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RNS INSTITUTE OF TECHNOLOGY

Autonomous Institution, Affiliated to VTU

Estd : 2001

2024 Scheme
I Semester B.E. Degree Examination-February 2025
Engineering Mathematics-1(Mechanical Branch)

Time: 3 hrs **Max. Marks: 100**

Instructions to Candidates:

1. ***Answer any 5 full questions, selecting at least one question from each module.***
2. ***Use of handbook is permitted.***

Q.No.		Module-1	Marks	COs
Q1	a	With usual notion prove that $\tan\phi = r\frac{d\theta}{dr}$.	06	CO1
	b	Find the radius of curvature of the parabola $y^2 = 4ax$ at $(a, 2a)$.	07	CO1
	c	Find the pedal equation of the polar curve $r^n = a^n \cos n\theta$.	07	CO1
		OR		
Q2	a	Find the angle between the radius vector and the tangent to the polar curve $r^m = a^m \cos m\theta$	06	CO1
	b	With usual notation prove that $\frac{1}{p^2} = \frac{1}{r^2} + \frac{1}{r^4}\left(\frac{dr}{d\theta}\right)^2$.	07	CO1
	c	Derive the radius of curvature in Cartesian form as $$\rho = \frac{(1 + {y_1}^2)^{3/2}}{y_2}$$	07	CO1
		Module-2		
Q3	a	Using Maclaurin's series expansion show that $$\sqrt{1 + \sin 2x} = 1 + x - \frac{x^2}{2} - \frac{x^3}{6} + \frac{x^4}{24} - \cdots$$	06	CO2
	b	If $z = e^{ax+by} f(ax - by)$, prove that $b\frac{\partial z}{\partial x} + a\frac{\partial z}{\partial y} = 2abz$.	07	CO2
	c	Find the extreme values of the function $$f(x, y) = x^4 + y^4 - 2x^2 + 4xy - 2y^2.$$	07	CO2
		OR		
Q4	a	If $u = f(x - y, y - z, z - x)$ then prove that $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0$.	06	CO2
	b	If $u = x^2 + 3y^2 - z^3, v = 4x^2yz, w = 2z^2 - xy$, evaluate $\frac{\partial(u,v,w)}{\partial(x,y,z)}$ at $(1, -1, 0)$.	07	CO2
	c	A rectangular box opened at the top is to have volume of 32 cubic ft. Find the dimension of the box requiring least material for its construction.	07	CO2

Module-3

Q5

a Solve: $x\frac{dy}{dx} + y = x^3y^6$ — 06 — CO3

b Show that the family of parabolas $y^2 = 4a(x+a)$ is self-orthogonal. — 07 — CO3

c Solve: $x^2p^2 + xyp - 6y^2 = 0$. — 07 — CO3

OR

Q6

a Solve: $(x^2 + y^3 + 6x)\,dx + xy^2\,dy = 0$ — 06 — CO3

b Water at temperature $10°C$ takes 5 minutes to warm up to $20°C$ at room temperature of 40°C. Find the temperature of the water after 20 minutes. — 07 — CO3

c Obtain the general and singular solution of the equation — 07 — CO3

$$\sin px \cos y = \cos px \sin y + p$$

Module-4

Q7

a If P is the pull required to lift a load W by means of a pulley block, find a linear law of the form $P = mW + c$ connecting P and W using the following data: — 06 — CO4

P	12	25	21	25
W	50	70	100	120

Compute P when $W = 150\ Kg$.

b If θ is the angle between the two regression lines, show that — 07 — CO4

$$tan\theta = \frac{\sigma_x\sigma_y}{\sigma_x^2 + \sigma_y^2}\left(\frac{1-r^2}{r}\right)$$

c Calculate the coefficient of correlation for the following data — 07 — CO4

x	1	2	3	4	5	6	7	8	9
y	9	8	10	12	11	13	14	16	15

OR

Q8

a By the method of least squares, find the straight line that best fits the following data in the form $y = ax + b$ — 06 — CO4

x	1	2	3	4	5
y	14	27	40	55	68

b Ten participants in a contest are ranked by two Judges as follows: — 07 — CO4

x	1	6	5	10	3	2	4	9	7	8
y	6	4	9	8	1	2	3	10	5	7

Calculate the rank correlation between x and y.

c In a partially destroyed laboratory record, only the lines of regression of y on x and x on y are available as $4x - 5y + 33 = 0$ and $20x - 9y = 107$ respectively. Calculate $\bar{x}, \bar{y}$ and the coefficient of correlation between x and y. — 07 — CO4

Module-5				
Q9	**a**	Find the rank of a matrix $\begin{pmatrix} 0 & 1 & -3 & -1 \\ 1 & 0 & 1 & 1 \\ 3 & 1 & 0 & 2 \\ 1 & 1 & -2 & 0 \end{pmatrix}$	06	CO5
	b	Solve the system of equations by Gauss-Seidel method $$10x + y + z = 12, x + 10y + z = 12, x + y + 10z = 12$$	07	CO5
	c	Find the largest eigen value and the corresponding eigen vector of the matrix $\begin{pmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{pmatrix}$ by taking the initial vector as $X = [1, 0, 0]^T$ using Rayleigh's power method.	07	CO5
OR				
Q10	**a**	Find the values of λ and μ for which the system $$x + y + z = 6, x + 2y + 3z = 10, x + 2y + \lambda z = \mu$$ has (i) unique solution (ii) infinitely many solutions (iii) no solution.	06	CO5
	b	Apply Gauss Jordan method to solve the system of equations $$x + y + z = 10, 2x - y + 3z = 19, x + 2y + 3z = 22$$	07	CO5
	c	Test the following system of equations for consistency and hence solve $$5x + 3y + 7z = 4, 3x + 26y + 2z = 9, 7x + 2y + 10z = 5$$	07	CO5

