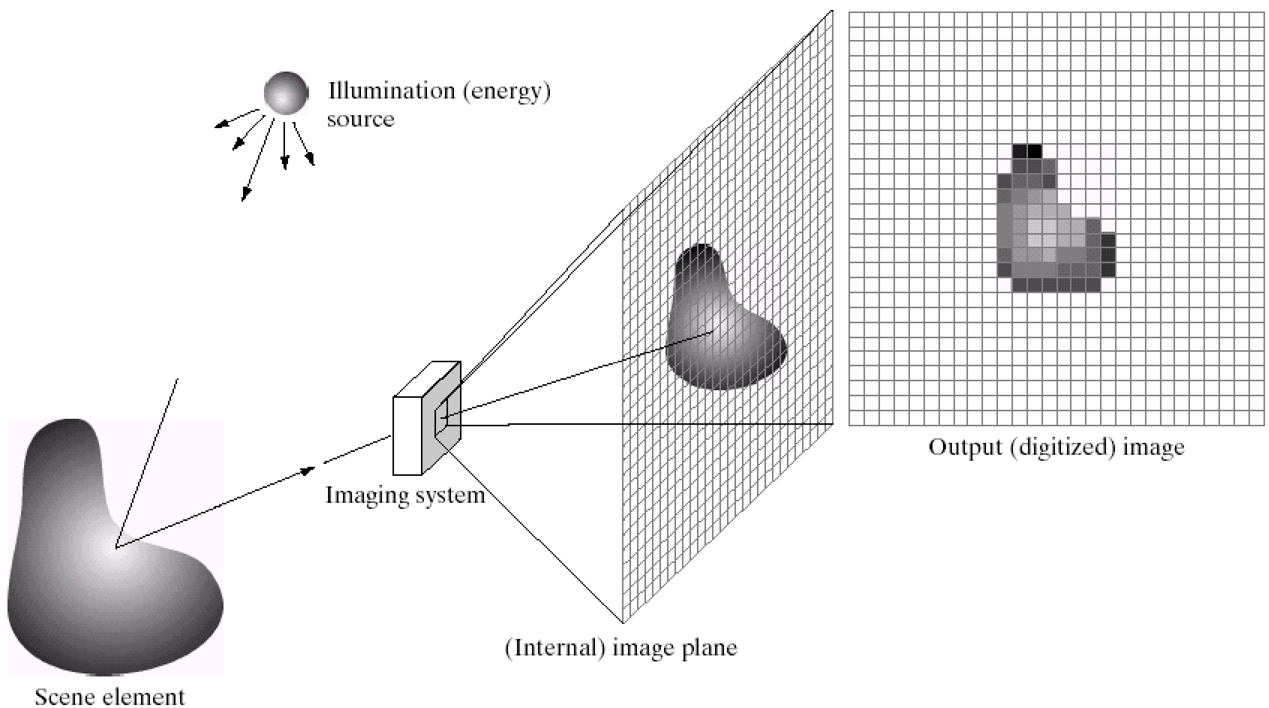
Digital Image Processing Operations

Session2 Overview

- This lecture will cover:
- Digitization
  - Quantization
  - Resolutions
- Pixel Relationships
  - Neighbourhood pixels
  - Connectivity
  - Adjacency
  - Path
  - Connected Components

What is a Digital Image?

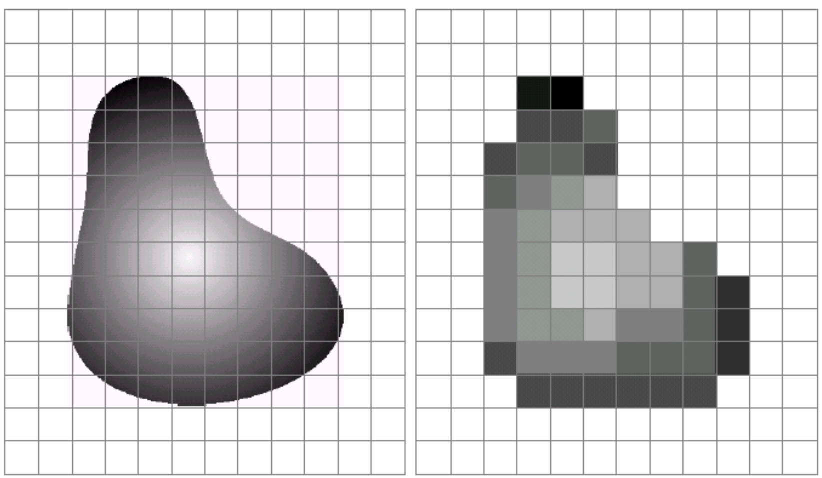
•A **digital image** is a representation of a two-dimensional image as a finite set of digital values, called picture elements or pixels

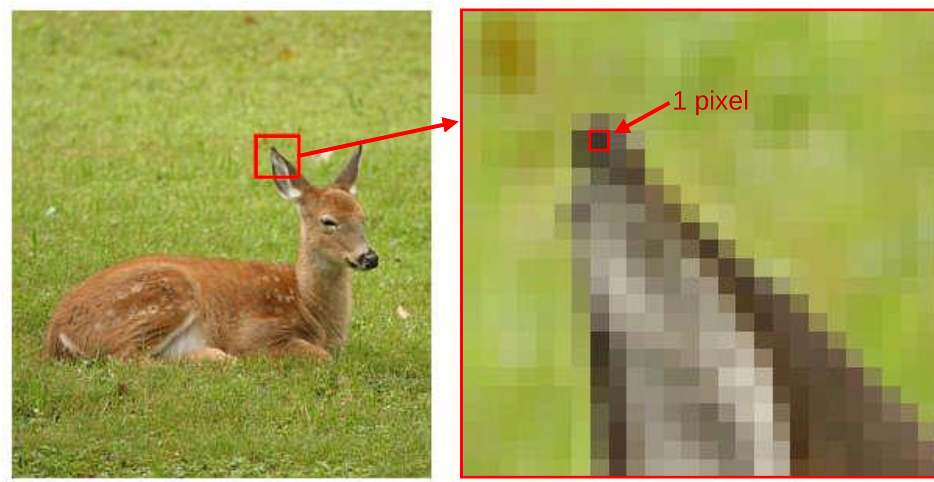


What is a Digital Image? (cont…)

•Pixel values typically represent gray levels, colours, heights, opacities etc

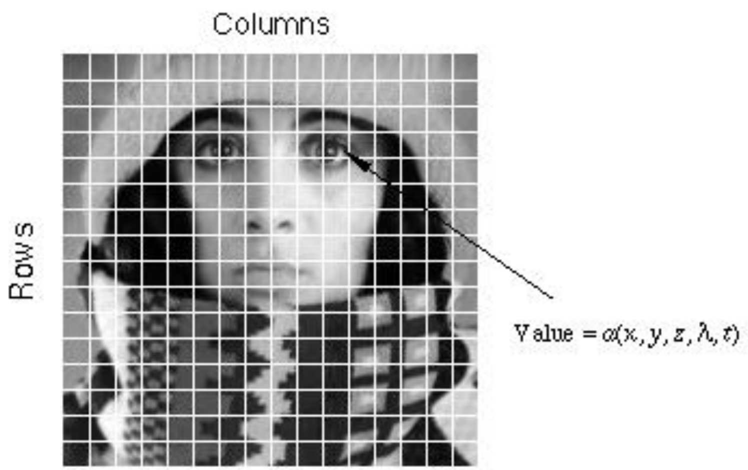
•**Remember** *digitization* implies that a digital image is an *approximation* of a real scene





