Assignment -2 **Poornima Kalyan Sundaram(2306047325)**

Problem 1: (5 *points*)
A color in the XYZ (or RGB) space is represented by three coordinates, call them (X, Y, Z). The corresponding projection of that color into the chromaticity space is represented by normalized chromaticity coordinates. (x_c, y_c). Prove that

$$Z = [(1- x_c - y_c) / y_c]\ Y$$

problem(1):

$$x_c = \frac{X}{X+Y+Z} \quad (1), \qquad y_c = \frac{Y}{X+Y+Z} \quad (2)$$

$$\Rightarrow X x_c + Y x_c + Z x_c = X.$$

$$\Rightarrow X = \frac{x_c(Y+Z)}{(1-x_c)} \quad \text{—— (3)}$$

~~yc~~ = Similarly (2) can be written as

$$y_c(X+Y+Z) = Y.$$

$$\Rightarrow y_c X + y_c Y + y_c Z = Y. \quad \text{—— (4)}$$

$$\Rightarrow X = \frac{Y(1-y_c) - y_c Z}{y_c}.$$

Combine (3) & (4)

$$\frac{x_c(Y+Z)}{1-x_c} = \frac{Y(1-y_c) - y_c Z}{y_c}.$$

$$x_c \cancel{y_c} Y + x_c \cancel{y_c} Z = Y - Y y_c - y_c Z - x_c Y + x_c y_c Y + x_c y_c Z$$

$$\Rightarrow Y(1-x_c-y_c) = y_c Z \Rightarrow \boxed{Z = \frac{(1-x_c-y_c)}{y_c} \cdot Y}$$

Problem 2: (*15 points*)
In a three dimensional color space such as XYZ, any color C with coordinates (X, Y, Z) can be expressed as a linear combination of the primaries P_1, P_2, P_3 with coordinates (X_1, Y_1, Z_1), (X_2, Y_2, Z_2) and (X_3, Y_3, Z_3) respectively. This may be expressed as

$$C\,(X, Y, Z) = \alpha_1 * P_1\,(X_1, Y_1, Z_1) + \alpha_2 * P_2\,(X_2, Y_2, Z_2) + \alpha_3 * P_3(X_3, Y_3, Z_3)$$

In this question you are asked to show that similarly, the normalized chromaticity coordinates of C can *also* be expressed as a linear combination of the normalized chromaticity coordinates of P_1, P_2, P_3. Proceed by answering the following:

- Find the normalized chromaticity coordinates of P_1, P_2, and P_3 in terms of given known quantities (*3 points*)
- Express the normalized chromaticity coordinates of the color C in terms of the chromaticity coordinates of P_1, P_2, and P_3 (*6 points*)
- Hence prove that the chromaticity coordinates of any color C (which is a linear combination of primaries P_1, P_2, and P_3 in XYZ color space) can be represented also as a linear combination of the chromaticity coordinates of the respective primaries. (*6 points*)

problem 2.

given:

$X = \alpha X_1 + \alpha X_2 + \alpha X_3$

$Y = \alpha Y_1 + \alpha Y_2 + \alpha Y_3$

$Z = \alpha Z_1 + \alpha Z_2 + \alpha Z_3$

(i)

Normalized Chromaticity coordinate of P_1 :-

$x_1 = \dfrac{X_1}{X_1+Y_1+Z_1}$, $y_1 = \dfrac{Y_1}{X_1+Y_1+Z_1}$

Normalized chromaticity coordinate of P_2 :-

$x_2 = \dfrac{X_2}{X_2+Y_2+Z_2}$, $y_2 = \dfrac{Y_2}{X_2+Y_2+Z_2}$

Normalized chromaticity coordinate of P_3 :-

$x_3 = \dfrac{X_3}{X_3+Y_3+Z_3}$, $y_3 = \dfrac{Y_3}{X_3+Y_3+Z_3}$

(ii) For C, normalized chromaticity coordinates are :-

$$x = \frac{X}{X+Y+Z}, \quad y = \frac{Y}{X+Y+Z} \longrightarrow (1)$$

In general;