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Scheme and Solutions

Faculty member's name	**Dr. Sampath Kumar R**
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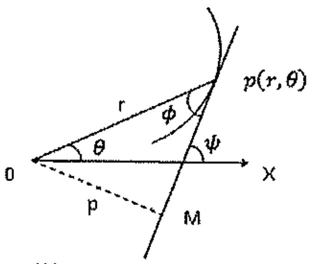
Signature	
No. of pages submitted: 9	

Course Title: Engineering Mathematics-1(Mechanical Stream)		**Course Code:BMATM101**
Question Number	**Solution**	**Marks Allocated**
Q 1. a)	Let $P(r,\theta)$ be any point on the polar curve $r = f(\theta)$. Let ψ be the angle from the X axis to the tangent. Let p be the per. dis from the origin to the tangent. By diagram, $\psi = \theta + \phi \Rightarrow \tan\psi = \tan(\theta + \phi)$ $\tan\psi = \frac{\tan\theta + \tan\phi}{1-\tan\theta . \tan\phi}$ --- (1) But $\tan\psi = \frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{\frac{d}{d\theta}(r\sin\theta)}{\frac{d}{d\theta}(r\cos\theta)} = \frac{\frac{dr}{d\theta}\sin\theta + r\cos\theta}{\frac{dr}{d\theta}\cos\theta - r\sin\theta}$ Divide by $\frac{dr}{d\theta}\cos\theta$ in nu and deno, $\tan\psi = \frac{\tan\theta + r\frac{d\theta}{dr}}{1-r\frac{d\theta}{dr}\tan\theta}$ --- (2) Equating components of (1) and (2), $\tan\phi = r\frac{d\theta}{dr}$  	Fig-1M 2M 2M 1M
Q 1. b)	$y^2 = 4ax$ Diff w. r. to x, $yy_1 = 2a$ --- (1) Diff again w. r. to x, $y_1^2 + yy_2 = 0$ --- (2) At $(a, 2a)$, $y_1 = \frac{2a}{2a} = 1$, $y_2 = -\frac{1}{2a}$ Radius of curvature $\rho = \frac{(1+y_1^2)^{3/2}}{y_2} = \frac{(1+1)^{3/2}}{\left(-\frac{1}{2a}\right)} = -2a(2)^{\frac{3}{2}} = -4\sqrt{2}a$ OR $\lvert\rho\rvert = 4\sqrt{2}a$	2M 2M 2M 1M
Q 1. c)	**To find:** ϕ $r^n = a^n \cos n\theta$ Take log on both sides, $n\log r = n\log a + \log\cos n\theta$ Differentiate w. r. to θ $\frac{n}{r}\frac{dr}{d\theta} = 0 + \frac{-n\sin n\theta}{\cos n\theta}$ $\frac{1}{r}\frac{dr}{d\theta} = -\tan n\theta$ $\cot\phi = \cot\left(\frac{\pi}{2} + n\theta\right)$ $\phi = \frac{\pi}{2} + n\theta$ **To find:** Pedal equation $p = r\sin\phi$ $p = r\sin\left(\frac{\pi}{2} + n\theta\right)$ $p = r\cos n\theta$ $p = r\left(\frac{r^n}{a^n}\right)$ $a^n p = r^{n+1}$	4M + 3M

Course Title: Engineering Mathematics-1(Mechanical Stream)		Course Code: BMATM101
Question Number	**Solution**	**Marks Allocated**
Q. 2.a)	$r^m = a^m \cos m\theta$ Take log on both sides, we get; $m \log r = m \log a + \log \cos m\theta$ Differentiate w. r. to θ $\frac{m}{r}\frac{dr}{d\theta} = \frac{-m \sin m\theta}{\cos m\theta} = -m \tan m\theta$ $\frac{1}{r}\frac{dr}{d\theta} = -\tan m\theta$ $cot\varphi = cot\left(\frac{\pi}{2} + m\theta\right) \Rightarrow \varphi = \frac{\pi}{2} + m\theta$	**1M** **2M** **2M** **1M**
Q. 2. b)	Let $P(r, \theta)$ – Any point on the polar curve $r = f(\theta)$. Let r and p - Radius vector and perpendicular distance from the origin respectively. By diagram, $\frac{p}{r} = \sin\phi \Rightarrow p = r \sin\phi$. $\frac{1}{p^2} = \frac{1}{r^2}\, cosec^2\phi = \frac{1}{r^2}(1 + \cot^2\phi)$ $= \frac{1}{r^2}\left(1 + \frac{1}{r^2}\left(\frac{dr}{d\theta}\right)^2\right) = \frac{1}{r^2} + \frac{1}{r^4}\left(\frac{dr}{d\theta}\right)^2$ Therefore, $\frac{1}{p^2} = \frac{1}{r^2} + \frac{1}{r^4}\left(\frac{dr}{d\theta}\right)^2$	**Fig 1M** **1M** **3M** **2M**
Q. 2. c)	$tan\psi = \frac{dy}{dx}$ OR $tan\psi = y_1 \Rightarrow \psi = tan^{-1}(y_1)$ Differentiating w. r. to x, $\frac{d\psi}{dx} = \frac{1}{1+y_1^2} \cdot y_2$ $\therefore$ radius of curvature is $\rho = \frac{ds}{d\psi} = \frac{ds}{dx} \cdot \frac{dx}{d\psi} = \sqrt{1 + y_1^2} \cdot \frac{1+y_1^2}{y_2}$ $\rho = \frac{(1+y_1^2)^{\frac{3}{2}}}{y_2}$ $\boxed{\rho = \frac{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\frac{d^2y}{dx^2}}}$	**Fig-1M** **1M** **2M** **2M** **1M**
Q.3. a)	$f(x) = \sqrt{1+\sin 2x} = \sqrt{1+2\sin x \cos x} = \sqrt{\sin^2 x + \cos^2 x + 2\sin x \cos x} = \sqrt{(\sin x + \cos x)^2}$ $y = \sin x + \cos x$; $y_1(0) = 1$ $y_1 = \cos x - \sin x$; $y_1(0) = 1$ $y_2 = -\sin x - \cos x = -y$; $y_2(0) = -1$ $y_3 = -y_1$; $y_3(0) = -1$ $y_4 = -y_2$; $y_4(0) = 1$ $y_5 = -y_3$; $y_5(0) = 1$ By Maclaurin's series,	**1M** **3M**