Images taken from Gonzalez & Woods, Digital Image Processing (2002)

Digital Image



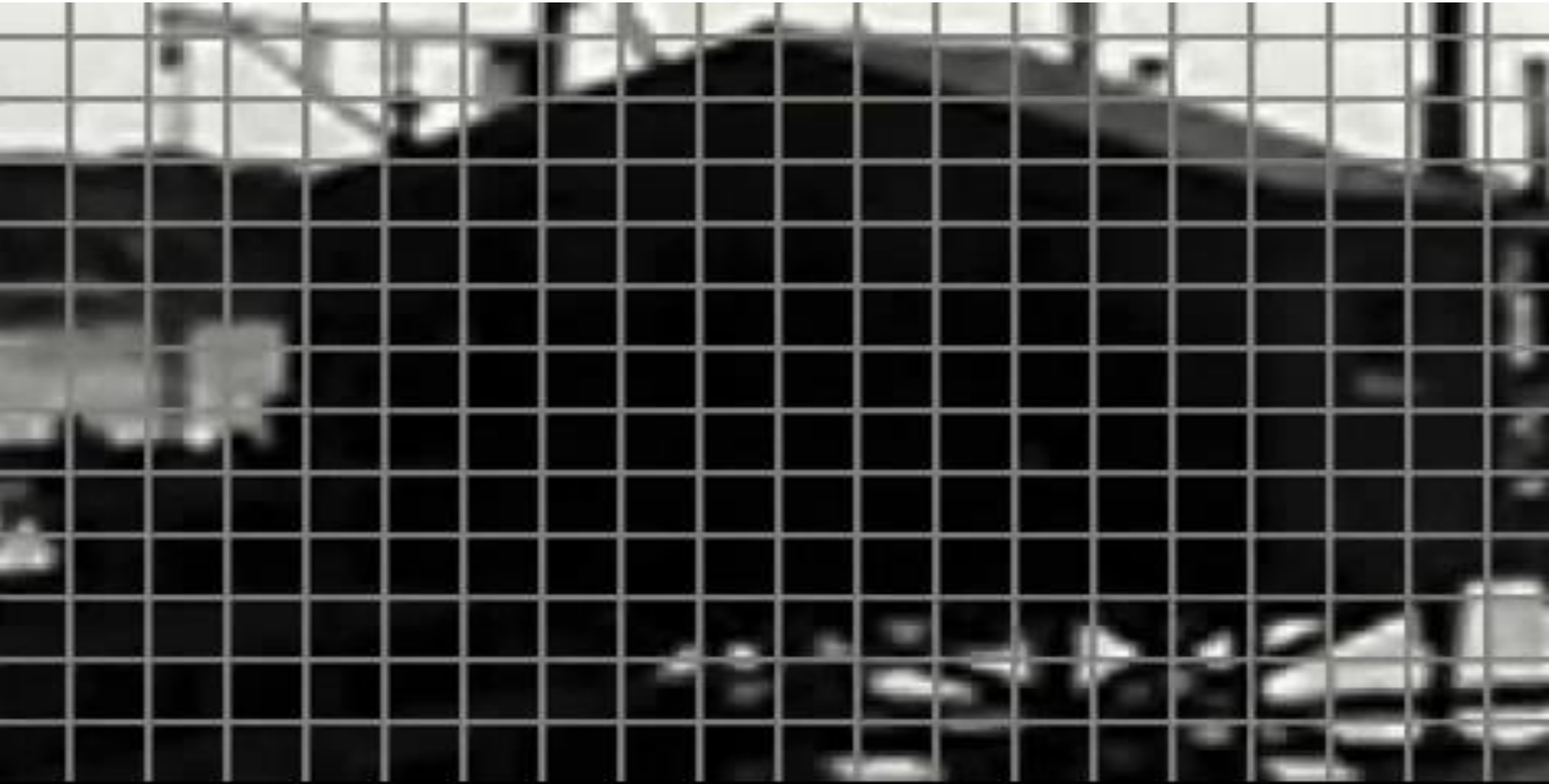
$$f(x,y) \cong \begin{bmatrix} f(0,0) & f(0,1) & f(0,N-1) \\ . & . & . \\ . & . & . \\ f(N-1,0) & .... & f(N-1,N-1) \end{bmatrix}_{N \times N}$$

- picture element or pixel or pel
- Typical image size:
  - 64 X 64, 128 X 128
  - 256 X 256, 640 X 480 1024 X 1024

