Mathematics II for Computer Science and Engineering stream

(Subject code: BMATS201)

Module 3: Numerical Methods

Syllabus:

Solution of algebraic and transcendental equations - Regula-Falsi and Newton-Raphson methods (only formulae). Problems.

Finite differences, Interpolation using Newton's forward and backward difference formulae, Newton's divided difference formula and Lagrange's interpolation formula (All formulae without proof). Problems.

Numerical integration: Trapezoidal, Simpson's $(1/3)_{rd}$ and $(3/8)_{th}$ rules (without proof). Problems.

3.1 Regula-Falsi method

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1. **Find an approximate value of the root of the equation $x^3 + x - 1 = 0$, using method of false position, correct to three decimal places.**

$f(x) = x^3 + x - 1$

$f(0) = -1 = -ve,\ f(1) = 1 = +ve$

A root lies in $(0, 1)$

Step 1:

$a = 0$	$b = 1$
$f(a) = -1$	$f(b) = 1$

$x_1 = \frac{af(b) - bf(a)}{f(b) - f(a)} = \frac{0(1) - 1(-1)}{1 - (-1)} = 0.5$

$f(0.5) = -0.3750 = -ve, f(1) = 1 = +ve$

A root lies in $(0.5, 1)$

Step 2:

$a = 0.5$	$b = 1$
$f(a) = -0.3750$	$f(b) = 1$

$x_2 = \frac{af(b) - bf(a)}{f(b) - f(a)} = \frac{(0.5)(1) - 1(-0.3750)}{1 - (-0.3750)} = 0.6364$

$f(0.6364) = -0.1058 = -ve, f(1) = 1 = +ve$

A root lies in $(0.6364, 1)$

Step 3:

$a = 0.6364$	$b = 1$
$f(a) = -0.1059$	$f(b) = 1$

$x_3 = \frac{af(b) - bf(a)}{f(b) - f(a)} = \frac{(0.6364)(1) - 1(-0.1059)}{1 - (-0.1059)} = 0.6712$

$f(0.6712) = -0.0264 = -ve, f(1) = +ve$

A root lies in $(0.6712, 1)$

Step 4:

$a = 0.6712$	$b = 1$
$f(a) = -0.0264$	$f(b) = 1$

$x_4 = \frac{af(b) - bf(a)}{f(b) - f(a)} = \frac{(0.6712)(1) - 1(-0.0264)}{1 - (-0.0264)} = 0.6796$

$f(0.6796) = -0.0065$

Conclusion: An approximate root of the given equation is **0.68**

2. **Find an approximate root of the equation $x^3 - 3x + 4 = 0$ using method of false position, correct to three decimal places which lie between -3 and -2. (Carryout three iterations).**

$f(x) = x^3 - 3x + 4$

$f(-3) = -14 = -ve,\ f(-2) = 2 = +ve$

A root lies in $(-3, -2)$

Step 1:

$a = -3$	$b = -2$
$f(a) = -14$	$f(b) = 2$

$$x_1 = \frac{af(b)-bf(a)}{f(b)-f(a)} = \frac{(-3)2-(-2)(-14)}{2-(-14)} = -2.125$$

$f(-2.125) = 0.7792 = +ve,\ f(-3) = -ve.$

A root lies in $(-3, -2.125)$

Step 2:

$a = -3$	$b = -2.125$
$f(a) = -14$	$f(b) = 0.7792$

$$x_2 = \frac{af(b)-bf(a)}{f(b)-f(a)} = \frac{(-3)(0.7792)-(-2.125)(-14)}{0.7792-(-14)} = -2.1711$$

$f(-2.1711) = 0.2794 = +ve,\ f(-3) = -ve$

A root lies in $(-3, -2.1711)$

Step 3:

$a = -3$	$b = -2.1711$
$f(a) = -14$	$f(b) = 0.2794$

$$x_3 = \frac{af(b)-bf(a)}{f(b)-f(a)} = \frac{(-3)(0.2794)-(-2.1711)(-14)}{0.2794-(-14)} = -2.1873$$

$f(-2.1873) = 0.0972$. A root lies in (-3, -2.1873)

Step 4:

$a = -3$	$b = -2.1873$
$f(a) = -14$	$f(b) = 0.0972$

$$x_4 = \frac{af(b)-bf(a)}{f(b)-f(a)} = \frac{(-3)(0.0972)-(-2.1873)(-14)}{0.0972-(-14)} = -2.1929$$

$f(-2.1929) = 0.0335$.

Conclusion: An approximate root of the given equation is $\mathbf{-2.1929}$

3. Find an approximate value of the root of the equation $xe^x = 3$, using the method of false position, carryout three iterations.

$f(x) = xe^x - 3$

$f(1) = -0.2817 = -ve, f(2) = 11.7781 = +ve$

A root lies in $(1, 2)$

Step 1:

$a = 1$	$b = 2$
$f(a) = -0.2817$	$f(b) = 11.7781$

$x_1 = \frac{af(b)-bf(a)}{f(b)-f(a)} = \frac{(1)(11.7781)-(2)(-0.2817)}{11.7781-(-0.2817)} = 1.0233$

$f(1.0233) = -0.1528 = -ve,\ f(2) = +ve$

A root lies in $(1.0233, 2)$

Step 2:

$a = 1.0233$	$b = 2$
$f(a) = -0.1528$	$f(b) = 11.7781$

$x_2 = \frac{af(b)-bf(a)}{f(b)-f(a)} = \frac{(1.0233)(11.7781)-(2)(-0.1528)}{11.7781-(-0.1528)} = 1.0358$

$f(1.0358) = -0.0818 = -ve,\ f(2) = +ve$

A root lies in $(1.0358, 2)$

Step 3:

$a = 1.0358$	$b = 2$
$f(a) = -0.0818$	$f(b) = 11.7781$

$x_3 = \frac{af(b)-bf(a)}{f(b)-f(a)} = \frac{(1.0358)(11.7781)-2(-0.0818)}{11.7781-(-0.0818)} = 1.0425$

$f(1.0425) = -0.0432 = -ve,\ f(2) = +ve$

A root lies in $(1.043, 2)$.

Step 4:

$a = 1.0425$	$b = 2$
$f(a) = -0.0432$	$f(b) = 11.7781$

$x_4 = \frac{af(b)-bf(a)}{f(b)-f(a)} = \frac{(1.0425)(11.7781)-2(-0.0432)}{11.7781-(-0.0432)} = 1.0460$

$f(1.0460) = -0.0228$

Conclusion: An approximate root of the given equation is $\mathbf{1.0460}$

4. Compute the real root of the equation $x \log_{10} x = 1.2$, by the method of Regula falsi method, correct to three decimal places, carryout three approximations.

$f(x) = x \log_{10} x - 1.2$

$f(2) = -0.5979 = -ve, \quad f(3) = 0.2313 = +ve$

A root lies in $(2, 3)$.

Step:1

$a = 2$	$b = 3$
$f(a) = -0.5979$	$f(b) = 0.2313$

$x_1 = \frac{af(b)-bf(a)}{f(b)-f(a)} = \frac{0.4626+1.7937}{0.2313+0.5979} = 2.7210$

$f(2.721) = -0.0171 = -ve, \; f(3) = 0.2313 = +ve$

Therefore, a root lies in $(2.721, 3)$.

Step:2

$a = 2.7210$	$b = 3$
$f(a) = -0.0171$	$f(b) = 0.2313$

$x_2 = \frac{af(b)-bf(a)}{f(b)-f(a)} = \frac{0.6293+0.0513}{0.2484} = 2.7399$

$f(2.7399) = -0.0006 = -ve, \; f(3) = +ve$

Therefore, a root lies in $(2.7399, 3)$.

Step:3

$a = 2.7399$	$b = 3$
$f(a) = -0.0006$	$f(b) = 0.2313$

$x_3 = \frac{af(b)-bf(a)}{f(b)-f(a)} = \frac{0.6337+0.0018}{0.2319} = 2.7404$

Conclusion: An approximate root of the given equation is 2.7404

5. **Compute the real root of the equation $\cos x = 3x - 1$, by the method of Regula falsi method, correct to three decimal places, carryout three approximations.**

$f(x) = \cos x - 3x + 1$ (Note: Change degree mode to radian mode.)

$f(0) = 2$, $f(1) = 0.5403 - 2 = -1.4597$

Therefore, a root lies in $(0, 1)$.

Step:1 $a = 0,\ b = 1$

$f(a) = 2,\ f(b) = -1.4597$

$x_1 = \frac{af(b)-bf(a)}{f(b)-f(a)} = \frac{0-2}{-1.4597-2} = 0.5780$

$f(0.5780) = 0.1036 = +ve, f(1) = -ve$

Therefore, a root lies in $(0.5780, 1)$.

Step:2 $a = 0.5780,\ b = 1$

$f(a) = 0.1036,\ f(b) = -1.4597$

$x_2 = \frac{af(b)-bf(a)}{f(b)-f(a)} = \frac{(0.5780)(-1.4597)-1(0.1036)}{-1.4597-0.1036} = 0.6060$

$f(0.6060) = 0.004 = +ve,\ f(1) = -ve$

Therefore, a root lies in $(0.6060, 1)$.