$f(x)=f(0)+\frac{x}{1!}f'(0)+\frac{x^2}{2!}f''(0)+\cdots=y(0)+\frac{x}{1!}y_1(0)+\frac{x^2}{2!}y_2(0)+\cdots$ **1M**

$\sqrt{1+\sin 2x}=1+\frac{x}{1!}(1)+\frac{x^2}{2!}(-1)+\frac{x^3}{3!}(-1)+\frac{x^4}{4!}(1)+\cdots$

$\Rightarrow \sqrt{1+\sin 2x}=1+x-\frac{x^2}{2!}-\frac{x^3}{3!}+\frac{x^4}{4!}+\cdots$ **1M**

Q.3. b)

$z=e^{ax+by}f(ax-by)$ ----- (1)

Differentiate (1) partially w. r. to x

$\frac{\partial z}{\partial x}=ae^{ax+by}f'(ax-by)+ae^{ax+by}f(ax-by)=ae^{ax+by}f'(ax-by)+az$ **2M**

Differentiate (1) partially w. r. to y

$\frac{\partial z}{\partial y}=-be^{ax+by}f'(ax-by)+be^{ax+by}f(ax-by)=-be^{ax+by}f'(ax-by)+bz$ **2M**

$\therefore b\frac{\partial z}{\partial x}+a\frac{\partial z}{\partial y}=abe^{ax+by}f'(ax-by)+abz-abe^{ax+by}f'(ax-by)+abz$

$b\frac{\partial z}{\partial x}+a\frac{\partial z}{\partial y}=2abz$ **3M**

Q.3. c)

$(x,y)=x^4+y^4-2x^2+4xy-2y^2\Rightarrow \qquad p=\frac{\partial f}{\partial x}=4x^3-4x+4y,$

$q=\frac{\partial f}{\partial y}=4y^3+4x-4y;\ r=\frac{\partial^2 f}{\partial x^2}=12x^2-4,\ t=\frac{\partial^2 f}{\partial y^2}=12y^2-4,\ s^2=\frac{\partial^2 f}{\partial x\partial y}=4$ **3M**

$rt-s^2=(12x^2-4)(12y^2-4)-16$

$p=0\Rightarrow 4x^3-4x+4y=0\Rightarrow x^3-x+y=0$ ---- (1)

$q=0\Rightarrow 4y^3+4x-4y=0\Rightarrow y^3-y+x=0$ ---- (2) **1M**

$(1)+(2)\Rightarrow y=-x$

In (2), $y=-x\Rightarrow -x^3+x+x=0,\ x(2-x^2)=0$

Therefore, Critical points are (0,0), $(\sqrt{2},-\sqrt{2}),(-\sqrt{2},\sqrt{2})$ **1M**

Critical points	$rt-s^2=(12x^2-4)(12y^2-4)-16$	$r=12x^2-4$	Remark
$(0,0)$	0	----	Doubtful
$(\sqrt{2},-\sqrt{2})$	$400-16$, Positive	Positive	Minimum
$(-\sqrt{2},\sqrt{2})$	$400-16$, Positive	Positive	Minimum

1M

Minimum value $=f(\sqrt{2},-\sqrt{2})=f(-\sqrt{2},\sqrt{2})=4+4-4-8-4=-8$ **1M**

Q. 4.a)

$p=x-y$	$q=y-z$	$r=z-x$
$\frac{\partial p}{\partial x}=1$	$\frac{\partial q}{\partial x}=0$	$\frac{\partial r}{\partial x}=-1$
$\frac{\partial p}{\partial y}=-1$	$\frac{\partial q}{\partial y}=1$	$\frac{\partial r}{\partial y}=0$
$\frac{\partial p}{\partial z}=0$	$\frac{\partial q}{\partial z}=-1$	$\frac{\partial r}{\partial z}=1$