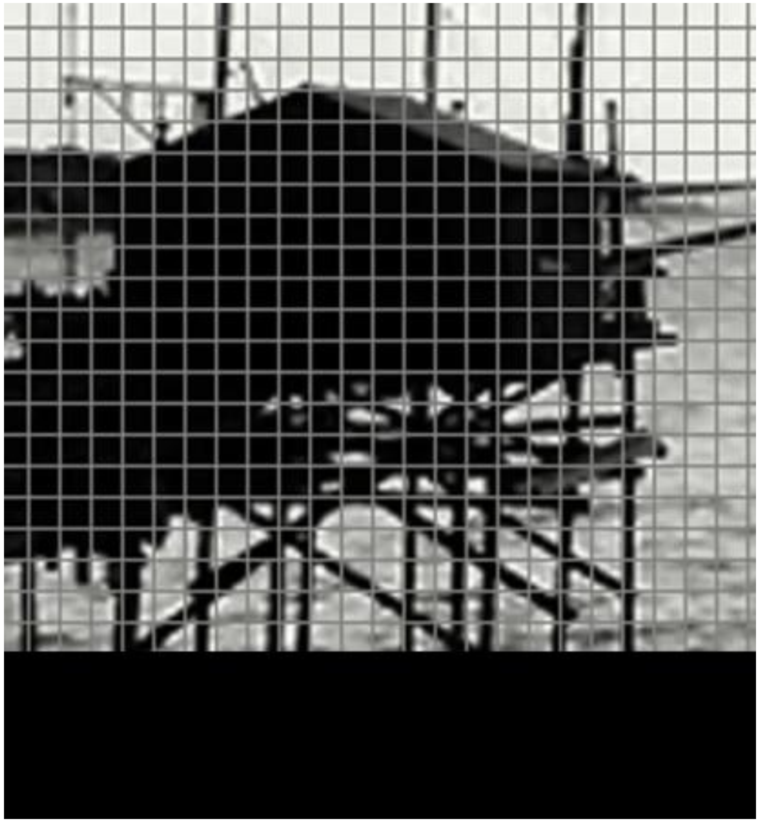
Gray scale Image

Sampling - Grids

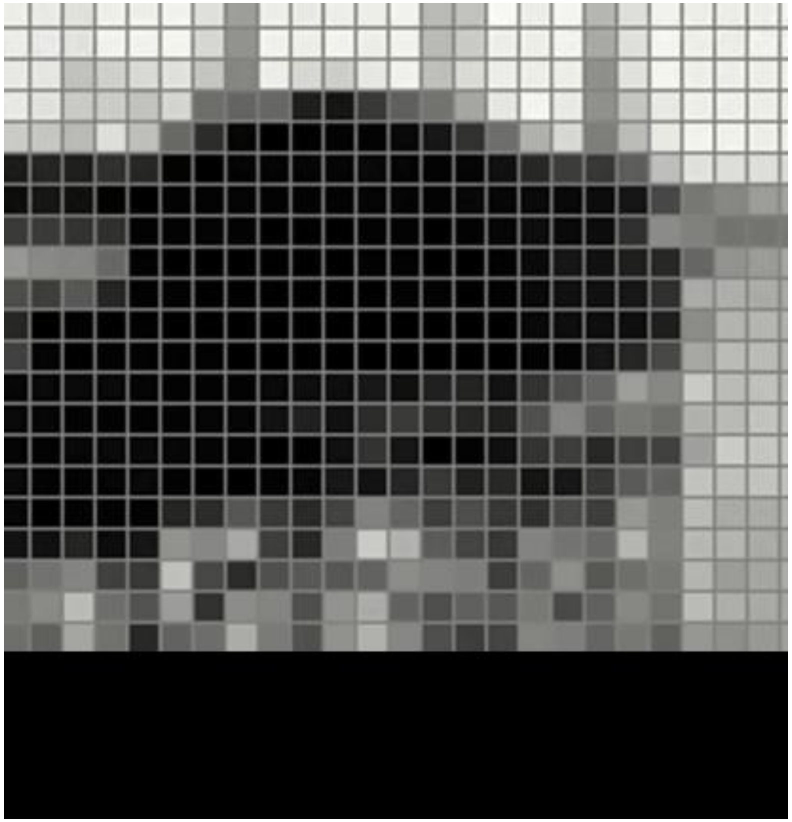
Sampling



Digitization



Original Image



Digitized Image

Reduction in Quantization level reduces quality

Digital Image

- The value of the function $f\ (x,\ y)$ at every point indexed by a row and a column is called *grey value* or *intensity* of the image.

- Resolution is an important characteristic of an imaging system. It is the ability of the imaging system to produce the smallest discernible details, i.e., the smallest sized object clearly, and differentiate it from the neighboring small objects that are present in the image.

Useful definitions

- Image resolution depends on two factors—optical resolution of the lens and spatial resolution. A useful way to define resolution is the smallest number of line pairs per unit distance.

- Spatial resolution depends on two parameters—the number of pixels of the image and the number of bits necessary for adequate intensity resolution, referred to as the bit depth.

- Bit depth The number of bits necessary to encode the pixel value is called *bit depth*. Bit depth is a power of two.

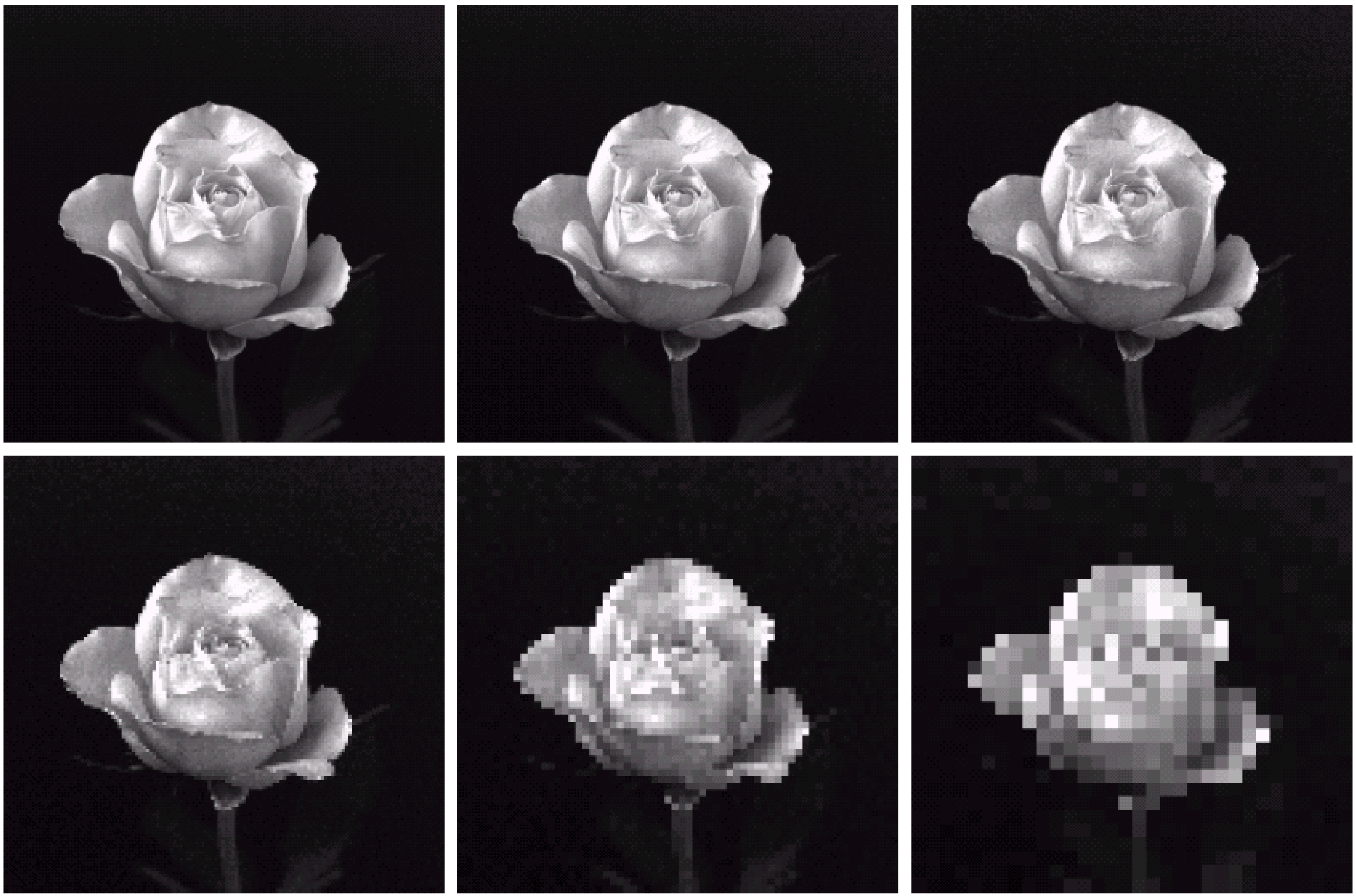
Intensity Level Resolution

- *Intensity level resolution* refers to the number of intensity levels used to represent the image
  - The more intensity levels used, the finer the level of detail discernible in an image
  - Intensity level resolution is usually given in terms of the number of bits used to store each intensity level

Number of Bits	**Number of Intensity Levels**	**Examples**
1	2	0, 1
2	4	00, 01, 10, 11
4	16	0000, 0101, 1111
8	256	00110011, 01010101
16	65,536	1010101010101010

