

A **z-score** reflects how many standard deviations above or below the mean a raw score is.

The **z-score** is positive if the data value lies above the mean and negative if the data value lies below the mean.

copy

z-score formula

$$z = \frac{x - \mu}{\sigma}$$

$$(z) = \frac{x - \mu}{\sigma}$$

Analyzing the data

Suppose SAT scores among college students are normally distributed with a mean of 500 and a standard deviation of 100. If a student scores a 700, what would be her z-score?

Analyzing the data

Suppose SAT scores among college students are normally distributed with a mean of 500 and a standard deviation of 100. If a student scores a 700, what would be her z-score?

$$z = \frac{700 - 500}{100} = 2$$

Analyzing the data

Suppose SAT scores among college students are normally distributed with a mean of 500 and a standard deviation of 100. If a student scores a 700, what would be her z-score?

$$z = \frac{700 - 500}{100} = 2$$

Her z-score would be 2 which means her score is two standard deviations above the mean.

Analyzing the data

What will be the miles per gallon for a Toyota Camry when the average mpg is 23, it has a z value of 1.5 and a standard deviation of 2?

Analyzing the data

What will be the miles per gallon for a Toyota Camry when the average mpg is 23, it has a z value of 1.5 and a standard deviation of 2?

Using the formula for z-scores: $z = \dfrac{x-\mu}{\sigma}$

Analyzing the data

What will be the miles per gallon for a Toyota Camry when the average mpg is 23, it has a z value of 1.5 and a standard deviation of 2?

Using the formula for z-scores: $z = \frac{x - \mu}{\sigma}$

$$1.5 = \frac{x - 23}{2}$$

Analyzing the data

What will be the miles per gallon for a Toyota Camry when the average mpg is 23, it has a z value of 1.5 and a standard deviation of 2?

Using the formula for z-scores: $z = \dfrac{x-\mu}{\sigma}$

$$1.5 = \frac{x-23}{2} \qquad 3 = x - 23 \qquad x = 26$$

The Toyota Camry would be expected to use 26 mpg of gasoline.