2M

$\frac{\partial u}{\partial x}=\frac{\partial u}{\partial p}\frac{\partial p}{\partial x}+\frac{\partial u}{\partial q}\frac{\partial q}{\partial x}+\frac{\partial u}{\partial r}\frac{\partial r}{\partial x}=\frac{\partial u}{\partial p}(1)+\frac{\partial u}{\partial q}(0)+\frac{\partial u}{\partial r}(-1)=\frac{\partial u}{\partial p}-\frac{\partial u}{\partial r}$ ---- (1)

$\frac{\partial u}{\partial y}=\frac{\partial u}{\partial p}\frac{\partial p}{\partial y}+\frac{\partial u}{\partial q}\frac{\partial q}{\partial y}+\frac{\partial u}{\partial r}\frac{\partial r}{\partial y}=\frac{\partial u}{\partial p}(-1)+\frac{\partial u}{\partial q}(1)+\frac{\partial u}{\partial r}(0)=\frac{\partial u}{\partial q}-\frac{\partial u}{\partial p}$ ---- (2)

$\frac{\partial u}{\partial z}=\frac{\partial u}{\partial p}\frac{\partial p}{\partial z}+\frac{\partial u}{\partial q}\frac{\partial q}{\partial z}+\frac{\partial u}{\partial r}\frac{\partial r}{\partial z}=\frac{\partial u}{\partial p}(0)+\frac{\partial u}{\partial q}(-1)+\frac{\partial u}{\partial r}(1)=\frac{\partial u}{\partial r}-\frac{\partial u}{\partial q}$ ---- (3) **3M**

(1) + (2) + (3) gives, $\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}+\frac{\partial u}{\partial z}=\frac{\partial u}{\partial p}-\frac{\partial u}{\partial q}+\frac{\partial u}{\partial q}-\frac{\partial u}{\partial r}+\frac{\partial u}{\partial r}-\frac{\partial u}{\partial p}=0$ **1M**

Q.4. b)

$u = x + 3y^2 - z^3$	$v = 4x^2yz$	$w = 2z^2 - xy$
$\frac{\partial u}{\partial x} = 1$	$\frac{\partial v}{\partial x} = 8xyz$	$\frac{\partial w}{\partial x} = -y$
$\frac{\partial u}{\partial y} = 6y$	$\frac{\partial v}{\partial y} = 4x^2z$	$\frac{\partial w}{\partial y} = -x$
$\frac{\partial u}{\partial z} = -3z^2$	$\frac{\partial v}{\partial z} = 4x^2y$	$\frac{\partial w}{\partial z} = 4z$

3M

2M

$$\frac{\partial(u,v,w)}{\partial(x,y,z)} = \begin{vmatrix} u_x & v_x & w_x \\ u_y & v_y & w_y \\ u_z & v_z & w_z \end{vmatrix} = \begin{vmatrix} 1 & 8xyz & -y \\ 6y & 4x^2z & -x \\ -3z^2 & 4x^2y & 4z \end{vmatrix}$$ 1M

At $(1,-1,0)$, $= \begin{vmatrix} 1 & 0 & 1 \\ -6 & 0 & -1 \\ 0 & -4 & 0 \end{vmatrix} = 20$ 1M

Q. 4. C)

S.A. of the rectangular box open at the top is minimum$\Rightarrow xy + 2yz + 2zx$ is min.

Volume of the rectangular box= 32 cubic units.$\Rightarrow xyz = 32$ 1M

Auxiliary equation is $F = (xy + 2yz + 2zx) + \lambda(xyz - 32)$

$F_x = 0 \Rightarrow y + 2z + yz\lambda = 0$ -- (1) & $F_y = 0 \Rightarrow x + 2z + xz\lambda = 0$ --- (2)

$F_z = 0 \Rightarrow 2y + 2x + xy\lambda = 0$ -------- (3) 3M

$x \times (1) - y \times (2) \Rightarrow 2z(x - y) = 0 \Rightarrow \boldsymbol{x = y}$ 1M

$y \times (2) - z \times (3) \Rightarrow xy - 2yz + 2yz - 2zx = 0$

$\Rightarrow x(y - 2z) = 0 \Rightarrow \boldsymbol{y = 2z}$ 1M

Therefore, $x = y = 2z$ and $xyz = 32 \Rightarrow 2z.2z.z = 32$

$\Rightarrow 4z^3 = 32 \Rightarrow z^3 = 8 \Rightarrow z = 2.$ 1M

Therefore, $x = 4, y = 4, z = 2$. Dimensions of the rectangular box are 4, 4, 2

Q. 5.a)

Divide the Given DE, by x on both sides, $\frac{dy}{dx} + \left(\frac{1}{x}\right)y = x^2y^6$ 2+1M

Divide by y^6 on both sides, $\frac{1}{y^6}\frac{dy}{dx} + \left(\frac{1}{x}\right)\frac{1}{y^5} = x^2$ ---- (1)

If $\frac{1}{y^5} = t$ then $-\frac{5}{y^6}\frac{dy}{dx} = \frac{dt}{dx}$ 1M

Put $\frac{1}{y^5} = t$, $\frac{1}{y^6}\frac{dy}{dx} = -\frac{1}{5}\frac{dt}{dx}$ in (1)