Step:3 $a = 0.6060,\ b = 1$

$f(a) = 0.004$, $f(b) = -1.4597$

$x_3 = \frac{af(b)-bf(a)}{f(b)-f(a)} = \frac{(0.6060)(-1.4597)-(1)(0.004)}{-1.4597-0.004} = 0.6070$

Conclusion: An approximate root of the given equation is **0.6070**

6. Compute the fourth root of 12 correct to three decimal places using the method of false position.

$x^4 - 12 = 0,\ f(x) = x^4 - 12$

$f(1) = -11 = -ve, f(2) = 4 = +ve.$

A root lies in $(1, 2)$

Step 1

$a = 1$	$b = 2$
$f(a) = -11$	$f(b) = 4$

$x_1 = \frac{af(b)-bf(a)}{f(b)-f(a)} = \frac{(1)(4)-(2)(-11)}{4-(-11)} = 1.7333$

$f(1.7333) = -2.9740 = -ve, f(2) = +ve$

A root lies in $(1.7333, 2)$

Step 2

$a = 1.7333$	$b = 2$
$f(a) = -2.9740$	$f(b) = 4$

$x_1 = \frac{af(b)-bf(a)}{f(b)-f(a)} = \frac{(1.7333)(4)-(2)(-2.9740)}{4-(-2.9740)} = 1.8470$

$f(1.8470) = -0.3623 = -ve, f(2) = +ve$

A root lies in $(1.8470, 2)$

Step 3

$a = 1.8470$	$b = 2$
$f(a) = -0.3623$	$f(b) = 4$

$x_1 = \frac{af(b)-bf(a)}{f(b)-f(a)} = \frac{(1.8470)(4)-(2)(-0.3623)}{4-(-0.3623)} = 1.8597$

$f(1.8597) = -0.0389 = -ve, f(2) = +ve$

A root lies in $(1.8597, 2)$

Step 4

$a = 1.8597$	$b = 2$
$f(a) = -0.0389$	$f(b) = 4$

$x_4 = \frac{af(b)-bf(a)}{f(b)-f(a)} = \frac{(1.8597)(4)-(2)(-0.0389)}{4-(-0.0389)} = 1.8610$

Conclusion: The fourth root of 12 is **1.8610**

7. **Using Regula-falsi method compute the real root of the equation $xe^x = \cos x$, correct to three decimal places.**

$xe^x - \cos x = 0,\ f(x) = xe^x - \cos x$

$f(0) = -1 = -ve,\ f(1) = 2.1780$

A root lies in $(0, 1)$

Step 1

$a = 0$	$b = 1$
$f(a) = -1$	$f(b) = 2.1780$

$$x_1 = \frac{af(b) - bf(a)}{f(b) - f(a)} = \frac{(0)(2.1780) - (1)(-1)}{2.1780 - (-1)} = 0.3146$$

$f(0.3146) = -0.52 = -ve,\ f(2) = +ve$

A root lies in $(0.3146, 1)$

Step 2

$a = 0.3146$	$b = 1$
$f(a) = -0.52$	$f(b) = 2.1780$

$$x_2 = \frac{af(b) - bf(a)}{f(b) - f(a)} = \frac{(0.3146)(2.1780) - (1)(-0.52)}{2.1780 - (-0.52)} = 0.4467$$

$f(0.4467) = -0.2036 = -ve, f(1) = +ve$

A root lies in $(0.4467, 1)$

Step 3

$a = 04467$	$b = 1$
$f(a) = -0.2036$	$f(b) = 2.1780$

$$x_1 = \frac{af(b) - bf(a)}{f(b) - f(a)} = \frac{(0.4467)(2.1780) - (1)(-0.2036)}{2.1780 - (-0.2036)} = 0.4940$$

$f(0.4940) = -0.071 = -ve, f(1) = +ve$

A root lies in $(0.4940, 1)$

Step 4

$a = 0.4940$	$b = 1$
$f(a) = -0.0708$	$f(b) = 2.1780$

$$x_4 = \frac{af(b) - bf(a)}{f(b) - f(a)} = \frac{(0.4940)(2.1780) - (1)(-0.0708)}{2.1780 - (-0.0708)} = 0.5099$$

Conclusion: An approximate real root of the given equation is 0.5099

8. Find the fourth root of 32 by Regula falsi method correct to three decimal places.

$x^4 - 32 = 0, \; f(x) = x^4 - 32$

$f(2) = -16 = -ve, f(3) = 49 = +ve.$

A root lies in $(2, 3)$

Step 1

$a = 2$	$b = 3$
$f(a) = -16$	$f(b) = 49$

$x_1 = \frac{af(b)-bf(a)}{f(b)-f(a)} = \frac{(2)(49)-(3)(-16)}{49-(-16)} = 2.2462$

$f(2.2462) = -6.5459 = -ve, \; f(3) = +ve$

A root lies in $(2.2462, 3)$

Step 2

$a = 2.2462$	$b = 3$
$f(a) = -6.5459$	$f(b) = 49$

$x_1 = \frac{af(b)-bf(a)}{f(b)-f(a)} = \frac{(2.2462)(49)-(3)(-6.5459)}{49-(-6.5459)} = 2.3350$

$f(2.3350) = -2.2716 = -ve, \; f(3) = +ve$

A root lies in $(2.3350, 3)$

Step 3

$a = 2.3350$	$b = 3$
$f(a) = -2.2716$	$f(b) = 49$

$x_1 = \frac{af(b)-bf(a)}{f(b)-f(a)} = \frac{(2.3350)(49)-(3)(-2.2716\,)}{49-(-2.2716\,)} = 2.3645$

$f(2.3645) = -0.7442 = -ve, \; f(3) = +ve$

A root lies in $(2.3645, 3)$

Step 4

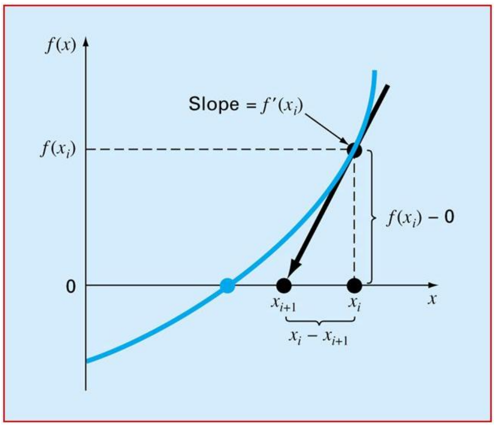
$a = 2.3645$	$b = 3$
$f(a) = -0.7442$	$f(b) = 49$

$x_4 = \frac{af(b)-bf(a)}{f(b)-f(a)} = \frac{(2.3645)(49)-(3)(-0.7442\,)}{49-(-0.7442\,)} = 2.3740$

Conclusion: The fourth root of 32 is 2.3740

3.2 Newton Raphson method

Introduction:

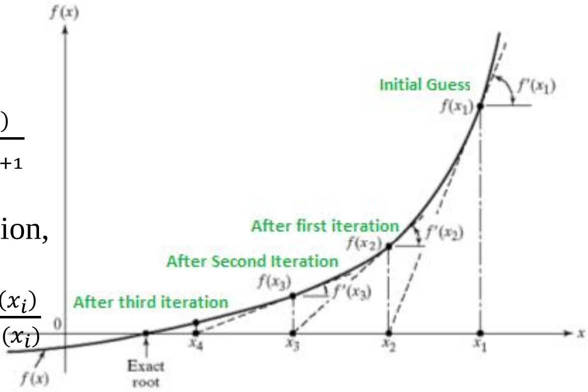


By diagram,

$f'(x_i) = \frac{f(x_i)}{x_i - x_{i+1}}$

On simplification,

$x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}$



Working rule:

- ❖ If $f(a)$ and $f(b)$ are opposite in sign, then a real root lies in (a, b).
- ❖ Choose $x_0 = a$ if $f(a)$ is closer to zero, otherwise choose $x_0 = b$.
- ❖ By Newton's Raphson method, $x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}$

First iteration:	Second iteration:	Third iteration:
$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$	$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$	$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$

- ❖ Stop if two consecutive values are almost same.

Problems:

1. **Find the real root of the equation $x^4 - x - 10 = 0$ correct to three decimal places using Newton – Raphson method.**

Let $f(x) = x^4 - x - 10$

$f(1) = -10, f(2) = 4.$

$f(2)$ is nearer to zero.

Let $x_0 = 2.$

By Newton's Raphson method,

$f'(x) = 4x^3 - 1$

$$\boldsymbol{x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}}$$

First iteration:

$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$

$= x_0 - \frac{x_0^4 - x_0 - 10}{4x_0^3 - 1}$

$= 2 - \frac{2^4 - 2 - 10}{4(2)^3 - 1}$

$= 1.8710$

Second iteration:

$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$

$= 1.8710 - \frac{2^4 - 2 - 10}{4(2)^3 - 1}$

$= 1.8710 - 0.0749$

$= 1.8558$

Third iteration:

$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$

$= 1.8558 - \frac{(1.8558)^4 - 1.8558 - 10}{4(1.8558)^3 - 1}$

$= 1.8558 - 0$

$= 1.8558$

Conclusion: The real root of the given equation is **1.8558**

2. Find the real root of the equation $3x = \cos x + 1$ correct to three decimal places using Newton's Raphson method.

Let $f(x) = 3x - \cos x - 1$

$f(0) = -2,\ f(1) = 1.46$

$f(1)$ is nearer to zero.

