MATHEMATICS

SECTION A

February 6, 2024

1 Vectors

1. Find the angle between the line $\vec{r} = \left(2\hat{i} - \hat{j} + 3\hat{k}\right) + \lambda\left(3\hat{i} - \hat{j} + 2\hat{k}\right)$ and the plane $\vec{r} \cdot \left(\hat{i} + \hat{j} + \hat{k}\right) = 3$.

2 Matrices

2. Find the value of $(x - y)$ from the matrix equation $2\begin{pmatrix} x & 5 \\ 7 & y-3 \end{pmatrix} + \begin{pmatrix} -3 & -4 \\ 1 & 2 \end{pmatrix} = \begin{pmatrix} 7 & 6 \\ 15 & 14 \end{pmatrix}$

3. Using elementary row transformations, find the inverse of the matrix $\begin{pmatrix} 3 & 0 & -1 \\ 2 & 3 & 0 \\ 0 & 4 & 1 \end{pmatrix}$.

4. Using matrices, solve the following system of linear equations:

$$2x + 3y + 10z = 4$$
$$4x - 6y + 5z = 1$$
$$6x + 9y - 20z = 2$$

5. Find the equation of the plane passing through $(-1, 3, 2)$ and perpendicular to the planes $x + 2y + 3z = 5$ and $3x + 3y + z = 0$.

3 Probability

6. If A and B are independent events with $P(A) = \frac{3}{7}$ and $P(B) = \frac{2}{5}$, then find $P(A' \cap B')$.

7. A card from a pack of 52 playing cards is lost. From the remaining cards of the pack, two cards are drawn at random (without replacement) and both are found to be spades. Find the probability of the lost card being a spade.

4 Differentiation

8. Find the differential equation representing the family of curves $y = -A\cos 3x + B\sin 3x$.

9. Find the differential of the function $\cos^{-1}(\sin 2x)$ w.r.t. x.

10. Solve the following differential equation :

$$\left(y + 3x^2\right)\frac{dx}{dy} = x$$

11. If $y = (\sin x^x) + \sin^{-1}\left(\sqrt{1 - x^2}\right)$, then find $\frac{dy}{dx}$

5   Integration

12. Find:
$$\int \frac{x-1}{(x-2)(x-3)}\,dx$$

13. Find:
$$\int e^x \left(\frac{2+\sin 2x}{2\cos^2 x}\right) dx$$

14. Evaluate:
$$\int_1^5 \left(|x-1| + |x-2| + |x-4|\right) dx$$

15. Find:
$$\int \cos 2x \cos 4x \cos 6x \, dx$$

6   Geometry

16. Prove that the radius of the right circular cylinder of greatest curved surface area which can be inscribed in a given cone is half of that of the cone.

7   Function

17. Find the interval in which the function f given by $f(x) = \sin 2x + \cos 2x, 0 \le x \le \pi$ is strictly decreasing.

8   Optimization

18. A company manufactures two types of novelty souvenirs made of plywood. Souvenirs of type A require 5 minutes each for cutting and 10 minutes each for assembling. Souvenirs of type B require 8 minutes each for cutting and 8 minutes each for assembling. There are 3 hours 20 minutes available for cutting and 4 hours for assembling. The profit for type A souvenirs is ₹ 100 each and for type B souvenirs, profit is ₹ 120 each. How many souvenirs of each type should the company manufacture in order to maximise the profit ? Formulate the problem as a LPP and then solve it graphically.