Uncertainty and knowledge Reasoning

Unit5

Uncertainty

Let action A_t = leave for airport t minutes before flight
Will A_t get me there on time?
Problems:
1. Partial observability (road state, other drivers' plans, etc.)
2. Noisy sensors (traffic reports)
3. Uncertainty in action outcomes (flat tire, etc.)
4. Immense complexity of modeling and predicting traffic

Hence a purely logical approach either
1. Risks falsehood: "A_{25} will get me there on time", or
2. Leads to conclusions that are too weak for decision making:

"A_{25} will get me there on time if there's no accident on the bridge and it doesn't rain and my tires remain intact etc."
(A_{1440} might reasonably be said to get me there on time but I'd have to stay overnight in the airport …)
What is right thing to do? It should maximize the performance measure.

Uncertainty

- *The right thing to do—the **rational decision***
  - Depends on both the relative importance of various goals and
  - *The likelihood* that they will be achieved.
    - Degree to which they will be achieved

Uncertainty

- Agents may need to handle **uncertainty**
  - Due to partial observability,
  - Nondeterminism,
    - Or a combination of the two.
    - An agent may never know for certain the state it's in
    - Where it will end up after a sequence of actions.
- A **belief state**— Rational agent, Logical agent
  - A representation of the set of all possible world states that it might be in
  - Generating a contingency plan that handles
    - Every possible eventuality that its sensors may report during execution.

Uncertainty

- Interpret partial sensor information,
  - A logical agent must consider *every logically possible* explanation for the observations, no matter how unlikely.
  - It results in impossible large and complex belief-state representations.
- A correct contingent plan that handles every eventuality
  - Will grow arbitrarily large and must consider arbitrarily unlikely contingencies.
- Sometimes there is no plan that is guaranteed to achieve the goal
  - Yet the agent must act.
  - It must have some way to compare the merits of plans that are not guaranteed.

Uncertainty

- Toothache ⇒ Cavity .
  - The problem is that this rule is wrong. Not all patients with toothaches have cavities;
  - some of them have gum disease, an abscess, or one of several other problems:
- Toothache ⇒ Cavity ∨ GumProblem ∨ Abscess . . .
  - Unfortunately, in order to make the rule true, we have to add an almost unlimited list of possible problems.
- We could try turning the rule into a causal rule: Cavity ⇒ Toothache
  - But this rule is not right either; not all cavities cause pain.

Probability

Probabilistic assertions summarize effects of
- laziness: failure to enumerate exceptions, qualifications, etc.
- ignorance: lack of relevant facts, initial conditions, etc.

Subjective probability:
- Probabilities relate propositions to agent's own state of knowledge

  e.g., P(A_{25} | no reported accidents) = 0.06

These are not assertions about the world

Probabilities of propositions change with new evidence:

e.g., P(A_{25} | no reported accidents, 5 a.m.) = 0.15

Making decisions under uncertainty

Suppose I believe the following:

P(A_{25} gets me there on time | …) = 0.04
P(A_{90} gets me there on time | …) = 0.70
P(A_{120} gets me there on time | …) = 0.95
P(A_{1440} gets me there on time | …) = 0.9999

- Which action to choose?

  Depends on my preferences for missing flight vs. time spent waiting, etc.
  - Utility theory is used to represent and infer preferences
  - Decision theory = probability theory + utility theory

Methods for handling uncertainty

- Default or nonmonotonic logic:
  - Assume my car does not have a flat tire
  - Assume A_{25} works unless contradicted by evidence
- Issues: What assumptions are reasonable? How to handle contradiction?
- Rules with fudge factors:
  - A_{25} $\mid\rightarrow_{0.3}$ get there on time
  - *Sprinkler* $\mid\rightarrow_{0.99}$ *WetGrass*
  - *WetGrass* $\mid\rightarrow_{0.7}$ *Rain*
- Issues: Problems with combination, e.g., *Sprinkler* causes *Rain*??
- Probability
  - Model agent's degree of belief
  - Given the available evidence,
  - A_{25} will get me there on time with probability 0.04

Syntax

- Basic element: random variable
- Similar to propositional logic: possible worlds defined by assignment of values to random variables.
- Boolean random variables

  e.g., *Cavity* (do I have a cavity?)
- Discrete random variables

  e.g., *Weather* is one of <*sunny,rainy,cloudy,snow*>

Syntax

- Atomic event: A complete specification of the state of the world about which the agent is uncertain

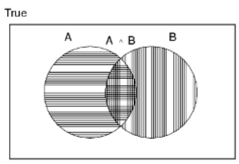
  E.g., if the world consists of only two Boolean variables *Cavity* and *Toothache*, then there are 4 distinct atomic events:

  Cavity = false $\wedge$ *Toothache = false*
  Cavity = false $\wedge$ *Toothache = true*
  Cavity = true $\wedge$ *Toothache = false*
  Cavity = true $\wedge$ *Toothache = true*
- Atomic events are mutually exclusive and exhaustive

Syntax

- Elementary proposition constructed by
  - Assignment of a value to a random variable: e.g., *Weather = sunny*, *Cavity = false*
  - abbreviated as $\neg$*cavity*
- Complex propositions formed from
  - elementary propositions and standard logical connectives
  - e.g. (*Weather = sunny*) $\vee$ (*Cavity = false*)
- Domain values must be exhaustive and mutually exclusive