$-\frac{1}{5}\frac{dt}{dx} + \frac{t}{x} = x^2$

Multiply by -5 on both sides,

$\frac{dt}{dx} - 5\frac{t}{x} = -5x^2$ This is an LDE in t with $P = -5/x, Q = -5x^2$ 1M

$IF = e^{\int -\frac{5}{x}dx} = e^{-5\log x} = \frac{1}{x^5}$ and General solution is $t.IF = \int Q.IF\, dx + c$

$t\frac{1}{x^5} = \int -5x^2\frac{1}{x^5}dx + c \Rightarrow \frac{t}{x^5} = -5\int x^{-3}\,dx \Rightarrow \frac{1}{x^5y^5} = \frac{5}{2x^2} + c$ 1M

Q.5. b)

$y^2 = 4a(x + a)$ ---- (1) Diff. w. r. to x, we get $2y\frac{dy}{dx} = 4a$

By substituting in (1), $y^2 = 2y\frac{dy}{dx}\left(x + \frac{y}{2}\frac{dy}{dx}\right)$

$y^2 = 2xy\frac{dy}{dx} + y^2\left(\frac{dy}{dx}\right)^2 \Rightarrow y = 2xy_1 + y(y_1)^2$ ------- (2) 3M

Replace $\frac{dy}{dx}$ by $-\frac{dx}{dy}$ in equation (2), $y = 2x\left(-\frac{dx}{dy}\right) + y\left(-\frac{dx}{dy}\right)^2$ 3M

$y = -2x\left(\frac{dx}{dy}\right) + y\left(\frac{dx}{dy}\right)^2 \Rightarrow y\left(\frac{dy}{dx}\right)^2 = -2x\left(\frac{dy}{dx}\right) + y \Rightarrow y = y(y_1)^2 + 2x(y_1)$ -(3)

Since (2) = (3), The given family of parabolas is self-orthogonal. 1M

Question Number	Solution	Marks Allocated
Q. 5.c)	$x^2p^2 + xyp - 6y^2 = 0$ $(xp+3y)(xp-2y) = 0$	2M
	Case $xp+3y=0$: $xp + 3y = 0$ $x\frac{dy}{dx} = -3y$ $\frac{1}{y}dy = \frac{-3}{x}dx$ On integrating, $\log y = -3\log x + \log c$ $\log y + 3\log x = \log c$ $\log yx^3 = \log cx$ $yx^3 = c$ $x^3y - c = 0$ **Case $xp-2y=0$:** $xp - 2y = 0$ $x\frac{dy}{dx} = 2y$ $\frac{1}{y}dy = \frac{2}{x}dx$ On integrating, $\log y = 2\log x + \log c$ $\log y = \log x^2 + \log c$ $y = cx^2$ $y - cx^2 = 0$	3M
	Therefore, the general solution is $(x^3y-c)(y-cx^2) = 0$	2M
Q.6.a)	$(x^2+y^3+6x)\,dx + xy^2\,dy = 0$ ---(1) $M = x^2+y^3+6x$, $\frac{\partial M}{\partial y} = 3y^2$; $N = xy^2$, $\frac{\partial N}{\partial x} = y^2$ Since $\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}$, this is not an exact D.E.	2M
	$\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x} = 2y^2$, close to N. Therefore, $\frac{1}{N}\left(\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}\right) = \frac{1}{xy^2}(2y^2) = \frac{2}{x} = f(x)$ $[say]$	1M
	$I.F = e^{\int f(x)\,dx} = e^{2\int\frac{1}{x}dx} = x^2$ and Multiply by x^2 on both the sides of equ. (1) $\left(x^{-3}y^2 - x^{-4}e^{1/x^3}\right)dx - x^{-2}y\,dy = 0$ This is an exact D.E.	1M
	G.S is $\int_{y-constant} M\,dx + \int(Terms\ of\ N\ not\ containing\ x)\ dy = c$ $\int_{y-constant}\left(x^{-3}y^2 - x^{-4}e^{1/x^3}\right)dx + \int(0)\ dy = c \Rightarrow -\frac{y^2}{2x^2} + \frac{e^{\frac{1}{x^3}}}{3} = c$	2M
Q.6. b)	By Newton's law of cooling, $\frac{d\theta}{dt} = -k(\theta-\theta_0) \Longrightarrow \frac{d\theta}{\theta-\theta_0} = -k\,dt$ On integrating, $log\,(\theta-\theta_0) = -kt + c' \Longrightarrow \theta - \theta_0 = ce^{-kt}$ By data, $\theta_0 = 40^0C$. Therefore, $\theta - 40 = ce^{-kt}$ ------ (1)	3M
	If $t=0, \theta=10$ $(1) \Rightarrow 10-40 = ce^0$ Therefore, $c = -30$ If $t=5, \theta=20$ $(1) \Rightarrow 20-40 = -30e^{-5k}$ $e^{5k} = \frac{30}{20} \Rightarrow\ 5k = \log\frac{3}{2}$ $k = \frac{1}{5}\log\frac{3}{2} = 0.0811$	2M
	On substituting in (1), $\theta - 40 = -30\,e^{-0.0811t}$ Put $t = 20$, $\theta - 40 = -30e^{-1.622} \Longrightarrow \theta - 40 = -5.9251 \Longrightarrow\ \theta = 34.07^0C$ The temperature of water after 20 minutes is 34.07^0C.	2M
Q.6. c)	$\sin(px-y) = p \Longrightarrow px - y = \sin^{-1}p \Longrightarrow y = px - \sin^{-1}p$. Clairaut's equation. General solution is $y = cx - \sin^{-1}c$ ------ (1)	2M
	Differentiate partially w.r.to c, we get $0 = x - \frac{1}{\sqrt{1-c^2}} \Longrightarrow \frac{1}{\sqrt{1-c^2}} = x$	2M
	$\sqrt{1-c^2} = \frac{1}{x} \Longrightarrow 1 - c^2 = \frac{1}{x^2} \Longrightarrow c^2 = 1 - \frac{1}{x^2}\ \Longrightarrow\ c = \frac{\sqrt{x^2-1}}{x}$	2M