$f'(x) = 3 + \sin x$

Let $x_0 = 1$

By Newton's Raphson method,

$$\boldsymbol{x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}}$$

First iteration:

$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$

$= x_0 - \frac{3x_0 - \cos x_0 - 1}{3 + \sin x_0}$

$= 1 - \frac{3(1) - \cos 1 - 1}{3 + \sin 1}$

$= 1 - 0.3800$

$= 0.6200$

Second iteration:

$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$

$= 0.6200 - \frac{3(0.6200) - \cos(0.6200) - 1}{3 + \sin(0.6200)}$

$= 0.62 - 0.012$

$= 0.6071$

Third iteration:

$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$

$= 0.6071 - \frac{3(0.6071) - \cos(0.6071) - 1}{3 + \sin(0.6071)}$

$= 0.6071 - 0$

$= 0.6071$

Conclusion: The real root of the given equation is 0.6071

3. Using Newton's iteration method, find the real root of the equation $x\log_{10}x = 1.2$ that lies near 2.5 correct to three decimal places.

Let $f(x) = x\log_{10}x - 1.2$

$= \frac{1}{\log 10}(x\log x) - 1.2$

$= 0.4343\ x\log x - 1.2$

$f'(x) = \frac{1}{\log 10}\left(x\frac{1}{x} + \log x\right)$

$= 0.4343\ (1 + \log x)$

Let $x_0 = 2.5$

By Newton's Raphson method,

$$\boldsymbol{x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}}$$

First iteration:

$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$

$= x_0 - \frac{0.4343\ x_0\log x_0 - 1.2}{0.4343\ (1+\log x_0)}$

$= 2.5 - \frac{0.4343\ (2.5)\log 2.5 - 1.2}{0.4343\ (1+\log 2.5)}$

$= 2.5 + 0.2465$

$= 2.7465$

Second iteration:

$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$

$= 2.7465 - \frac{0.4343\ (2.7465)\log 2.7465 - 1.2}{0.4343\ (1+\log 2.7465)}$

$= 2.7465 - 0.0059$

$= 2.7406$

Third iteration:

$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$

$= 2.7406 - \frac{0.4343\ (2.7406)\log 2.7406 - 1.2}{0.4343\ (1+\log 2.7406)}$

$= 2.7406 + 0$

$= 2.7406$

Conclusion: The real root of the given equation is 2.7406

4. **Using Newton's iteration method, find the real root of the equation $xe^x - 2 = 0$ correct to three decimal places.**

Let $f(x) = xe^x - 2$

$f(0) = -2, \ f(1) = 0.718$

Let $x_0 = 1$

$f'(x) = xe^x + e^x$

By Newton's Raphson method, $\boldsymbol{x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}}$

First iteration:

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$$= x_0 - \frac{x_0 e^{x_0} - 2}{x_0 e^{x_0} + e^{x_0}}$$

$$= 1 - \frac{e-2}{2e}$$

$$= 0.8679$$

Second iteration:

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

$$= 0.8679 - \frac{0.8679e^{0.8679} - 2}{0.8679e^{0.8679} + e^{0.8679}}$$

$$= 0.8679 - 0.0151$$

$$= 0.8528$$

Third iteration:

$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$$

$$= 0.8528 - \frac{0.8578e^{0.8578} - 2}{0.8578e^{0.8578} + e^{0.8578}}$$

$$= 0.8528 - 0.0002$$

$$= 0.8526$$

Fourth iteration:

$$x_4 = x_3 - \frac{f(x_3)}{f'(x_3)}$$

$$= 0.8526 - \frac{0.8526e^{0.8526} - 2}{0.8578e^{0.8526} + e^{0.8526}}$$

$$= 0.8526 - 0$$

$$= 0.8526$$

Since $x_3 = x_4$, STOP.

Conclusion: The real root of the given equation is 0.8526

5. **Using Newton – Raphson method, find the real root of the equation $x \tan x + 1 = 0$ near $x = \pi$ correct to three decimal places.**

Let $f(x) = x \tan x + 1$

$f'(x) = x \sec^2 x + \tan x$

$= x + x \tan^2 x + \tan x$

Let $x_0 = \pi$

By Newton's Raphson method,

$$x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}$$

First iteration:

$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$

$= x_0 - \frac{x_0 \tan x_0 + 1}{x_0 + x_0 \tan^2 x_0 + \tan x_0}$

$= \pi - \frac{\pi \tan \pi + 1}{\pi + \pi \tan^2 \pi + \tan \pi}$

$= \pi - 0.3183$

$= 2.8233$

Second iteration:

$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$

$= 2.8233 - \frac{(2.8233)\tan(2.8233)+1}{2.8233+(2.8233)\tan^2(2.8233)+\tan(2.8233)}$

$= 2.8233 - 0.0249$

$= 2.7984$

Third iteration:

$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$

$= 2.7984 - \frac{(2.7984)\tan(2.7984)+1}{2.7984+(2.7984)\tan^2(2.7984)+\tan(2.7984)}$

$= 2.7984 - 0$

$= 2.7984$

Since $x_2 = x_3$, STOP.

Conclusion: The real root of the given equation is **2.7984**

6. Find the real root of the equation $x \sin x + \cos x = 0$ near $x = \pi$ by Newton-Raphson method, correct to three decimal places.

Let $f(x) = x \sin x + \cos x$

$f'(x) = x \cos x + \sin x - \sin x = x \cos x$

Let $x_0 = \pi$

By Newton's Raphson method, $\boldsymbol{x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}}$

First iteration:

$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$

$= x_0 - \frac{x_0 \sin x_0 + \cos x_0}{x_o \cos x_0}$

$= \pi - 0.3183$

$= 2.8233$

Second iteration:

$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$

$= 2.8233 - \frac{2.8233 \sin(2.8233) + \cos(2.8233)}{2.8233 \cos(2.8233)}$

$= 2.8233 - 0.0247$

$= 2.7986$

Third iteration:

$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$

$= 2.7986 - \frac{2.7986 \sin(2.7986) + \cos(2.7986)}{2.7986 \cos(2.7986)}$

$= 2.7986 - 0.0002$

$= 2.7984$

Fourth iteration:

$x_4 = x_3 - \frac{f(x_3)}{f'(x_3)}$

$= 2.7984 - \frac{2.7984 \sin(2.7984) + \cos(2.7984)}{2.7984 \cos(2.7984)}$

$= 2.7984 - 0$

$= 2.7984$

Since $x_3 = x_4$, STOP.

Conclusion: The real root of the given equation is **2.7984**

7. **Find the real root of the equation $\cos x = xe^x$, which is nearer to $x = 0.5$ by Newton-Raphson method, correct to three decimal places.**

Let $f(x) = \cos x - xe^x$	$f'(x) = -\sin x - xe^x - e^x$
Let $x_0 = 0.5$	

By Newton's Raphson method, $x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}$

First iteration:

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$$= x_0 - \frac{\cos x_0 - x_0 e^{x_0}}{-\sin x_0 - x_0 e^{x_0} - e^{x_0}}$$

$$= 0.5 + 0.0180$$

$$= 0.5180$$

Second iteration:

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

$$= 0.5180 - \frac{\cos(0.5180) - 0.5180e^{0.5180}}{-\sin(0.5180) - 0.5180e^{0.5180} - e^{0.5180}}$$

$$= 0.5180 - 0.0002$$

$$= 0.5178$$

Third iteration:

$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$$

$$= 0.5178 - \frac{\cos(0.5178) - 0.5178e^{0.5178}}{-\sin(0.5178) - 0.5178e^{0.5178} - e^{0.5178}}$$

$$= 0.5178 - 0$$

$$= 0.5178$$

Since $x_2 = x_3, STOP.$

Conclusion: The real root of the given equation is $\mathbf{0.5178}$

8. Find the real root of the equation $x^3 - 4x - 9 = 0$ correct to three decimal places using Newton – Raphson method.

Let $f(x) = x^3 - 4x - 9$ | $f'(x) = 3x^2 - 4$

$f(2) = -9,\ f(3) = 6.$

$f(3)$ is nearer to zero.

Let $x_0 = 3.$

By Newton's Raphson method, $\boldsymbol{x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}}$

First iteration:

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$$= 3 - \frac{3^3 - 4(3) - 9}{3(3)^2 - 4}$$

$$= 3 - 0.2609$$

$$= 2.7391$$

Second iteration:

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

$$= 2.7391 - \frac{2.7391^3 - 4(2.7391) - 9}{3(2.7391)^2 - 4}$$

$$= 2.7391 - 0.0321$$

$$= 2.7070$$

Third iteration:

$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$$

$$= 2.7070 - \frac{2.7070^3 - 4(2.7070) - 9}{3(2.7070)^2 - 4}$$

$$= 2.7070 - 0.0005$$

$$= 2.7065$$

Fourth iteration:

$$x_4 = x_3 - \frac{f(x_3)}{f'(x_3)}$$

$$= 2.7065 - \frac{2.7065^3 - 4(2.7065) - 9}{3(2.7065)^2 - 4}$$

$$= 2.7065 - 0$$

$$= 2.7065$$

Conclusion: The real root of the given equation is **2.7065**

3.3 Newton's forward and backward interpolation formula for equal intervals

Introduction:

Evaluating y at any point $\begin{cases} within\ (x_0, x_n)\ is\ called\ interpolation. \\ Out\ side\ (x_0, x_n)\ is\ called\ extrapolation. \end{cases}$

To evaluate y at any point $\begin{cases} nearer\ to\ x_0, use\ forward\ interpolation\ formula. \\ nearer\ to\ x_n, use\ backward\ interpolation\ formula \end{cases}$

Notation:

	Forward difference	Backward difference
First	Δy	∇y
Second	$\Delta^2 y$	$\nabla^2 y$
Third	$\Delta^3 y$	$\nabla^3 y$

Difference table:

x	y	I Diff.	II Diff.	III Diff.
1	24			
		96		
3	120		120	
		216		48
5	336		168	
		384		
7	720			

$$y_0 = 24,\ \ \Delta y_0 = 96,\ \ \Delta^2 y_0 = 120,\ \ \Delta^3 y_0 = 48$$

$$y_n = 720, \nabla y_n = 384, \nabla^2 y_n = 168, \nabla^3 y_n = 48$$

Newton's forward interpolation formula:

$$y(x) = y_0 + p\Delta y_0 + \frac{p(p-1)}{2!}\Delta^2 y_0 + \frac{p(p-1)(p-2)}{3!}\Delta^3 y_0 + \cdots \quad \text{Where, } p = \frac{x-x_0}{h}$$

Newton's Backward interpolation formula:

$$y(x) = y_n + q\nabla y_n + \frac{q(q+1)}{2!}\nabla^2 y_n + \frac{q(q+1)(q+2)}{3!}\nabla^3 y_n + \cdots \quad \text{Where, } q = \frac{x-x_n}{h}$$

1. **$\sin 45^0 = 0.7071, \sin 50^0 = 0.7660, \sin 55^0 = 0.8192, \sin 60^0 = 0.8660$, find $\sin 52^0$, using Newton's forward interpolation formula.**

$x = 52^0$ is nearer to the initial value. Use Newton's forward interpolation formula.

