Homework Assignment 3
CSc 301 Scientific Computing

1. Splines can be used to approximate a "parametric curve" $\left(x(t), y(t)\right)$ by using a spline for each of the functions separately and then plotting the resulting function y vs. x. Draw whatever you like on a graph paper, place a few points on your drawing and take down the coordinates of the points (it doesn't have to be many points: 5 to 12 points is enough, depending on the drawing if chosen wisely). Create two arrays with these data and find the spline approximations $S_y(t)$ and $S_x(t)$ for each of the functions $y(t)$ and $x(t)$ with parameter t representing the array index. Plot the resulting S_y vs. S_x. Despite only using a few points for the entire drawing, the resulting plot should be nice and smooth. Experiment with different "end conditions". Explain your choice. (P.S. you don't have to draw it on the paper. You can generate the points from mouse-clicking if that's easier).

2. The first U.S. postage stamp was issued in 1885, with the cost to mail a letter set at 2 cents. In 1917, the cost was raised to 3 cents but then was returned to 2 cents in 1919. The record of price changes can be found on Wikipedia: https://en.wikipedia.org/wiki/History_of_United_States_postage_rates).
   (a) Determine the Newton interpolation polynomial for these data;
   (b) Determine the not-a-knot cubic spline for these data;
   (c) Considering only the data up to 2016, predict when will it cost 50 cents to mail a letter using both types of interpolation (Newton and spline). Compare with the actual record for 2017 and 2018. Explain the results.

3. Derive Hermite cubic spline for a periodic function $f(x)$, given that $f(x_n) = 0$, $f'(x_n) = (-1)^n$ for $x_n = \pi \cdot n$.
   (a) Use the derived spline to approximate $f(x) = \sin x$ at $x = \frac{\pi}{2} + \pi k$. Plot the actual function and the resultant spline on the same plot.
   (b) Use the derived spline to approximate $f(x) = \cos x$ at the same locations $x = \frac{\pi}{2} + \pi k$. Plot the result.
   (c) Does the spline approximate the considered periodic functions well at these locations? Explain.