	Substitute the value of c in (1). We get the required singular solution $y=\sqrt{x^2-1}-\sin^{-1}\frac{\sqrt{x^2-1}}{x}$	1M

Q.7.a)

W	P	W^2	WP
50	12	2500	600
70	15	4900	1050
100	21	10000	2100
120	25	14400	3000
$\sum W$=340	$\sum P$=73	$\sum W^2$=31800	$\sum WP$=6750

3M

The normal equations. 2M

$$\left.\begin{array}{r}\sum P=m\sum W+nc\\ \sum WP=m\sum W^2+c\sum W\end{array}\right\}\Rightarrow \begin{array}{l}73=340m+4c\\ 6750=31800m+340c\end{array}$$

Solving the normal equation, we get $m=0.1879$ and $c=2.2785$ 1M

Hence the line of best fit is $P=c+mW=0.1879W+2.2785$

When $w=150$kg, $P=2.2785+0.1879\times150=30.4635$

Q. 7. b) The equation to the regression line of y on x is

$y-\bar{y}=r\frac{\sigma_y}{\sigma_x}(x-\bar{x})$ comparing with $y=mx+c$

The slope of the line is $m_1=r\frac{\sigma_y}{\sigma_x}$ 2M

The equ to the regression line of x on y is $x-\bar{x}=r\frac{\sigma_x}{\sigma_y}(y-\bar{y})$

rearranging $y-\bar{y}=\frac{\sigma_y}{r\sigma_x}(x-\bar{x})$. The slope of the line is $m_2=\frac{1}{r}\frac{\sigma_y}{\sigma_x}$ 2M

we know that the angle between the two straight line is given by

$$tan\theta=\frac{m_2-m_1}{1+m_1m_2}=\frac{\frac{1}{r}\frac{\sigma_y}{\sigma_x}-r\frac{\sigma_y}{\sigma_x}}{1+\frac{1}{r}\frac{\sigma_y}{\sigma_x}.r\frac{\sigma_y}{\sigma_x}}=\frac{\left(\frac{1-r^2}{r}\right)\frac{\sigma_y}{\sigma_x}}{1+\frac{\sigma_y^2}{\sigma_x^2}}=\frac{\left(\frac{1-r^2}{r}\right)\frac{\sigma_y}{\sigma_x}}{\frac{\sigma_x^2+\sigma_y^2}{\sigma_x^2}}$$ 2M

$$=\left(\frac{1-r^2}{r}\right)\frac{\sigma_y}{\sigma_x}\times\frac{\sigma_x^2}{\sigma_x^2+\sigma_y^2}=\left(\frac{1-r^2}{r}\right)\times\frac{\sigma_x\sigma_y}{\sigma_x^2+\sigma_y^2}$$ 1M

$$tan\theta=\left(\frac{1-r^2}{r}\right)\times\frac{\sigma_x\sigma_y}{\sigma_x^2+\sigma_y^2}$$

Q.7.c)

x	y	xy	x^2	y^2
1	9	9	1	81
2	8	16	4	64
3	10	30	9	100
4	12	48	16	144
5	11	55	25	121
6	13	78	36	169
7	14	98	49	196
8	16	128	64	256
9	15	135	81	225
$\sum x=45$	$\sum y=108$	$\sum xy=597$	$\sum x^2=285$	$\sum y^2=1356$

4M

$$r=\frac{n\sum xy-\sum x\sum y}{\sqrt{n\sum x^2-(\sum x)^2}\sqrt{n\sum y^2-(\sum y)^2}}=\frac{513}{\sqrt{540}\sqrt{540}}=0.95$$ 3M

Question Number	Solution	Marks Allocated

Q.8. a) Here $n = 5$. Construct the table for the given data.

x	y	x^2	xy
1	14	1	14
2	27	4	54
3	40	9	120
4	55	16	220
5	68	25	340
$\sum x$=15	$\sum y$=204	$\sum x^2$=55	$\sum xy$=748

3M

the normal equations. **1M**

$$\left.\begin{aligned}\sum y &= a\sum x + nb\\ \sum xy &= a\sum x^2 + b\sum x\end{aligned}\right\} \Rightarrow \begin{aligned}204 &= 15a + 5b\\ 748 &= 55a + 15b\end{aligned}$$ **1M**

Solving these equations we get $a = 13.6$ and $b = 0$ **1M**

$\therefore$ required best st line is $y = ax + b = 13.6x + 0 = 13.6x$.