By data, $p = \frac{x-x_0}{h} = \frac{52-45}{5} = 1.4$

Difference table:

x	y	I Diff.	II Diff.	III Diff.
45^0	0.7071			
		0.0589		
50^0	0.7660		-0.0057	
		0.0532		-0.0007
55^0	0.8192		-0.0064	
		0.0468		
60^0	0.8660			

By table, $y_0 = 0.7071,\ \Delta y_0 = 0.0589,\ \Delta^2 y_0 = -0.0057,\ \Delta^3 y_0 = -0.0007$

By Newton's forward interpolation formula,

$y(x) = y_0 + p\Delta y_0 + \frac{p(p-1)}{2!}\Delta^2 y_0 + \frac{p(p-1)(p-2)}{3!}\Delta^3 y_0$

$y(52) = 0.7071 + 1.4(0.0589) + \frac{1.4(1.4-1)}{2!}(-0.0057) + \frac{1.4(1.4-1)(1.4-2)}{3!}(-0.0007)$

$\sin 52^0 = 0.7071 + 0.0825 - 0.0002 + 0$

Therefore, $\quad \mathbf{\sin 52^0 = 0.7894}$

2. The area A of a circle of diameter d is given for the following values:

d	80	85	90	95	100
A	5026	5674	6362	7088	7854

Calculate the area of a circle of diameter 105.

$x = 105$ is nearer to the end value. Use Newton's backward difference table.

$p = \frac{x-x_n}{h} = \frac{105-100}{5} = 1$

Difference table:

x	y	I Diff.	II Diff.	III Diff.	IV Diff.
80	5026				
		648			
85	5674		40		
		688		-2	
90	6362		38		4
		726		2	
95	7088		40		
		766			
100	7854				

By table, $y_n = 7854, \nabla y_n = 766, \nabla^2 y_n = 40, \nabla^3 y_n = 2, \nabla^4 y_n = 4$

By Newton's backward interpolation formula,

$$y(x) = y_n + p\nabla y_n + \frac{p(p+1)}{2!}\nabla^2 y_n + \frac{p(p+1)(p+2)}{3!}\nabla^3 y_n + \frac{p(p+1)(p+2)(p+3)}{4!}\nabla^4 y_n$$

$$y(105) = 7854 + 1(766) + \frac{1.2}{2!}(40) + \frac{1.2.3}{3!}(2) + \frac{1.2.3.4}{3!}(4)$$

Area $= 7854 + 766 + 40 + 2 + 4 = 8666$

3. Find $y(8)$ from $y(1) = 24,\ y(3) = 120,\ y(5) = 336,\ y(7) = 720$ by using Newton's backward difference interpolation formula.

$x = 8$ is nearer to the end value.

Use Newton's backward interpolation formula.

$p = \frac{x - x_n}{h} = \frac{1}{2} = 0.5$

Difference table:

x	y	I Diff.	II Diff.	III Diff.
1	24			
		96		
3	120		120	
		216		48
5	336		168	
		384		
7	720			

By table, $y_n = 720, \nabla y_n = 384, \nabla^2 y_n = 168, \nabla^3 y_n = 48$

By Newton's backward interpolation formula,

$$y(x) = y_n + p\nabla y_n + \frac{p(p+1)}{2!}\nabla^2 y_n + \frac{p(p+1)(p+2)}{3!}\nabla^3 y_n$$

$$y(8) = 720 + (0.5)(384) + \frac{0.5(0.5+1)}{2}(168) + \frac{0.5(0.5+1)(0.5+2)}{6}(48)$$

$$= 720 + 192 + 63 + 15$$

$$= 990$$

4. Using Newton's appropriate interpolation formula, find the values of y at $x = 8$ and at $x = 22$ from the following table:

x	0	5	10	15	20	25
y	7	11	14	18	24	32

Step 1

$x = 8$ is nearer to the initial value.

Use Newton's forward interpolation formula. $p = \frac{x-x_0}{h} = \frac{8-0}{5} = 1.6$

x	y	I Diff.	II Diff.	III Diff.	IV Diff.	V Diff.
0	**7**					
		4				
5	11		**−1**			
		3		**2**		
10	14		1		**−1**	
		4		1		**0**
15	18		2		**−1**	
		6		**0**		
20	24		**2**			
		8				
25	**32**					

By table, $y_0 = 7, \Delta y_0 = 4, \Delta^2 y_0 = -1, \Delta^3 y_0 = 2, \Delta^4 y_0 = -1, \Delta^5 y_0 = 0$

By Newton's forward interpolation formula,

$$y(x) = y_0 + p\Delta y_0 + \frac{p(p-1)}{2!}\Delta^2 y_0 + \frac{p(p-1)(p-2)}{3!}\Delta^3 y_0 + \frac{p(p-1)(p-2)(p-3)}{4!}\Delta^4 y_0$$

$$y(8) = 7 + 1.6(4) + \frac{1.6(1.6-1)}{2}(-1) + \frac{1.6(1.6-1)(1.6-2)}{6}(2) + \frac{1.6(1.6-1)(1.6-2)(1.6-3)}{24}(-1)$$

$$= 7 + 6.4 - 0.48 - 0.1280 - 0.0224 \quad = 12.7696$$

Step 2

$x = 22$ is nearer to the end value.

Use Newton's backward interpolation formula. $p = \frac{x-x_n}{h} = \frac{22-25}{5} = -0.6$

By table, $y_n = 32, \nabla y_n = 8, \nabla^2 y_n = 2, \nabla^3 y_n = 0, \nabla^4 y_n = -1, \nabla^5 y_n = 0$

By Newton's backward interpolation formula,

$$y(x) = y_n + p\nabla y_n + \frac{p(p+1)}{2!}\nabla^2 y_n + \frac{p(p+1)(p+2)}{3!}\nabla^3 y_n + \frac{p(p+1)(p+2)(p+3)}{4!}\nabla^4 y_n$$

$$y(22) = 32 + (-0.6)(8) + \frac{-0.6(-0.6+1)}{2}(2) + 0 + \frac{-0.6(-0.6+1)(-0.6+2)(-0.6+3)}{24}(-1)$$

$$= 32 - 4.8 - 0.24 + 0 + 0.0336 \quad = 26.9936$$

5. Find the number of men getting wages below 3500 from the following data:

Wages (₹)	0-1000	1000-2000	2000-3000	3000-4000
Frequency	9	30	35	42

$x = 3500$ is nearer to the end value.

Use Newton's backward interpolation formula.

$$p = \frac{x - x_n}{h} = \frac{3500 - 4000}{1000} = -0.5$$

Wages below(₹)	1000	2000	3000	4000
Cumulative Frequency	9	39	74	116
	y_0	y_1	y_2	y_3

Difference table:

x	y	I Diff.	II Diff.	III Diff.
1000	9			
		30		
2000	39		5	
		35		2
3000	74		7	
		42		
4000	116			

By table, $y_n = 116, \nabla y_n = 42, \nabla^2 y_n = 7, \nabla^3 y_n = 2$

By Newton's backward interpolation formula,

$$y(x) = y_n + p\nabla y_n + \frac{p(p+1)}{2!}\nabla^2 y_n + \frac{p(p+1)(p+2)}{3!}\nabla^3 y_n$$

$$y(3500) = 116 + (-0.5)(42) + \frac{-0.5(-0.5+1)}{2}(7) + \frac{-0.5(-0.5+1)(-0.5+2)}{6}(2)$$

$$= 116 - 21 - 0.8750 - 0.1250$$

$$= 94$$

6. **From the data given below, find the number of students who obtained (i) less than 40 marks (ii) Between 40 and 45 marks.**

x	0-40	41-50	51-60	61-70	71-80
y	31	42	51	35	31

Solution:

$x\ (Below)$	40	50	60	70	80
$y(Cumulative)$	31	73	124	159	190
	y_0	y_1	y_2	y_3	y_4

Difference table:

x	y	I Diff.	II Diff.	III Diff.	IV Diff.
40	**31**				
		42			
50	73		**9**		
		51		**-25**	
60	124		-16		**37**
		35		12	
70	159		-4		
		31			
80	190				

$x = 45$ is nearer to the initial value. $p = \frac{x-x_0}{h} = \frac{45-40}{10} = 0.5$

By table, $y_0 = 184, \Delta y_0 = 20, \Delta^2 y_0 = 2, \Delta^3 y_0 = 0$

Newton's forward interpolation formula:

$$y(x) = y_0 + p\Delta y_0 + \frac{p(p-1)}{2!}\Delta^2 y_0 + \frac{p(p-1)(p-2)}{3!}\Delta^3 y_0$$

$$y(45) = 31 + 0.5(42) + \frac{0.5(0.5-1)}{2}(9) + \frac{0.5(0.5-1)(0.5-2)}{2}(-25) + \frac{0.5(0.5-1)0.5-2)(0.5-3)}{2}(37)$$

$$= 31 + 21 - 1.125 - 1.5625 - 1.445$$

$$= 47.8675 \cong 48$$

Conclusion:

The number of students who obtained less than 45 marks is 48.