Spatial Resolution..contd

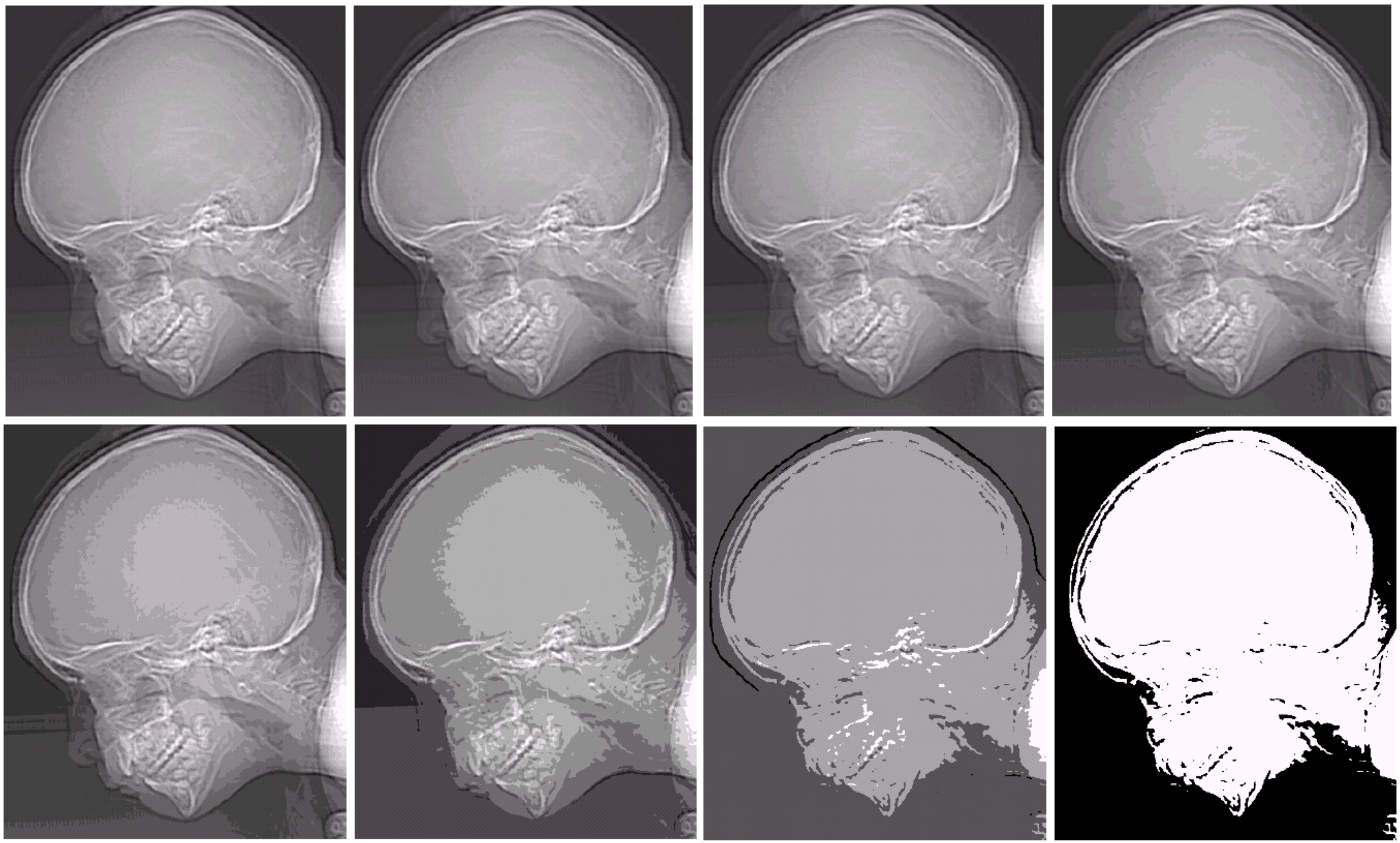
1024 * 1024 512 * 512 256 * 256



128 * 128 64 * 64 32 * 32

Intensity Level Resolution (cont…)

256 grey levels (8 bits per pixel) 128 grey levels (7 bpp) 64 grey levels (6 bpp) 32 grey levels (5 bpp)



16 grey levels (4 bpp) 8 grey levels (3 bpp) 4 grey levels (2 bpp) 2 grey levels (1 bpp)

Session3 Overview

- Digitization
  - Quantization
  - Resolutions
- This lecture will cover:
- Pixel Relationships
  - Neighbourhood pixels
  - Connectivity
  - Adjacency
  - Path
  - Connected Components

Neighborhoods of a pixel

- A pixel p at location (x,y) has horizontal And vertical neighbors.

	(x-1,y)	
(x,y-1)	p (x,y)	(x,y+1)
	(x+1,y)	

- This set of four pixels is called 4-neighbors Of p=N_4(p).
- Each of these neighbors is at a unit distance From p.
- If p is a boundary pixel then it will have less Number of neighbors.

P				

Diagonal & 8-neighbors.

A pixel p has four diagonal neighbors=$N_D(p)$

(x-1,y-1)		(x-1,y+1)
	p (x,y)	
(x+1,y-1)		(x+1,y+1)

The points of $N_4(p)$ and $N_D(p)$ together are
Called 8-neighbors of p.
$N_8(p) = N_4(p) \cup N_D(p)$

If p is a boundary pixel then both $N_D(p)$ and
And $N_8(p)$ will have less number of pixels.

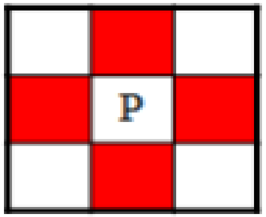
Neighbours of a Pixel

Neighbors of a Pixel

- The 4- neighbors of pixel p are: $\mathbf{N_4(p)}$

  Any pixel p(x,y) has two vertical and two horizontal neighbors, given by:

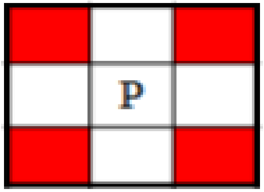
  (x+1, y), (x-1, y), (x, y+1), (x, y-1)



- The 4- diagonal neighbors are: $\mathbf{N_D(p)}$

  given by:

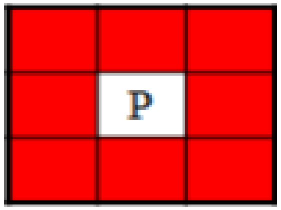
  (x+1, y+1), (x+1, y-1), (x-1, y+1), (x-1, y-1)



- The 8-neighbors are : $\mathbf{N_8(p)}$

  8-neighbors of a pixel pare its vertical, horizontal and 4 diagonal neighbors denoted by $N_8(p)$

  $$\mathbf{N_8(p) = N_4(p) \cup N_D(p)}$$



What is connectivity ?