Q. 8. b) $n = 10$

$d_i = x_i - y_i$	-5	2	-4	2	2	0	1	-1	2	1
d_i^2:	25	4	16	4	4	0	1	1	4	1

4M

$n = 10,\ \sum d_i^2 = 60.$ **1M**

Rank correlation coefficient is given by $r = 1 - \frac{6\sum d^2}{n^3-n}$ **2M**

$$r = 1 - \frac{6(60)}{10^3 - 10} = 0.64$$

Q. 8. c) Since the regression lines pass through $(\bar{x},\bar{y})$

we have $4\bar{x} - 5\bar{y} + 33 = 0$ ------- (1)

$20\bar{x} - 9\bar{y} = 107$ ------- (2)

Solving the equations (1) and (2) we get $\bar{x} = 13,\quad \bar{y} = 17$ **1M**

The lines of regression of y on x, $4x - 5y + 33 = 0 \Rightarrow y = \frac{4}{5}x + \frac{33}{5}$ **2M**

Regression coefficient of y on x is $b_{yx} = r\frac{\sigma_y}{\sigma_x} = \frac{4}{5}$

The lines of regression of x on y, $20x - 9y = 107 \Rightarrow x = \frac{9}{20}y + \frac{107}{9}$ **1M**

Regression coefficient of x on y is $b_{xy} = r\frac{\sigma_x}{\sigma_y} = \frac{9}{20}$ **2M**

$$\therefore\ r = \sqrt{b_{xy} \times b_{yx}} = \sqrt{\frac{4}{5} \times \frac{9}{20}} = 0.6$$ **1M**

Q.9.a)

$$\text{Let } A = \begin{pmatrix}0 & 1 & -3 & -1\\ 1 & 0 & 1 & 1\\ 3 & 1 & 0 & 2\\ 1 & 1 & -2 & 0\end{pmatrix} \sim \begin{pmatrix}1 & 0 & 1 & 1\\ 0 & 1 & -3 & -1\\ 3 & 1 & 0 & 2\\ 1 & 1 & -2 & 0\end{pmatrix}$$

$R_1 \leftrightarrow R_2$ $\qquad$ $R_3 \to R_3 - 3R_1, R_4 \to R_4 - R_1$ **3M**

$$\sim \begin{pmatrix}1 & 0 & 1 & 1\\ 0 & 1 & -3 & -1\\ 0 & 1 & -3 & -1\\ 0 & 1 & -3 & -1\end{pmatrix} \sim \begin{pmatrix}1 & 0 & 1 & 1\\ 0 & 1 & -3 & -1\\ 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0\end{pmatrix}$$

$R_3 \to R_3 - R_2, R_4 \to R_4 - R_2$

This is an echelon form. The no. of non-zero rows is 2. $\therefore$ the rank of the matrix is 2.

The given system of equations is diagonally dominant system. **2M**

Q. 9.b)

$x = \frac{1}{10}(12 - y - z)$; $y = \frac{1}{10}(12 - z - x)$ and $z = \frac{1}{10}(12 - x - y)$ — 1M

Consider the initial approximations as $x_0 = 0, y_0 = 0, z_0 = 0$.

First iteration: $x_1 = \frac{1}{10}(12 - y_0 - z_0) = \frac{1}{10}(12 - 0 - 0) = 1.2$

$y_1 = \frac{1}{10}(12 - z_0 - x_1) = \frac{1}{10}(12 - 0 - 1.2) = 1.32$ — 1M

$z_1 = \frac{1}{10}(12 - x_1 - y_1) = \frac{1}{10}(12 - 1.2 - 1.32) = 1.452$

Second iteration: $x_2 = \frac{1}{10}(12 - y_1 - z_1) = \frac{1}{10}(12 - 1.32 - 1.452) = 1.2$ — 2M

$y_2 = \frac{1}{10}(12 - z_1 - x_2) = \frac{1}{10}(12 - 1.452 - 1.2) = 0.9348$

$z_2 = \frac{1}{10}(12 - x_2 - y_2) = \frac{1}{10}(12 - 1.2 - 0.9348) = 0.9865$

Third iteration: $x_3 = \frac{1}{10}(12 - y_2 - z_2) = \frac{1}{10}(12 - 0.9348 - 0.9865) = 1.0079$ — 2M

$y_3 = \frac{1}{10}(12 - z_2 - x_3) = \frac{1}{10}(12 - 0.9865 - 1.0079) = 1.0005$

$z_3 = \frac{1}{10}(12 - x_3 - y_3) = \frac{1}{10}(12 - 1.0079 - 1.0005) = 0.9992$

$\therefore$ approx. solu of the given system of equs is $x = 1.0079, y = 1.0005, z = 0.9992$ — 2M

Q. 9. c)

Take $(1 \quad 0 \quad 0)^T$ as an initial eigen vector. — 1M

$$\begin{pmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 2 \\ -1 \\ 0 \end{pmatrix} = 2\begin{pmatrix} 1 \\ -0.5 \\ 0 \end{pmatrix}$$ — 1M

$$\begin{pmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{pmatrix}\begin{pmatrix} 1 \\ -0.5 \\ 0 \end{pmatrix} = \begin{pmatrix} 2.5 \\ -2 \\ 0.5 \end{pmatrix} = 2.5\begin{pmatrix} 1 \\ -0.8 \\ 0.2 \end{pmatrix}$$ — 1M