The number of students who obtained less than 40 marks is 31.

The number of students who obtained between 40 and 45 marks is $48 - 31 = 17$.

7. **Use an appropriate interpolation formula to compute $f(42)$ using the following data.**

x	40	50	60	70	80	90
y	184	204	226	250	276	304

Difference table:

x	y	I Diff.	II Diff.	III Diff.
40	184			
		20		
50	204		2	
		22		0
60	226		2	
		24		0
70	250		2	
		26		0
80	276		2	
		28		
90	304			

$x = 42$ is in the first half of the given range. $p = \frac{x-40}{10} = \frac{42-40}{10} = 0.2$

By table, $y_0 = 184, \Delta y_0 = 20, \Delta^2 y_0 = 2, \Delta^3 y_0 = 0$

Newton's forward interpolation formula:

$$y(x) = y_0 + p\Delta y_0 + \frac{p(p-1)}{2!}\Delta^2 y_0 + \frac{p(p-1)(p-2)}{3!}\Delta^3 y_0$$

$$y(8) = 184 + 0.2(20) + \frac{0.2(0.2-1)}{2}(2) + 0$$

$$= 184 + 4 - 0.16 \qquad = 187.84$$

Home work

8. The population of a town is given by the following table:

Year	1951	1961	1971	1981	1991
Population	19.6	39.65	58.81	72.21	94.61

Using Newton's forward and backward interpolation formula, calculate the increase in population from the year 1955 and 1985.

Answer: 52.005

9. The following table gives the melting point of an alloy of lead and zinc, where t is the temperature in Celsius and p is the percentage of lead in the alloy:

p	60	70	80	90
t	226	250	276	304

Find the melting point of the alloy containing 84% of lead, using Newton's interpolation formula.

Answer: 286.96

10. The following table gives the distances in miles of the visible horizon for the given heights in feet above the earth's surface:

x (Height)	100	150	200	250	300	350	400
y $(Distance)$	10.63	13.03	15.04	16.81	18.42	19.90	21.27

Find the values of y when (i) $x = 160$ ft, (ii) $x = 410$ ft, using Newton's interpolation formula.

Answer: 13.46 and 21.53

3.4 Newton's divided difference formula

Divided difference table:

x	$f(x)$	$I\ D.D$	$II\ D.D$	$III\ D.D$
x_0	$f(x_o)$			
		$f(x_0,x_1) = \dfrac{f(x_1)-f(x_0)}{x_1-x_0}$		
x_1	$f(x_1)$		$f(x_0,x_1,\ x_2) = \dfrac{f(x_1,\ x_2)-f(x_0,\ x_1)}{x_2-x_0}$	
		$f(x_1,x_2) = \dfrac{f(x_2)-f(x_1)}{x_2-x_1}$		$f(x_0,x_1,x_2,x_3) = \dfrac{f(x_1,x_2,x_3)-f(x_0,x_1,x_2)}{x_3-x_0}$
x_2	$f(x_2)$		$f(x_1,x_2,\ x_3) = \dfrac{f(x_2,\ x_3)-f(x_1,\ x_2)}{x_3-x_1}$	
		$f(x_2,x_3) = \dfrac{f(x_3)-f(x_2)}{x_3-x_2}$		
x_3	$f(x_3)$			

Newton's divided difference formula:

$$y = f(x_0) + (x-x_0)f(x_0,x_1) + (x-x_0)(x-x_1)f(x_0,x_1,x_2)$$
$$+(x-x_0)(x-x_1)(x-x_2)\ f(x_0,x_1,x_2,x_3) + \cdots$$

1. **Using Newton's divided difference formula, evaluate $f(8)$ from the following table:**

x	4	5	7	10	11	13
$f(x)$	48	100	294	900	1210	2028

Divided difference table:

x	$f(x)$	$I\ D.D$	$II\ D.D$	$III\ D.D$	$IV\ D.D$
4	48				
		$\frac{100-48}{5-4}=52$			
5	100		$\frac{97-52}{7-4}=15$		
		$\frac{294-100}{7-5}=97$		$\frac{21-15}{10-4}=1$	
7	294		$\frac{202-97}{10-5}=21$		0
		$\frac{900-294}{10-7}=202$		$\frac{27-21}{11-5}=1$	
10	900		$\frac{310-202}{11-7}=27$		0
		$\frac{1210-900}{11-10}=310$		$\frac{33-27}{13-7}=1$	
11	1210		$\frac{409-310}{13-10}=33$		
		$\frac{2028-1210}{13-11}=409$			
13	2028				

By table, $f(x_0)=48, f(x_0,x_1)=52\ , f(x_0,x_1,x_2)=15, f(x_0,x_1,x_2,x_3)=1.$

By data, $x=8$

By Newton's divided difference formula,

$$y=f(x_0)+(x-x_0)f(x_0,x_1)+(x-x_0)(x-x_1)f(x_0,x_1,x_2)$$
$$+(x-x_0)(x-x_1)(x-x_2)\, f(x_0,x_1,x_2,x_3)$$
$$=48+(8-4)(52)+(8-4)(8-5)(15)+(8-4)(8-5)(8-7)(1)$$
$$=48+4(52)+(4)(3)(15)+(4)(3)(1)(1)=48+208+180+12$$
$$=448$$

2. **Find u_2 from the data $u_{10} = 355, u_0 = -5, u_8 = -21, u_1 = -14$ and $u_4 = -125$ by using Newton's divided difference formula.**

Divided difference table:

x	$f(x)$	$I\ D.D$	$II\ D.D$	$III\ D.D$
0	-5			
		$\frac{-14+5}{1-0} = -9$		
1	-14		$\frac{-37+9}{4-0} = -7$	
		$\frac{-125+14}{4-1} = -37$		$\frac{9+7}{8-0} = 2$
4	-125		$\frac{26+37}{8-1} = 9$	
		$\frac{-21+125}{8-4} = 26$		$\frac{27-9}{10-1} = 2$
8	-21		$\frac{188-26}{10-4} = 27$	
		$\frac{355+21}{10-8} = 188$		
10	355			

By table, $f(x_0) = -5, f(x_0, x_1) = -9\,, f(x_0, x_1, x_2) = -7, f(x_0, x_1, x_2, x_3) = 2.$

By data, $x = 2$

By Newton's divided difference formula,

$$y = f(x_0) + (x - x_0)f(x_0, x_1) + (x - x_0)(x - x_1)f(x_0, x_1, x_2)$$
$$+(x - x_0)(x - x_1)(x - x_2)\, f(x_0, x_1, x_2, x_3)$$

$$= -5 + (2 - 0)(-9) + (2 - 0)(2 - 1)(-7) + (2 - 0)(2 - 1)(2 - 4)(2)$$

$$= -5 - 18 - 14 - 8$$

$$= -45$$

3. Find $f(10)$ on the curve passing through the points $(4,-43), (7,83), (9,327)$ and $(12,1053)$ by using Newton's divided difference formula.

Divided difference table:

x	$f(x)$	$I\ D.D$	$II\ D.D$	$III\ D.D$
4	-43			
		$\frac{83+43}{7-4} = 42$		
7	83		$\frac{122-42}{9-4} = 16$	
		$\frac{327-83}{9-7} = 122$		$\frac{24-16}{12-4} = 1$
9	327		$\frac{242-122}{12-7} = 24$	
		$\frac{1053-327}{12-9} = 242$		
12	1053			

By table, $f(x_0) = -43, f(x_0,x_1) = 42\ , f(x_0,x_1,x_2) = 16, f(x_0,x_1,x_2,x_3) = 1$

By data, $x = 10$

By Newton's divided difference formula,

$$y = f(x_0) + (x - x_0)f(x_0,x_1) + (x - x_0)(x - x_1)f(x_0,x_1,x_2)$$
$$+(x - x_0)(x - x_1)(x - x_2)\ f(x_0,x_1,x_2,x_3)$$
$$= -43 + (10 - 4)(42) + (10 - 4)(10 - 7)(16) + (10 - 4)(10 - 7)(10 - 9)(1)$$
$$= -43 + 252 + 288 + 18$$
$$= 515$$

4. **Using Newton's divided difference formula, evaluate $f(-2)$ from the following data:**

x	-4	-1	0	2	5
$f(x)$	1245	33	5	9	1335

Divided difference table:

x	$f(x)$	$I\ D.D$	$II\ D.D$	$III\ D.D$	$IV\ D.D$
-4	1245				
		$\frac{33-1245}{-1+4} = 404$			
-1	33		$\frac{-28+404}{0+4} = 94$		
		$\frac{5-33}{0+1} = -28$		$\frac{10-94}{2+4} = -14$	
0	5		$\frac{2+28}{2+1} = 10$		$\frac{13+14}{5+4} = 3$
		$\frac{9-5}{2-0} = 2$		$\frac{442+28}{5+1} = 13$	
2	9		$\frac{442-2}{5-0} = 88$		
		$\frac{1335-9}{5-2} = 442$			
5	1335				

