Data Analytics

Interactive Visualization, Probabilistic Modeling, and Machine Learning with Mathematica



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Bachelor of Arts and Science in Integrated Innovation

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