$$\begin{pmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{pmatrix}\begin{pmatrix} 1 \\ -0.8 \\ 0.2 \end{pmatrix} = \begin{pmatrix} 2.8 \\ -2.8 \\ 1.2 \end{pmatrix} = 2.8\begin{pmatrix} 1 \\ -1 \\ 0.43 \end{pmatrix}$$ — 1M

$$\begin{pmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{pmatrix}\begin{pmatrix} 1 \\ -1 \\ 0.43 \end{pmatrix} = \begin{pmatrix} 3 \\ -3.43 \\ 1.86 \end{pmatrix} = 3.43\begin{pmatrix} 0.87 \\ -1 \\ 0.54 \end{pmatrix}$$ — 1M

$$\begin{pmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{pmatrix}\begin{pmatrix} 0.87 \\ -1 \\ 0.54 \end{pmatrix} = \begin{pmatrix} 2.74 \\ -3.41 \\ 2.08 \end{pmatrix} = 3.41\begin{pmatrix} 0.76 \\ -1 \\ 0.65 \end{pmatrix}$$ — 1M

After 5 iterations, largest eigen value is 3.41

The corresponding eigen vector is $\begin{pmatrix} 0.76 \\ -1 \\ 0.65 \end{pmatrix}$. — 1M

Q. 10.a)

Augmented matrix is

$$(A, B) = \begin{pmatrix} 1 & 1 & 1 & 6 \\ 1 & 2 & 3 & 10 \\ 1 & 2 & \lambda & \mu \end{pmatrix} \sim \begin{pmatrix} 1 & 1 & 1 & 6 \\ 0 & 1 & 2 & 4 \\ 0 & 1 & \lambda - 1 & \mu - 6 \end{pmatrix} \sim \begin{pmatrix} 1 & 1 & 1 & -3 \\ 0 & -2 & -5 & 7 \\ 0 & 0 & \lambda - 3 & \mu - 10 \end{pmatrix}$$

$R_2 \to R_2 - R_1$, $R_3 \to R_3 - R_2$ This is in echelon form. — 3M

	$R_3 \to R_3 - R_1$ (i) If $\lambda \neq 3$, then $\rho(A) = \rho(A,B) = 3$. Gives unique solution. (ii) If $\lambda = 3, \mu = 10$ then $\rho(A) = \rho(A,B) = 2$. infinitely many solutions. (iii) If $\lambda = 3, \mu \neq 10$ then $\rho(A) \neq \rho(A,B)$. has no solution.	
Q. 10. b)	The augmented matrix associated to the given system of equations is $(A:B) = \begin{pmatrix} 1 & 1 & 1 & 10 \\ 2 & -1 & 3 & 19 \\ 1 & 2 & 3 & 22 \end{pmatrix} \sim \begin{pmatrix} 1 & 1 & 1 & 10 \\ 0 & -3 & 1 & -1 \\ 0 & 1 & 2 & 12 \end{pmatrix} \sim \begin{pmatrix} 1 & 1 & 1 & 10 \\ 0 & 1 & -1/3 & 1/3 \\ 0 & 1 & 2 & 12 \end{pmatrix}$ $R_2 \to R_2 - 2R_1, R_3 \to R_3 - R_1$ $\quad R_2 \to \frac{R_2}{-3}$ $\quad R_3 \to R_3 - R_2, R_1 \to R_1 - R_2$ $\sim \begin{pmatrix} 1 & 0 & 4/3 & 29/3 \\ 0 & 1 & -1/3 & 1/3 \\ 0 & 0 & 7/3 & 35/3 \end{pmatrix}$ $\quad \sim \begin{pmatrix} 1 & 0 & 4/3 & 29/3 \\ 0 & 1 & -1/3 & 1/3 \\ 0 & 0 & 1 & 5 \end{pmatrix}$ $R_3 \to \frac{3}{7} R_3$ $\quad R_2 \to R_2 + \frac{1}{3} R_3, R_1 \to R_1 - \frac{4}{3} R_3$ $\sim \begin{pmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 5 \end{pmatrix}$ Therefore, $x = 3,\ y = 2,\ z = 5$.	3M 3M 3M 1M
Q.10.c)	$(A,B) = \begin{pmatrix} 5 & 3 & 7 & 4 \\ 3 & 26 & 2 & 9 \\ 7 & 2 & 10 & 5 \end{pmatrix}$ $\quad R_2 \to 5R_2 - 3R_1, R_3 \to 5R_3 - 7R_1$ $\sim \begin{pmatrix} 5 & 3 & 7 & 4 \\ 0 & 121 & -11 & 33 \\ 0 & -11 & 1 & -3 \end{pmatrix}$ $R_3 \to 11R_3 + R_2$ $\quad \sim \begin{pmatrix} 5 & 3 & 7 & 4 \\ 0 & 121 & -11 & 33 \\ 0 & 0 & 0 & 0 \end{pmatrix}$ This is in echelon form. Number of non-zero rows is 2. $\rho(A) = \rho(A,B) = 2$. Therefore, the given system of equations is consistent and has an infinite no. of solutions. Reduced system of equations is $5x + 3y + 7z = 4$ ---- (1) $121y - 11z = 33$ ---- (2) Choose $z = k$ then $y = \frac{3+k}{11}$ and $x = \frac{7-16k}{11}$. Therefore, $x = \frac{7-16k}{11}, y = \frac{3+k}{11}, z = k$.	3M 2M 2M

NOTE: Marks can be awarded for alternate methods.