By table, $f(x_0) = 1245, f(x_0, x_1) = -404\ , f(x_0, x_1, x_2) = 94,$

$$f(x_0, x_1, x_2, x_3) = -14, \quad f(x_0, x_1, x_2, x_3, x_4) = 3$$

By data, $x = -2$

By Newton's divided difference formula,

$$y = f(x_0) + (x - x_0)f(x_0, x_1) + (x - x_0)(x - x_1)f(x_0, x_1, x_2)$$

$$+(x - x_0)(x - x_1)(x - x_2)\ f(x_0, x_1, x_2, x_3)$$

$$+(x - x_0)(x - x_1)(x - x_2)(x - x_3)\ f(x_0, x_1, x_2, x_3, x_4)$$

$$y = 1245 + (-2 + 4)(-404) + (-2 + 4)(-2 + 1)(94)$$

$$+(-2 + 4)(-2 + 1)(-2 - 0)(-14)$$

$$+(-2 + 4)(-2 + 1)(-2 - 0)(-2 - 2)(3)$$

$$= 1245 - 2(404) - 2(94) - (4)(14) - 16(3)$$

$$= 1245 - 808 - 188 - 56 - 48$$

$$= 145$$

5. **Determine $f(x)$ as a polynomial in x for the data given below by using Newton's divided difference formula:**

x	2	4	5	6	8	10
$f(x)$	10	96	196	350	868	1746

Divided difference table:

x	$f(x)$	$I\ D.D$	$II\ D.D$	$III\ D.D$	$IV\ D.D$
2	10				
		$\frac{96-10}{4-2}=43$			
4	96		$\frac{100-43}{5-2}=19$		
		$\frac{196-96}{5-4}=100$		$\frac{27-19}{6-2}=2$	
5	196		$\frac{154-100}{6-4}=27$		$\frac{2-2}{8-2}=0$
		$\frac{350-196}{6-5}=154$		$\frac{35-27}{8-4}=2$	
6	350		$\frac{259-154}{8-5}=35$		$\frac{2-2}{10-4}=0$
		$\frac{868-350}{8-6}=259$		$\frac{45-35}{10-5}=2$	
8	868		$\frac{439-259}{10-6}=45$		
		$\frac{1746-868}{10-8}=439$			
10	1746				

By table, $f(x_0)=10$, $f(x_0,x_1)=43$, $f(x_0,x_1,x_2)=19$,

$f(x_0,x_1,x_2,x_3)=2$, $\quad f(x_0,x_1,x_2,x_3,x_4)=0$

By Newton's divided difference formula,

$$y=f(x_0)+(x-x_0)f(x_0,x_1)+(x-x_0)(x-x_1)f(x_0,x_1,x_2)$$
$$+(x-x_0)(x-x_1)(x-x_2)\,f(x_0,x_1,x_2,x_3)$$
$$+(x-x_0)(x-x_1)(x-x_2)(x-x_3)\,f(x_0,x_1,x_2,x_3,x_4)$$
$$=10+(x-2)(43)+(x-2)(x-4)(19)+(x-2)(x-4)(x-5)(2)$$
$$+(x-2)(x-4)(x-5)(x-6)(0)$$
$$=10+43x-86+(x-2)(19x-76+(x-4)(2x-10))$$
$$=10+43x-86+(x-2)(2x^2-18x+40+19x-76)$$
$$=43x-76+(x-2)(2x^2+x-36)=2x^3-3x^2+5x-4$$

Home work:

6. Using Newton's divided difference formula, evaluate $f(5.5)$ from the following data:

x	0	1	4	5	6
$f(x)$	1	14	15	6	3

Answer: 3.09

7. Given $u_{20} = 24.37, u_{22} = 49.28, u_{29} = 162.86$ and $u_{32} = 240.5$, find u_{28} by Newton's divided difference formula.
Answer: 141.94

3.5 Lagrange's interpolation formula

Introduction:

Consider

x	x_0	x_1	x_2	x_3
y	y_0	y_1	y_2	y_3

By Lagrange's interpolation formula,

$$y = \frac{(x-x_1)(x-x_2)(x-x_3)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)}y_0 + \frac{(x-x_0)(x-x_2)(x-x_3)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)}y_1$$
$$+\frac{(x-x_0)(x-x_1)(x-x_3)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)}y_2 + \frac{(x-x_0)(x-x_1)(x-x_2)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)}y_3$$

1. Find y at $x = 5$ if $y(1) = -3,\ y(3) = 9, y(4) = 30, y(6) = 132$ using Lagrange's interpolation formula.

   By data,

$x_0 = 1$	$x_1 = 3$	$x_2 = 4$	$x_3 = 6$
$y_0 = -3$	$y_1 = 9$	$y_2 = 30$	$y_3 = 132$

   Also $x = 5$.

   By Lagrange's interpolation formula,

$$f(x) = \frac{(x-x_1)(x-x_2)(x-x_3)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)}y_0 + \frac{(x-x_0)(x-x_2)(x-x_3)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)}y_1$$
$$+\frac{(x-x_0)(x-x_1)(x-x_3)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)}y_2 + \frac{(x-x_0)(x-x_1)(x-x_2)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)}y_3$$
$$= \frac{(5-3)(5-4)(5-6)}{(1-3)(1-4)(1-6)} \times (-3) + \frac{(5-1)(5-4)(5-6)}{(3-1)(3-4)(3-6)} \times 9$$
$$+\frac{(5-1)(5-3)(5-6)}{(4-1)(4-3)(4-6)} \times 30 + \frac{(5-1)(5-3)(5-4)}{(6-1)(6-3)(6-4)} \times 132$$
$$= -0.2 - 6 + 40 + 35.2$$
$$= 69$$

2. Find the interpolation polynomial using Lagrange's interpolation formula to the data given below and hence find $f(3)$.

x	0	1`	2	5
y	2	3	12	147

By data, $x_0 = 0, x_1 = 1, x_2 = 2, x_3 = 5$
and $y_0 = 2, y_1 = 3, y_2 = 12, y_3 = 147$. Also $x = 3$.

By Lagrange's interpolation formula,

$$f(x) = \frac{(x-x_1)(x-x_2)(x-x_3)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)}y_0 + \frac{(x-x_0)(x-x_2)(x-x_3)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)}y_1$$

$$+\frac{(x-x_0)(x-x_1)(x-x_3)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)}y_2 + \frac{(x-x_0)(x-x_1)(x-x_2)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)}y_3$$

$$= \frac{(x-1)(x-2)(x-5)}{(0-1)(0-2)(0-5)}2 + \frac{(x-0)(x-2)(x-5)}{(1-0)(1-2)(1-5)}3$$

$$+\frac{(x-0)(x-1)(x-5)}{(2-0)(2-1)(2-5)}12 + \frac{(x-0)(x-1)(x-2)}{(5-0)(5-1)(5-2)}147$$

$$= -0.2(x^3 - 8x^2 + 17x - 10) + 0.75(x^3 - 7x^2 + 10x)$$

$$- 2(x^3 - 6x^2 + 5x) + 2.45(x^3 - 3x^2 + 2x)$$

$$= x^3 + x^2 - x + 2$$

Therefore, $f(3) = 3^3 + 3^2 - 3 + 2 = 35$

3. Find the value of $f(9)$ from the following data using Lagrange's interpolation formula.

x	5	7	11	13	17
y	150	392	1452	2366	5202

By data, $x_0 = 5, x_1 = 7, \quad x_2 = 11, \quad x_3 = 13, \quad x_4 = 17$
and $y_0 = 150, y_1 = 392, y_2 = 1452, y_3 = 2366, y_4 = 5202. \; x = 9.$

By Lagrange's interpolation formula,

$$f(x) = \frac{(x-x_1)(x-x_2)(x-x_3)(x-x_4)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)(x_0-x_4)}y_0 + \frac{(x-x_0)(x-x_2)(x-x_3)(x-x_4)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)(x_1-x_4)}y_1$$

$$+\frac{(x-x_0)(x-x_1)(x-x_3)(x-x_4)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)(x_2-x_4)}y_2 + \frac{(x-x_0)(x-x_1)(x-x_2)(x-x_4)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)(x_3-x_4)}y_3$$

$$+\frac{(x-x_0)(x-x_1)(x-x_2)(x-x_4)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)(x_3-x_4)}y_4$$

$$f(9) = \frac{(9-7)(9-11)(9-13)(9-17)}{(5-7)(5-11)(5-13)(5-17)}(150) + \frac{(9-5)(9-11)(9-13)(9-17)}{(7-5)(7-11)(7-13)(7-17)}(392)$$

$$+\frac{(9-5)(9-7)(9-13)(9-17)}{(11-5)(11-7)(11-13)(11-17)}(1452)$$

$$+\frac{(9-5)(9-7)(9-11)(9-17)}{(13-5)(13-7)(13-11)(13-17)}(2366)$$

$$+\frac{(9-5)(9-7)(9-11)(9-13)}{(17-5)(17-7)(17-11)(17-13)}(5202)$$

$$= 810$$

4. Find the value of $f(11)$ from the following data using Lagrange's interpolation formula.

x	2	5	8	14
y	94.8	87.9	81.3	68.7

By data, $x_0 = 2, x_1 = 5, \ \ x_2 = 8, \ \ \ x_3 = 14,$
and $y_0 = 94.8, y_1 = 87.9, y_2 = 81.3, y_3 = 68.7$. Also $x = 11$.