Two pixels are said to be connected if they are adjacent in some sense

-- They are neighbors(N_4,N_D or N_8) and

-- Their intensity values (gray levels) are similar

For a binary image B, two points p and q Will be connected if $q \in N(p)$ or $p \in N(q)$ and $B(p) = B(q)$.

	q	
	p	

		q
	p	

	p	q

.....etc

Connectivity

Let V be the set of gray levels used to define Connectivity for two points p,q $\in$ v, three types of Connectivity are defined

- 4-connectivity=>p,q $\in$ v & p $\in N_4(q)$
- 8-connectivity=>p,q $\in$ v & p $\in N_8(q)$
- M-connectivity (mixed connectivity)

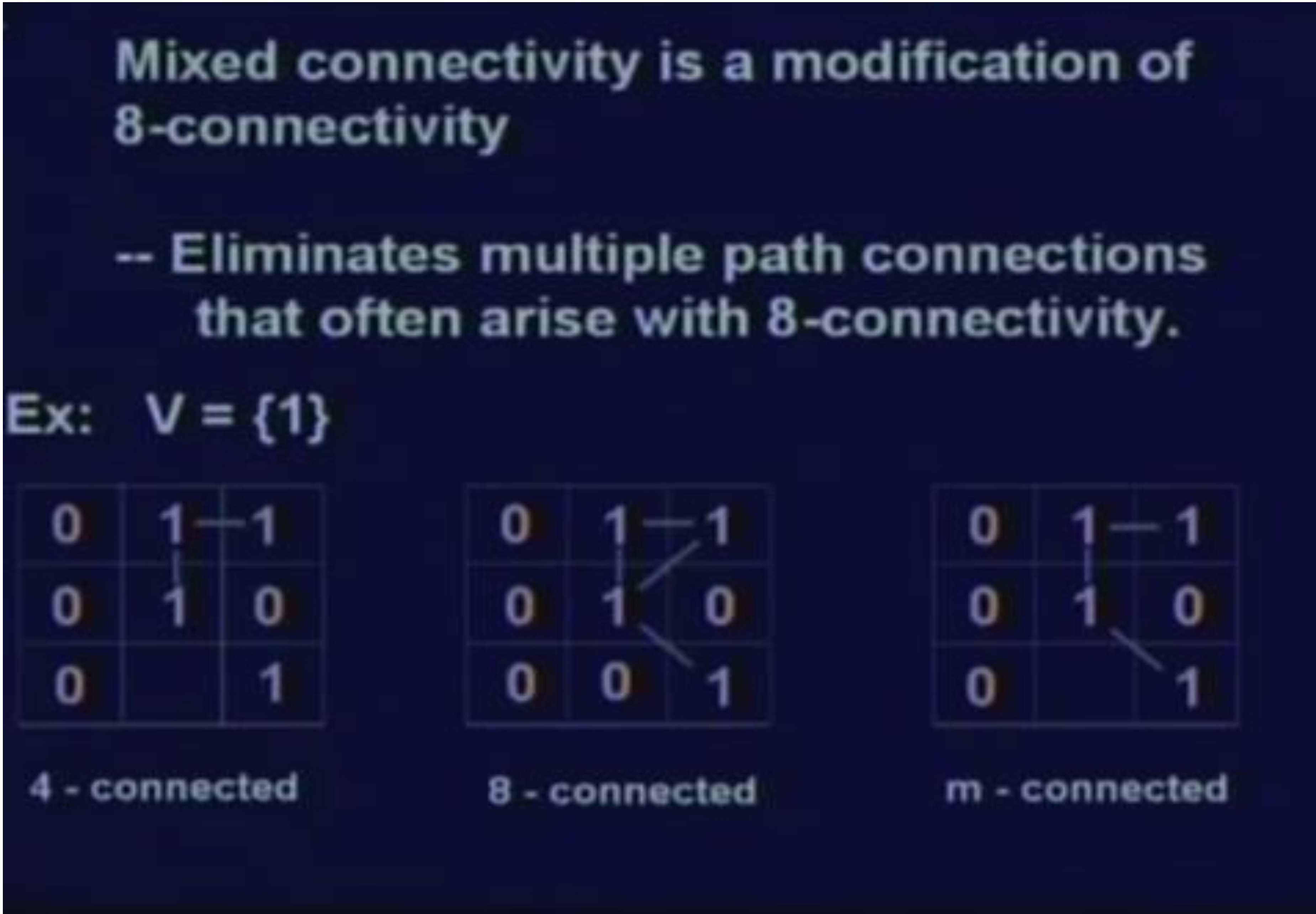
  p,q $\in$ v are m-connected if

  (i) $q \in N_4(p)$ Or

  (ii) $q \in N_D(p)$ and $N_4(p) \cap N_4(q) = \phi$

  $N_4(p) \cap N_4(q)$ => set of pixels that are 4-neighbors Of both p and q and whose values are from v.

Connectivity



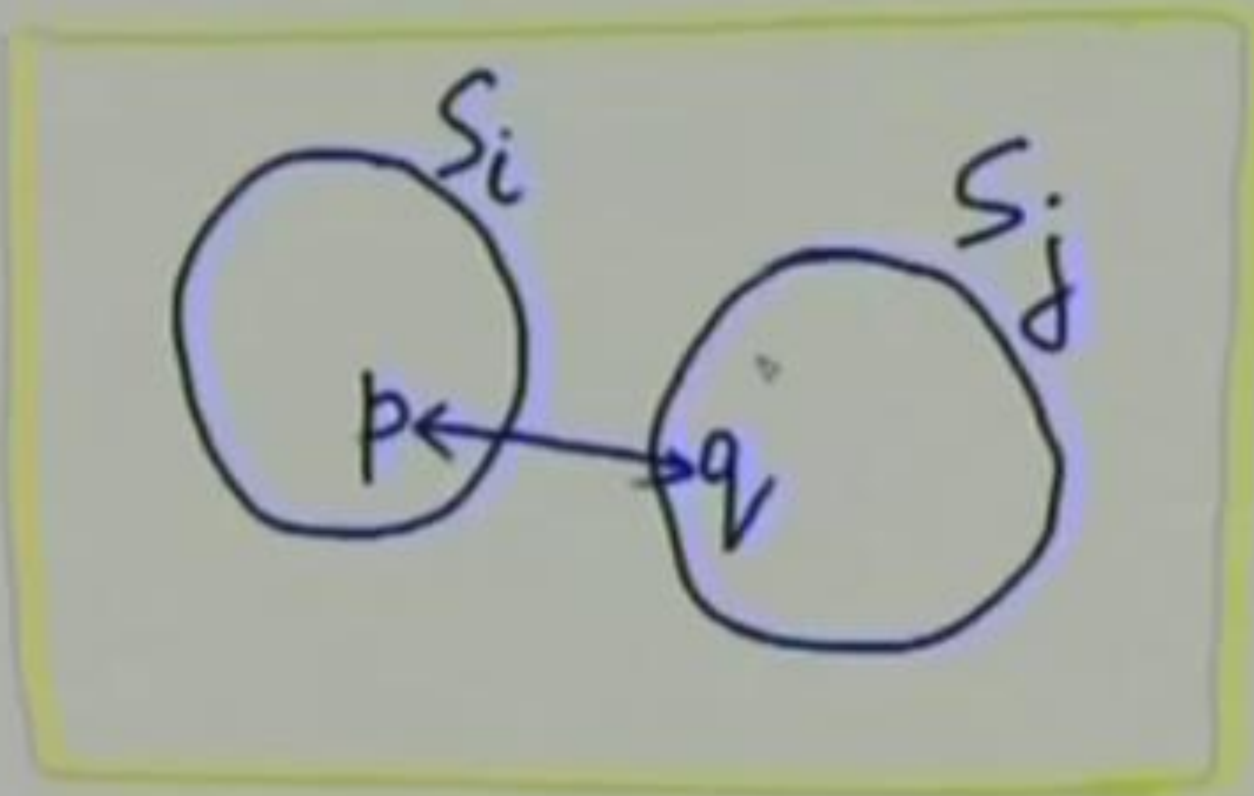
Adjacency

Two pixels p and q are adjacent if they are connected

- 4-adjacency
- 8-adjacency
- m-adjacency

-- depending on type of connectivity used.