$$Z = [(1-x_c-y_c)/y_c]\, Y \; [\longrightarrow \text{from (problem (1))}]$$

$$X = Y \cdot \frac{x_c}{y_c} \; [\rightarrow \text{from equation (1)}] \longrightarrow (2)$$

$$X+Y+Z = \alpha_1 X_1 + \alpha X_2 + \alpha X_3 + \alpha_1 Y_1 + \alpha Y_2 + \alpha Y_3 + \alpha_1 Z_1 + \alpha Z_2 + \alpha Z_3$$

$$= \alpha_1 (X_1+Y_1+Z_1) + \alpha_2 (X_2+Y_2+Z_2) + \alpha_3 (X_3+Y_3+Z_3) \longrightarrow \text{from (2)}$$

$$= \alpha_1 + \alpha_2 + \alpha_3$$

($\because$ R+G+B values equal to 1 in HSV plane)

Similarly

$$\& \; X_1 = x_1 * 1 = x_1, \quad X_2 = x_2 \; \& \; X_3 = x_3$$

$$\therefore x = \frac{\alpha X_1 + \alpha X_2 + \alpha X_3}{X+Y+Z}$$

$$x = \frac{\alpha_1 x_1 + \alpha_2 x_2 + \alpha_3 x_3}{\alpha_1 + \alpha_2 + \alpha_3}$$

Likewise

$$y = \frac{\alpha_1 y_1 + \alpha_2 y_2 + \alpha_3 y_3}{\alpha_1 + \alpha_3 + \alpha_3}$$

(iii) from (ii)

$$x = \frac{\alpha_1}{\alpha_1+\alpha_2+\alpha_3} x_1 + \frac{\alpha_2}{\alpha_1+\alpha_2+\alpha_3} x_2 + x_3 \frac{\alpha_3}{\alpha_1+\alpha_2+\alpha_3}$$

$$y = \frac{\alpha_1}{\alpha_1+\alpha_2+\alpha_3} y_1 + \frac{\alpha_2}{\alpha_1+\alpha_2+\alpha_3} y_2 + \frac{\alpha_3}{\alpha_1+\alpha_2+\alpha_3} y_3$$

$$\therefore C(x,y) = \frac{\alpha_1}{\alpha_1+\alpha_2+\alpha_3} C_1(x_1,y_1) + \frac{\alpha_2}{\alpha_1+\alpha_2+\alpha_3} C_2(x_2,y_2) + \frac{\alpha_3}{\alpha_1+\alpha_2+\alpha_3} C_3(x_3,y_3)$$

where C_1, C_2 & C_3 are chromaticity co-ordinates of the primaries

Problem 3: (*20 points*)

Consider a communication system that gives out only two symbols X and Y. Assume that the parameterization followed by the probabilities are $P(X) = x^2$ and $P(Y) = (1-x^2)$.

- Write down the entropy function and plot it as a function of *x*. (*1 point*)
- From your plot, for what value of *x* does the Entropy become a minimum? (*3 points*)
- Although the plot visually gives you the value of *x* for which the entropy in minimum, can you now mathematically find out the value(s) for which the entropy is a minimum? (*8 points*)
- Can you do the same for the maximum, that is can you mathematically find out value(s) of *x* for which the value is a maximum? (*8 points*)