By Lagrange's interpolation formula,

$$f(x) = \frac{(x-x_1)(x-x_2)(x-x_3)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)}y_0 + \frac{(x-x_0)(x-x_2)(x-x_3)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)}y_1$$

$$+\frac{(x-x_0)(x-x_1)(x-x_3)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)}y_2 + \frac{(x-x_0)(x-x_1)(x-x_2)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)}y_3$$

$$f(11) = \frac{(11-5)(11-8)(11-14)}{(2-5)(2-8)(2-14)}(94.8) + \frac{(11-2)(11-8)(11-14)}{(5-2)(5-8)(5-14)}(87.9)$$

$$+\frac{(11-2)(11-5)(11-14)}{(8-2)(8-5)(8-14)}(81.3) + \frac{(11-2)(11-5)(11-8)}{(14-2)(14-5)(14-8)}(68.7)$$

$$= 74.925$$

5. Find the value of $f(2)$ from the following data using Lagrange's interpolation formula.

x	1	3	4	6
y	4	40	85	259

By data, $x_0 = 1, x_1 = 3, \ \ x_2 = 4, \ \ \ x_3 = 6,$
and $y_0 = 4, y_1 = 40, y_2 = 85, y_3 = 259$. Also $x = 2$.

By Lagrange's interpolation formula,

$$f(x) = \frac{(x-x_1)(x-x_2)(x-x_3)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)}y_0 + \frac{(x-x_0)(x-x_2)(x-x_3)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)}y_1$$

$$+\frac{(x-x_0)(x-x_1)(x-x_3)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)}y_2 + \frac{(x-x_0)(x-x_1)(x-x_2)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)}y_3$$

$$f(2) = \frac{(2-3)(2-3)(2-6)}{(1-3)(1-4)(1-6)}(4) + \frac{(2-1)(2-4)(2-6)}{(3-1)(3-4)(3-6)}(40)$$

$$+\frac{(2-1)(2-3)(2-6)}{(4-1)(4-3)(4-6)}(85) + \frac{(2-1)(2-3)(2-4)}{(6-1)(6-3)(6-4)}(259)$$

$$= 15$$

6. Find the interpolating polynomial $f(x)$ by using Lagrange's interpolating formula and hence find $f(0.5)$ and $f(3.1)$ from the following data:
$f(0) = 3,\quad f(1) = 6,\quad f(2) = 11,\quad f(3) = 18,\quad f(4) = 27$

x	0	1	2	3	4
y	3	6	11	18	27

By data, $x_0 = 0, x_1 = 1, \ x_2 = 2, \quad x_3 = 3, \quad x_4 = 4$
And $\quad y_0 = 3, y_1 = 6, \ y_2 = 11, \ y_3 = 18, \ y_4 = 27.$

By Lagrange's interpolation formula,

$$f(x) = \frac{(x-x_1)(x-x_2)(x-x_3)(x-x_4)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)(x_0-x_4)} y_0 + \frac{(x-x_0)(x-x_2)(x-x_3)(x-x_4)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)(x_1-x_4)} y_1$$
$$+ \frac{(x-x_0)(x-x_1)(x-x_3)(x-x_4)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)(x_2-x_4)} y_2 + \frac{(x-x_0)(x-x_1)(x-x_2)(x-x_4)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)(x_3-x_4)} y_3$$
$$+ \frac{(x-x_0)(x-x_1)(x-x_2)(x-x_4)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)(x_3-x_4)} y_4$$

$$f(x) = \frac{(x-1)(x-2)(x-3)(x-4)}{(0-1)(0-2)(0-3)(0-4)}(3) + \frac{(x-0)(x-2)(x-3)(x-4)}{(1-0)(1-2)(1-3)(1-4)}(6)$$
$$+ \frac{(x-0)(x-1)(x-3)(x-4)}{(2-0)(2-1)(2-3)(2-4)}(11) + \frac{(x-0)(x-1)(x-2)(x-4)}{(3-0)(3-1)(3-2)(3-4)}(18)$$
$$+ \frac{(x-0)(x-1)(x-2)(x-3)}{(4-0)(4-1)(4-2)(4-3)}(27)$$
$$= x^2 + 2x + 3$$

Home work:

7. Using Lagrange's formula find $f(10)$ from the following data:

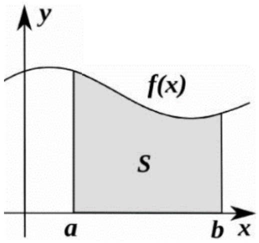
x	5	6	9	11
$f(x)$	12	13	14	16

8. Using Lagrange's interpolation formula, find a polynomial which passes through the points $(0, -12), (1, 0), (3, 6), (4, 12)$.
9. Using Lagrange's interpolation formula, fit a polynomial which passes through the points $(-1, 0), (1, 2), (2, 9), (3, 8)$ and hence estimate the value of y when $x = 2.2$ (MQP 1)

3.6 Numerical Integration

Introduction:

Numerical integration is the process of obtaining approximately the value of the definite integral $I = \int_a^b f(x)\,dx$ without actually integrating the function.



Suppose the interval $[a, b]$ is divided into 6 equal intervals of width h.

Trapezoidal Rule:

$$\int_a^b f(x)\,dx = \frac{h}{2}[y_0 + 2(y_1 + y_2 + \cdots + y_{n-1}) + y_n]$$

Simpson's one third rule

$$\int_a^b f(x)\,dx = \frac{h}{3}[(y_0 + y_6) + 4(y_1 + y_3 + y_5) + 2(y_2 + y_4)]$$

Simpson's three eighth rule

$$\int_a^b f(x)\,dx = \frac{3h}{8}[(y_0 + y_6) + 3(y_1 + y_2 + y_4) + 2(y_3)]$$

Note:

Rule	No. of intervals (n)
Simpson's one third rule	$n = 2k$
Simpson's three eighth rule	$n = 3k$

1. **Evaluate $\int_2^7 \frac{1}{x}\,dx$ using Trapezoidal rule by taking 5 equal sub intervals**

By data, $\frac{7-2}{5} = 1$.

x	2	3	4	5	6	7
y	1/2	1/3	1/4	1/5	1/6	1/7

By Trapezoidal rule,

$$\int_a^b f(x)\,dx = \frac{h}{2}[y_0 + 2(y_1 + y_2 + \cdots + y_{n-1}) + y_n]$$

$$\int_2^7 f(x)\,dx = \frac{1}{2}\left[\frac{1}{2} + 2\left(\frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \frac{1}{6}\right) + \frac{1}{7}\right] = 1.2714$$

2. **Use Trapezoidal rule with 6 sub intervals to approximate $\int_0^2 \frac{1}{16+x^2} dx$.**

By data, $h = \frac{b-a}{n} = \frac{2-0}{6} = \frac{2}{6} = \frac{1}{3}$.

x	0	1/3	2/3	1	4/3	5/3	2
y	0.0625	0.062	0.0608	0.0588	0.0562	0.0532	0.05

By Trapezoidal rule,

$$\int_a^b f(x)\,dx = \frac{h}{2}[y_0 + 2(y_1 + y_2 + \cdots + y_{n-1}) + y_n]$$

$$\int_0^2 \frac{1}{16+x^2} dx = \frac{\frac{1}{3}}{2}[0.0625 + 2(0.0608 + 0.0588 + 0.0562 + 0.0532) + 0.05]$$

$$= 0.1158$$

3. **Evaluate $\int_{1.4}^{2.2} y\,dx$ using Trapezoidal rule for the data**

x	1.4	1.6	1.8	2.0	2.2
$f(x)$	4.0552	4.9530	6.0436	7.3891	9.0250

By Trapezoidal rule,

$$\int_a^b f(x)\,dx = \frac{h}{2}[y_0 + 2(y_1 + y_2 + \cdots + y_{n-1}) + y_n]$$

$$\int_{1.4}^{2.2} y\,dx = \frac{0.2}{2}[4.0552 + 2(4.9530 + 6.0436 + 7.3891 + 0.0532) + 9.0250]$$

$$= 4.9852$$

4. **Evaluate $\int_0^6 \frac{1}{1+x^2} dx$ by taking 7 ordinates and by using (i) Simpson's 1/3 rule, (ii) Simpson's 3/8 rule. Compare with its actual value.**

$f(x) = \frac{1}{1+x^2}$, $a = 0, b = 6, n = 6,\ h = \frac{b-a}{n} = \frac{6-0}{6} = 1$

x	0	1	2	3	4	5	6
$y = 1/(1+x^2)$	1	0.5	0.2	0.1	0.0588	0.0385	0.0270

Simpson's one third rule

$\int_0^6 \frac{1}{1+x^2} dx \ = \frac{h}{3}[(y_0 + y_6) + 4(y_1 + y_3 + y_5) + 2(y_2 + y_4)]$

$= \frac{1}{3}[(1 + 0.0270) + 4(0.5 + 0.1 + 0.0385) + 2(0.2 + 0.0588)] = 1.3662$

Simpson's three eighth rule

$\int_0^6 \frac{1}{1+x^2} dx = \frac{3h}{8}[(y_0 + y_6) + 3(y_1 + y_2 + y_4 + y_5) + 2(y_3)]$

$= \frac{3}{8}[(1 + 0.0271) + 3(0.5 + 0.2 + 0.0588 + 0.0385) + 2(0.1)] = 1.3571$

Actual value

$\int_0^6 \frac{1}{1+x^2} dx = [tan^{-1}x]_0^6 = tan^{-1}6 - tan^{-1}0 = 1.4056$

5. Evaluate $\int_0^1 \frac{1}{1+x^2} dx$ by taking 7 ordinates and by using (i) Simpson's 1/3 rule, (ii) Simpson's 3/8 rule. Compare with its actual value.