Two image subsets S_i and S_j are adjacent if $\Im$ P $\in S_i$ and $\Im$ q $\in S_j$ such that p and q are adjacent



S_i
S_j
p
q

Two image subsets S1 and S2 are adjacent if some pixel in S1 is adjacent to some pixel in S2.

Consider the two image subsets, $S1$ and $S2$, shown in the following figure. For V={1}, determine whether these two subsets are (a) 4-adjacent, (b) 8-adjacent, or (c) m-adjacent.

$$
\begin{array}{c|cccc|cccc|c}
 & \multicolumn{4}{c|}{S_1} & \multicolumn{4}{c|}{S_2} & \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\
1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 \\
1 & 0 & 0 & 1 & 0 & 1 & 1 & 0 & 0 & 0 \\
0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\
\hline
0 & 0 & 1 & 1 & 1 & 0 & 0 & 1 & 1 & 1
\end{array}
$$

Solution:

Let p and q be as shown in Fig. Then:

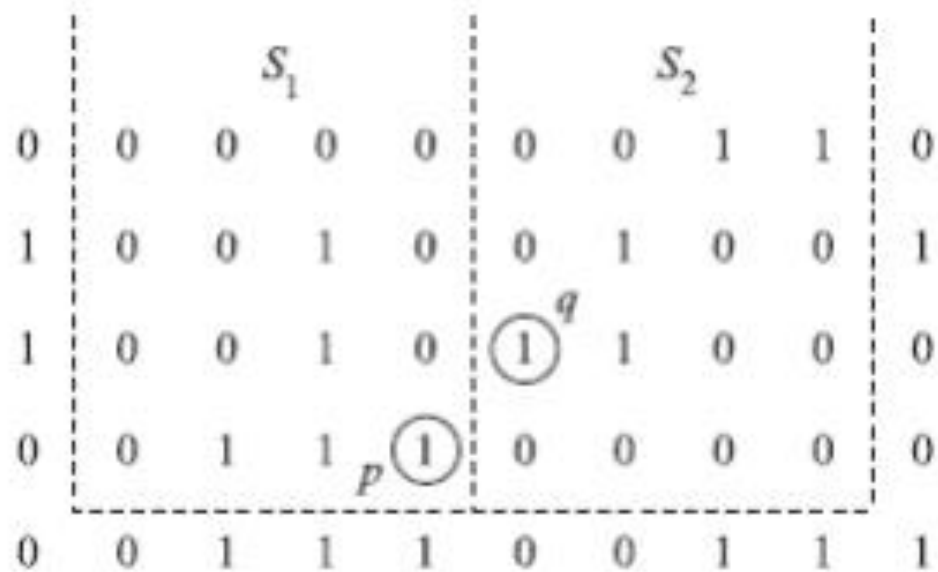
(a) $S1$ and $S2$ are **not 4-connected** because q is not in the set $N_4(p)$;

(b) $S1$ and $S2$ are **8-connected** because q is in the set $N_8(p)$;

(c) $S1$ and $S2$ are ***m*-connected** because

(i) q is in $N_D(p)$, and

(ii) the set $N_4(p) \cap N_4(q)$ is empty.



Path

A path from $p(x,y)$ to $q(s,t)$ is a sequence of distinct pixels.

$(x_0,y_0), (x_1,y_1)\ldots\ldots\ldots(x_n,y_n)$

Where

$(x_0,y_0) = (x,y), (x_n,y_n) = (s,t)$
(x_i,y_i) is adjacent to (x_{i-1},y_{i-1})
for $1 \leq i \leq n$

$n \Rightarrow$ length of the path.