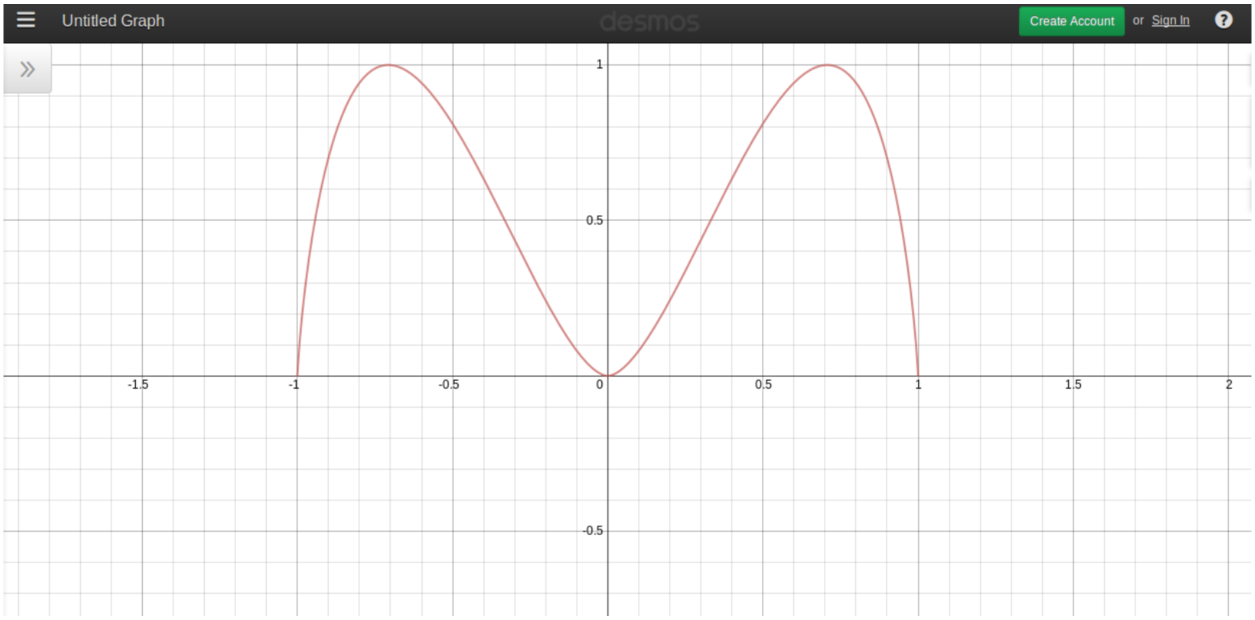
Problem ③

$$H = -\sum_{i=X,Y} P(i)\log_2 P(i)$$

$$= -(x^2\log_2 x^2 + (1-x^2)\log_2(1-x^2)]$$

$$= -[x^2\log_2 x^2 + \log_2(1-x^2) - x^2\log(1-x^2)]$$

(ii) Graph for the entropy function



For x= -1,1 and 0 the entropy is minimum.

(iii) & (iv). $H = -x^2 \log(x^2) - (1-x^2)\log(1-x^2)$

Find 1st order derivative,

$$H' = -\Big[2x\log(x^2) + \cancel{x^2}\frac{1}{\cancel{x^2}}\cdot\cancel{2x} + (-2x)\log(1-x^2) + \frac{\cancel{(1-x^2)}\cdot(-\cancel{2x})}{\cancel{(1-x^2)}}\Big]$$

$$H' = -[2x\log(x^2) - 2x\log(1-x^2)]$$

equate $H' = 0$, if $-2x = 0$; then $x = 0$

$\log x^2 - \log(1-x^2) = 0$

$\log x^2 = \log(1-x^2) \Rightarrow x = \pm 1$

also $x^2 = 1 - x^2$

$$2x^2 = 1 \Rightarrow x = \pm \frac{1}{\sqrt{2}}$$

Substitute $\frac{1}{\sqrt{2}}$ in H,

$$H = -\frac{1}{2}\log\frac{1}{2} - \frac{1}{2}\log\left(\frac{1}{2}\right)$$

$$= -1 \times -1 = 1$$

Substitute $-\frac{1}{\sqrt{2}}$ in H,

$$H = -\frac{1}{2}\log\frac{1}{2} - 1/2\log(1/2)$$

$$= 1$$

Substituting 0, 1 and -1 in H we get H = 0.

∴ Since H is positive with $1/\sqrt{2}$ & $\frac{-1}{\sqrt{2}}$, they are points of maxima.

H is non positive with 0 & -1, they are points of minima.

Analysis Part

Conclusions:

- DCT seems to perform better than DWT in some cases
- As the number of coefficients increase , the quality of image increases in both DWT and DCT

Execution:

Type “java <file name> <image filename> -1” for thr program to start executing. The existing window would close and a new image would appear for every 10 seconds with an increased number of coefficients.