$f(x) = \frac{1}{1+x^2}$, $a = 0, b = 1, n = 6$

$h = \frac{b-a}{n} = \frac{1-0}{6} = \frac{1}{6}$

x	0	1/6	2/6	3/6	4/6	5/6	1
$y = \dfrac{1}{1+x^2}$	1	0.9730	0.9	0.8	0.6923	0.5902	0.5
	y_0	y_1	y_2	y_3	y_4	y_5	y_6

Simpson's one third rule

$$\int_0^1 \frac{1}{1+x^2} dx = \frac{h}{3}[(y_0 + y_6) + 4(y_1 + y_3 + y_5) + 2(y_2 + y_4)]$$

$$= \frac{1/6}{3}[(1 + 0.5) + 4(0.9730 + 0.8 + 0.5902) + 2(0.9 + 0.6923)] = 0.7854$$

Simpson's three eighth rule

$$\int_0^1 \frac{1}{1+x^2} dx = \frac{3h}{8}[(y_0 + y_6) + 3(y_1 + y_2 + y_4 + y_5) + 2(y_3)]$$

$$= \frac{1/2}{8}[(1 + 0.5) + 3(0.9730 + 0.9 + 0.6923 + 0.5902) + 2(0.8)] = 0.7854$$

Actual value

$$\int_0^1 \frac{1}{1+x^2} dx = [tan^{-1}x]_0^1 = tan^{-1}1 - tan^{-1}0 = \frac{\pi}{4} = 0.7854$$

6. **Evaluate $\int_0^1 \frac{1}{1+x} dx$ by taking 7 ordinates and by using (i) Simpson's 1/3 rule, (ii) Simpson's 3/8 rule. Compare the results with its actual value.**

$f(x) = \frac{1}{1+x}$, $a = 0, b = 1, n = 6$

$h = \frac{b-a}{n} = \frac{1-0}{6} = \frac{1}{6}$

x	0	1/6	2/6	3/6	4/6	5/6	1
$y = \frac{1}{1+x}$	1	0.8571	0.75	0.6667	0.6	0.5455	0.5
	y_0	y_1	y_2	y_3	y_4	y_5	y_6

Simpson's one third rule

$$\int_0^1 \frac{1}{1+x} dx = \frac{h}{3}[(y_0 + y_6) + 4(y_1 + y_3 + y_5) + 2(y_2 + y_4)]$$

$$= \frac{\frac{1}{6}}{3}[(1 + 0.5) + 4(0.8571 + 0.6667 + 0.5455) + 2(0.75 + 0.6)]$$

$$= 0.6932$$

Simpson's three eighth rule

$$\int_0^1 \frac{1}{1+x} dx = \frac{3h}{8}[(y_0 + y_6) + 3(y_1 + y_2 + y_4 + y_5) + 2(y_3)]$$

$$= \frac{\frac{1}{2}}{8}[(1 + 0.5) + 3(0.8571 + 0.75 + 0.6 + 0.5455) + 2(0.6667)]$$

$$= 0.6932$$

Actual value

$$\int_0^1 \frac{1}{1+x} dx = [\log(1 + x)]_0^1$$

$$= \log 2$$

$$= 0.6932$$

7. **Evaluate $\int_4^{5.2} \log_e x \, dx$ by taking six equal strips and by using Simpson's 1/3 rule, Simpson's 3/8 rule and Weddle's rule. Compare the results with its actual value.**

$f(x) = \log_e x$, $a = 4, b = 5.2, n = 6$

$h = \frac{b-a}{n} = \frac{5.2-4}{6} = 0.2$

x	4	4.2	4.4	4.6	4.8	5	5.2
$y = \log_e x$	1.3863	1.4351	1.4816	1.5261	1.5686	1.6094	1.6487
	y_0	y_1	y_2	y_3	y_4	y_5	y_6

Simpson's one third rule

$$\int_4^{5.2} \log_e x \, dx = \frac{h}{3}[(y_0 + y_6) + 4(y_1 + y_3 + y_5) + 2(y_2 + y_4)]$$

$$= \frac{0.2}{3}[(1.3863 + 1.6487) + 4(0.4351 + 1.5261 + 1.6094) + 2(0.4816 + 1.5686)]$$

$$= 1.8279$$

Simpson's three eighth rule

$$\int_4^{5.2} \log_e x \, dx = \frac{3h}{8}[(y_0 + y_6) + 3(y_1 + y_2 + y_4 + y_5) + 2(y_3)]$$

$$= \frac{0.2}{8}[(1.3863 + 1.6487) + 3(0.4351 + 0.4816 + 1.5686 + 1.6094) + 2(1.5261)]$$

$$= 1.8278$$

Actual value

$$\int_4^{5.2} \log_e x \, dx = [x \log x - x]_4^{5.2} = 3.3730 - 1.5452 = 1.8278$$

8. Evaluate $\int_0^{0.6} e^{-x^2}\, dx$ by taking seven ordinates and by using (i) Simpson's 1/3 rule, (ii) Simpson's 3/8 rule.

$f(x) = e^{-x^2}$, $a = 0, b = 0.6, n = 6$

$h = \frac{b-a}{n} = \frac{0.6-0}{6} = 0.1$

x	0	0.1	0.2	0.3	0.4	0.5	0.6
$y = e^{x^2}$	1	0.99	0.9608	0.9139	0.8521	0.7788	0.6977
	y_0	y_1	y_2	y_3	y_4	y_5	y_6

Simpson's one third rule

$\int_0^{0.6} e^{-x^2}\, dx = \frac{h}{3}[(y_0 + y_6) + 4(y_1 + y_3 + y_5) + 2(y_2 + y_4)]$

$= \frac{0.1}{3}[(1 + 0.6977) + 4(0.99 + 0.9139 + 0.7788) + 2(0.9608 + 0.8521)]$

$= 0.5351$

Simpson's three eighth rule

$\int_0^{0.6} e^{-x^2}\, dx = \frac{3h}{8}[(y_0 + y_6) + 3(y_1 + y_2 + y_4 + y_5) + 2(y_3)]$

$= \frac{3(0.1)}{8}[(1 + 0.6977) + 3(0.99 + 0.9608 + 0.8521 + 0.7788) + 2(0.9139)]$

$= 0.5351$

9. Evaluate $\int_0^5 \frac{1}{4x+5} dx$ by using Simpson's 1/3rd rule by taking 11 ordinates. Hence find the value of $\log 5$.

$f(x) = \frac{1}{4x+5}$, $a = 0, b = 5, n = 10$

$h = \frac{b-a}{n} = \frac{5-0}{10} = 0.5$

x	0	0.5	1.0	1.5	2.0	2.5	3.0	3.5	4.0	4.5	5.0
$y = \frac{1}{4x+5}$	0.2	0.1429	0.1111	0.0909	0.0769	0.0667	0.0588	0.0526	0.0476	0.0435	0.04
	y_0	y_1	y_2	y_3	y_4	y_5	y_6	y_7	y_8	y_9	y_{10}

Simpson's one third rule

$$\int_0^5 \frac{1}{4x+5} dx = \frac{h}{3}[(y_0 + y_{10}) + 4(y_1 + y_3 + y_5 + y_7 + y_9) + 2(y_2 + y_4 + y_6 + y_8)]$$

$$= \frac{0.5}{3}[(0.2 + 0.04) + 4(0.1429 + 0.0909 + 0.0667 + 0.0526 + 0.0435) + +2(0.1111 + 0.0769 + 0.0588 + 0.0476)]$$

$$= 0.4025$$

To find: The value of $log\ 5$

$$\int_0^5 \frac{1}{4x+5} dx = \frac{1}{4}[\log(4x+5)]_0^5$$

$$= \frac{1}{4}(\log 25 - \log 5)$$

$$= \frac{1}{4}\log 5$$

$$= 0.4025$$

Therefore, $log\ 5 = 1.610$

10. Evaluate $\int_0^{0.3}(1-8x^3)^{\frac{1}{2}}\,dx$ by using Simpson's 3/8 rule, by taking 3 equal intervals.

Let $f(x) = (1-8x^3)^{\frac{1}{2}}$

Here $a = 0$ and $b = 0.3, n = 3.$

$h = \frac{b-a}{n} = \frac{0.3-0}{3} = 0.1$

x	0	0.1	0.2	0.3
y	1	0.996	0.9675	0.8854
	y_0	y_1	y_2	y_3

Simpson's three eighth rule

$$\int_0^{0.3}(1-8x^3)^{\frac{1}{2}}\,dx = \frac{3h}{8}[(y_0+y_6)+3(y_1+y_2+y_4+y_5)+2(y_3)]$$

$$= \frac{3(0.1)}{8}[(1+0.8854)+3(0.996+0.9675)]$$

$$= 0.2916$$

Home work

11. Evaluate $\int_{0.2}^{1.4}(\sin x - \log x + e^x)\,dx$ by taking seven ordinates using (i) Simpson's 1/3 rule, (ii) Simpson's 3/8 rule

Answer: 4.032, 4.030, 4.033

12. Using Simpson's 1/3rd rule, evaluate $\int_0^{\frac{\pi}{2}}\sqrt{\cos\theta}\,d\theta$ by taking 9 ordinates.

Answer: 1.1873