Connected Component

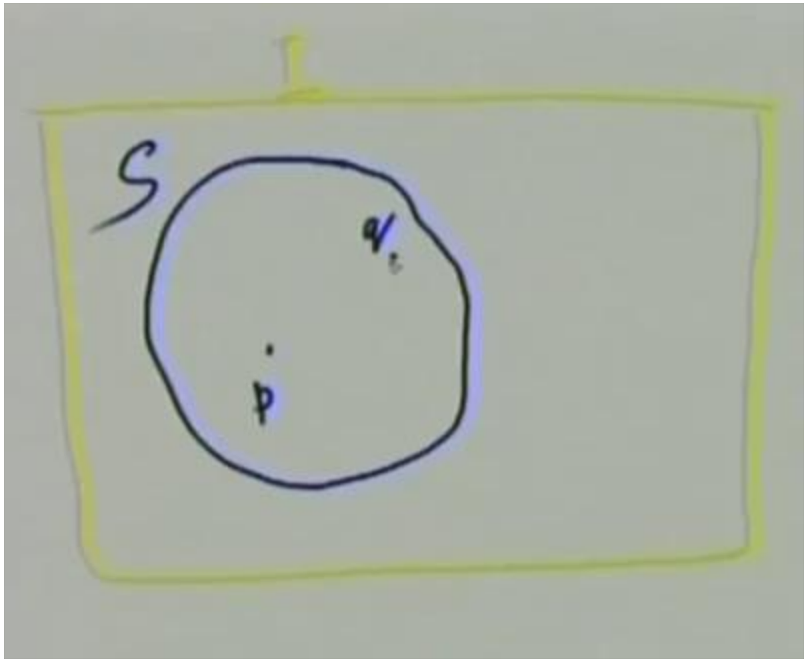
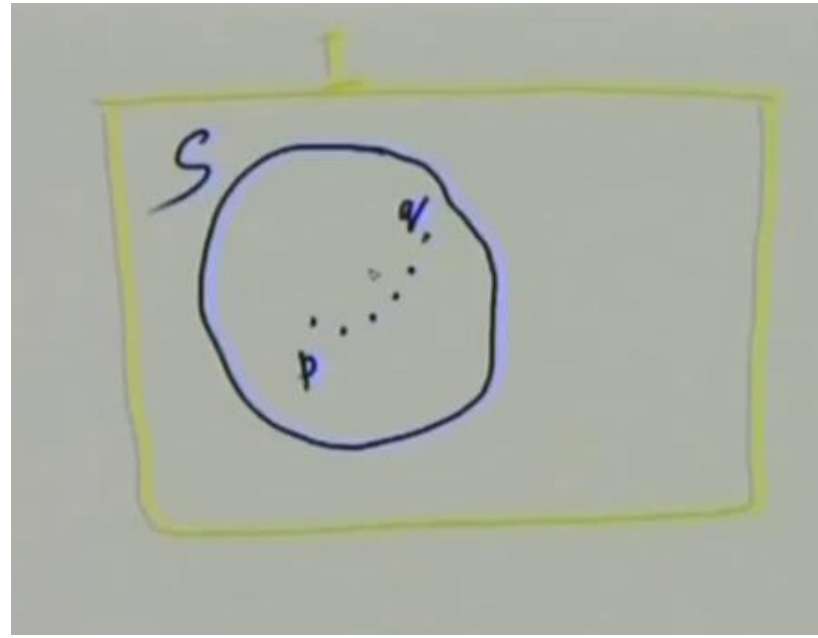
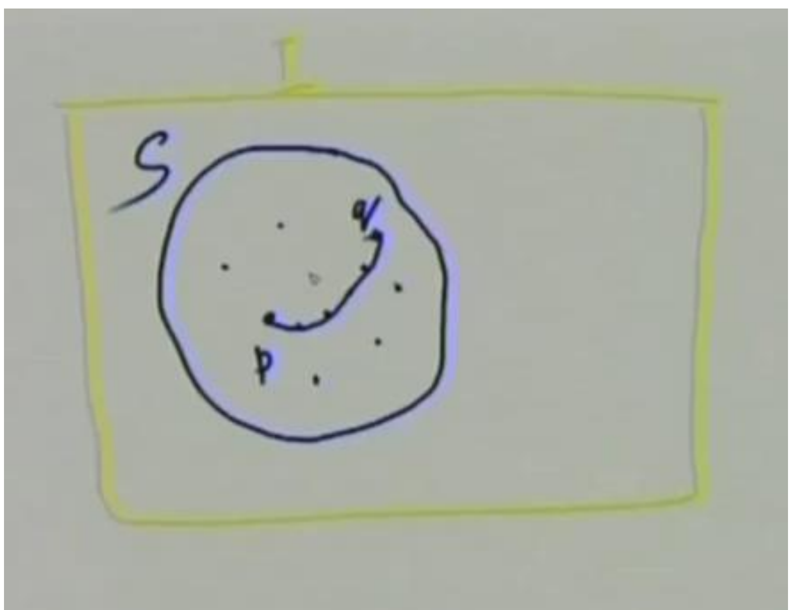
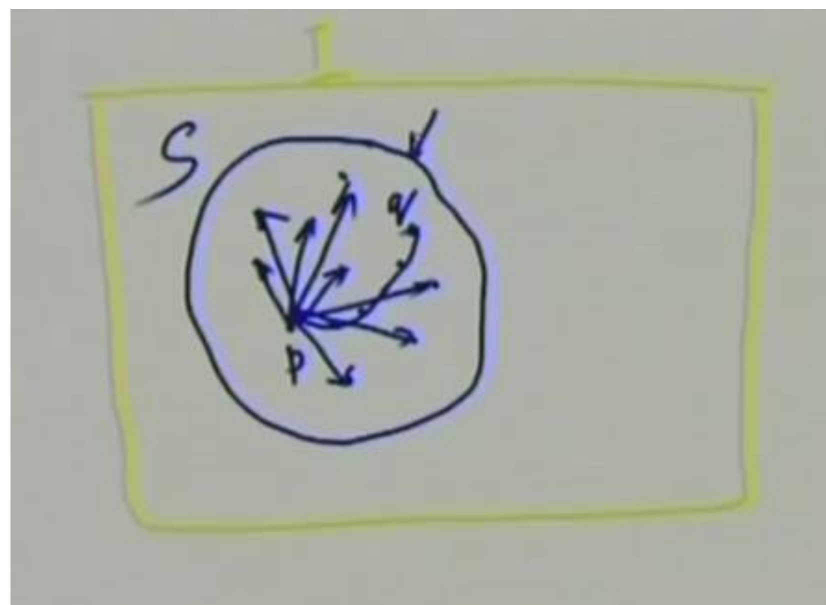
Let

$S \subseteq I$ and $p,q \in S$

Then p is connected to q in S if there is a path From p to q consisting entirely of pixels in S

For any $p \in S$, the set of pixels in S that are Connected to p is call a connected component of S.

=> Any two pixels of a connected component are connected to each other


I
S
q
p

I
S
q
p

I
S
q
p

I
S
q
p

Regions and boundaries

- Let R be a subset of pixels in an image. We call R a region of the image if R is a connected set.

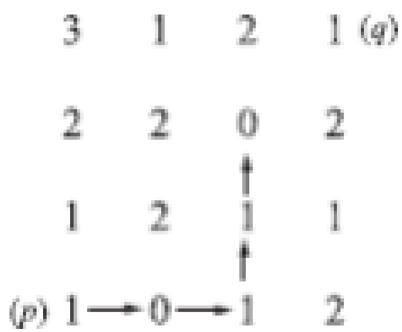
- The boundary (also called border or contour) of a region R is the set of pixels in the region that have one or more neighbours that are not in R

Consider the image segment shown.

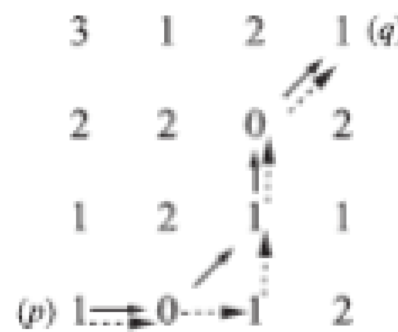
Let V={0, 1} and compute the lengths of the shortest 4-, 8-, and m-path between p and q. If a particular path does not exist between these two points, explain why.

	3	1	2	1 (q)
	2	2	0	2
	1	2	1	1
(p)	1	0	1	2

- When $V = \{0,1\}$, 4-path does not exist between p and q because it is impossible to get from p to q by traveling along points that are both 4-adjacent and also have values from V. Fig. *a* shows this condition; it is not possible to get to q.
- The shortest 8-path is shown in Fig. *b* its length is **4**.
- The length of the shortest m- path (shown dashed) is **5**.
- Both of these shortest paths are unique in this case.



(a)



(b)

Distance Measures

Distance Measures

Take three pixels

$P \approx (x,y)$ $q \approx (s,t)$ $z \approx (u,v)$

D is a distance function or metric if

$D(p,q) \geq 0$; $D(p,q) = 0$ iff $p = q$

$D(p,q) = D(q,p)$ → Symmetric

$D(p,z) \leq D(p,q) + D(q,z)$ → Inequality

- **Euclidean Distance:**

$$\boxed{D_e(p, q) = [(x-s)^2 + (y-t)^2]^{1/2}}$$

- **City Block Distance**

$$\boxed{D_4(p, q) = |x-s| + |y-t|}$$

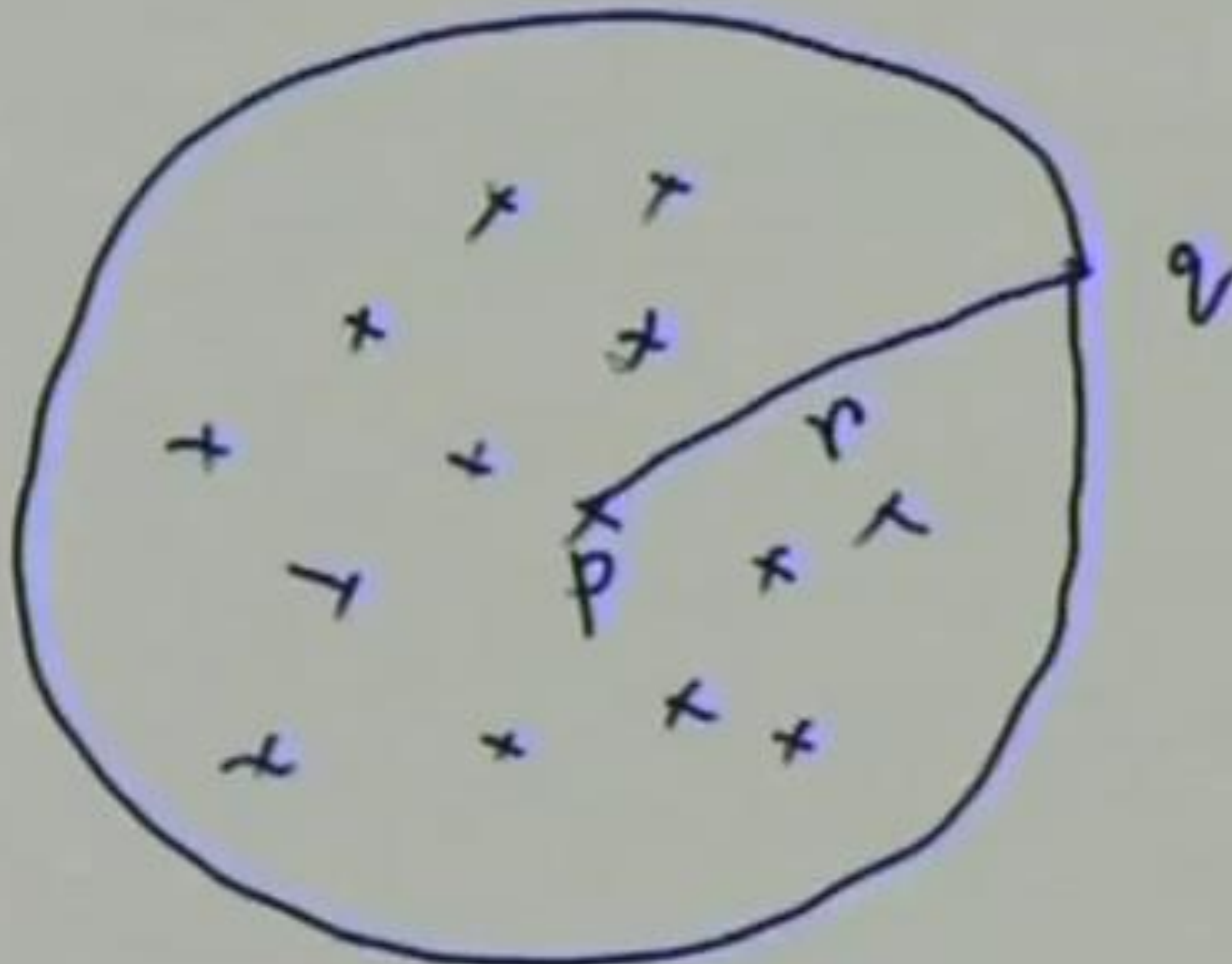
- **Chess Board Distance:**

$$\boxed{D_8(p, q) = \max(|x-s|, |y-t|)}$$

Euclidean Distance

$$D_e(p,q) = [(x-s)^2 + (y-t)^2]^{1/2}$$

Set of points $S = \{ q \mid D(p,q) \leq r \}$ are the points contained in a disk of radius r centered at p.

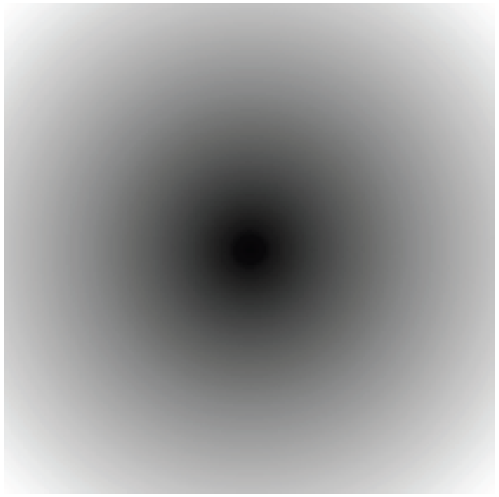


q
r
p

Distance Operations

2.8	2.2	2.0	2.2	2.8
2.2	1.4	1.0	1.4	2.2
2.0	1.0	0.0	1.0	2.0
2.2	1.4	1.0	1.4	2.2
2.8	2.2	2.0	2.2	2.8

Euclidean distances



City – Block Distance

D_4 distance or City-Block (Manhattan) Distance.

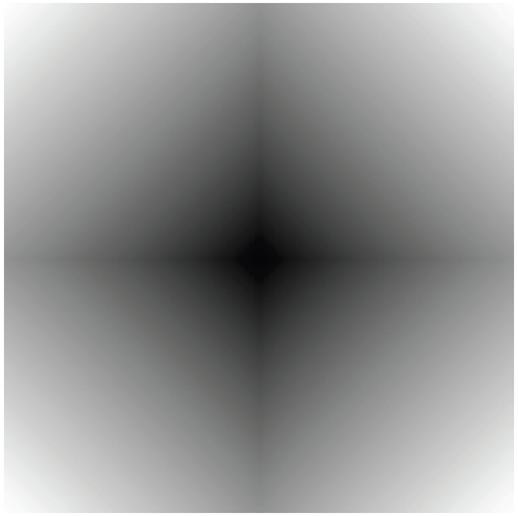
$$D_4(p,q) = | x-s | + | y-t |$$

Points having city block distance from p less than or equal to r from diamond centered at p.

Distance Operations

4	3	2	3	4
3	2	1	2	3
2	1	0	1	2
3	2	1	2	3
4	3	2	3	4

City-block distances
(1's are 4-neighbors)



Chess Board Distance

D_8 distance or chess board distance is defined as

$$D_8(p,q) = \max(|x-s|, |y-t|)$$

$S = \{ q \mid D_8(p,q) \leq r \}$ forms a square centered at p.

Points with $D_8 = 1$ are 8 neighbors of p

Distance Operations

2	2	2	2	2
2	1	1	1	2
2	1	0	1	2
2	1	1	1	2
2	2	2	2	2

Chessboard distances
(1's are 8-neighbors)

Problems

1. A)Let V={0,1}. Compute the D_e, D_4, D_8 distances between two pixels p and q. Let the pixel coordinates of p and q be (3,0) and (2,3) respectively, for the image shown below:

   B)compute the lengths of the shortest 4, 8, and m-path between p and q, in the image sample given below:

	0	1	2	3
0	0	1	1	1
1	1	0	0	1
2	1	1	1	1 q
3	1 p	1	1	1

De = $\sqrt{10}$

D4 = 1 + 3 = 4

D8 = max(1,3) = 3

Distance Operations

- Spatial relationships and distance calculations will be used in wide variety of image processing operations
  - Spatial filtering
  - Mathematical morphology
  - Image segmentation
  - Computer vision applications

  -- **Shape Matching**