Axioms of probability

- For any propositions *A, B*
  - $0 \leq P(A) \leq 1$
  - P(*true*) = 1 and P(*false*) = 0
  - $P(A \vee B) = P(A) + P(B) - P(A \wedge B)$
  - 

Prior probability

- Prior or unconditional probabilities of propositions
- e.g., P(*Cavity* = true) = 0.1 and P(*Weather* = sunny) = 0.72 correspond to belief prior to arrival of any (new) evidence
- Probability distribution gives values for all possible assignments:
  - **P**(*Weather*) = <0.72,0.1,0.08,0.1> (normalized, i.e., sums to 1)
    - ✓ *Weather* is one of *<sunny,rainy,cloudy,snow>*

Prior probability

- Joint probability distribution for a set of random variables gives the probability of every atomic event on those random variables
- **P**(*Weather,Cavity*) = a 4 × 2 matrix of values:

Weather =	sunny	rainy	cloudy	snow
Cavity = true	0.144	0.02	0.016	0.02
Cavity = false	0.576	0.08	0.064	0.08

- Every question about a domain can be answered by the joint distribution

Conditional probability

- Conditional or posterior probabilities
  e.g., P(*cavity* | *toothache*) = 0.8
  i.e., given that *toothache* is all I know
- Notation for conditional distributions:
  - **P**(*Cavity* | *Toothache*) = 2-element
    - ✓(vector of 2-element vectors)
- If we know more, e.g., *cavity* is also given, then we have
  - P(*cavity* | *toothache* , *cavity*) = 1

Conditional probability

- New evidence may be irrelevant, allowing simplification, e.g.,
  - P(*cavity* | *toothache, sunny*) = P(*cavity* | *toothache*) = 0.8
- This kind of inference is crucial
  - Sanctioned by domain knowledge

Conditional probability

- Definition of conditional probability:
  - $P(a \mid b) = P(a \wedge b) / P(b)$ if $P(b) > 0$
- Product rule gives an alternative formulation:
  - $P(a \wedge b) = P(a \mid b) P(b) = P(b \mid a) P(a)$
- A general version holds for whole distributions
  P(*Weather,Cavity*) = **P**(*Weather* | *Cavity*) **P**(*Cavity*)

Conditional probability

- Chain rule is derived by successive application of product rule:

$\mathbf{P}(X_1, \ldots, X_n) = \mathbf{P}(X_n \mid X_1, \ldots, X_{n-1})\ \mathbf{P}(X_1, \ldots, X_{n-1})$
$= \mathbf{P}(X_n \mid X_1, \ldots, X_{n-1})\ \mathbf{P}(X_{n-1} \mid X_1, \ldots, X_{n-2})\ \mathbf{P}(X_1, \ldots, X_{n-2})$

$= \pi_{i=1}{}^\wedge n\ \mathbf{P}(X_i \mid X_1, \ldots, X_{i-1})$

Inference by enumeration

- Cavity, Toothache, Catch the aching tooth.
- Start with the joint probability distribution:

	toothache		*¬ toothache*	
	catch	*¬ catch*	*catch*	*¬ catch*
cavity	.108	.012	.072	.008
¬ cavity	.016	.064	.144	.576

- For any proposition ϕ, sum the atomic events where it is true: $P(\phi) = \Sigma_{\omega:\omega \models \phi} P(\omega)$

Inference by enumeration

	toothache		*¬ toothache*	
	catch	*¬ catch*	*catch*	*¬ catch*
cavity	.108	.012	.072	.008
¬ cavity	.016	.064	.144	.576

- For any proposition ϕ, P(*toothache*)
  - sum the atomic events where it is true:
    - ✓ $P(\phi) = \Sigma_{\omega:\omega \models \phi} P(\omega)$
- P(*toothache*) = 0.108 + 0.012 + 0.016 + 0.064 = 0.2
- P(¬*cavity* | *toothache*) ?

Inference by enumeration

- Start with the joint probability distribution:
-
- For any proposition ϕ, sum the atomic events where it is true: $P(\phi) = \Sigma_{\omega:\omega \models \phi} P(\omega)$
- P(*toothache*) =
  - 0.108 + 0.012 + 0.016 + 0.064 = 0.2
- P(¬*cavity* | *toothache*) ?

Inference by enumeration

	toothache		*¬ toothache*	
	catch	*¬ catch*	*catch*	*¬ catch*
cavity	.108	.012	.072	.008
¬ cavity	.016	.064	.144	.576

- Can also compute conditional probabilities:

P(¬*cavity* | *toothache*) $= \dfrac{P(\neg cavity \wedge toothache)}{P(toothache)}$

$= \dfrac{0.016+0.064}{0.108 + 0.012 + 0.016 + 0.064}$

= 0.4

P(*cavity* | *toothache*) = 0.6

= α * P(¬*cavity* ∧ *toothache*)

Normalization

	toothache		*¬ toothache*	
	catch	*¬ catch*	*catch*	*¬ catch*
cavity	.108	.012	.072	.008
¬ cavity	.016	.064	.144	.576

P(¬*cavity* | *toothache*) = 0.6
Normalization = <0.6,0.4>

General idea:
compute distribution on query variable by fixing evidence variables and summing over hidden variables
Hidden Variable = Catch
Query Variable = Cavity
Evidence Variable = Toothache

Normalization

	toothache		¬ toothache	
	catch	¬ catch	catch	¬ catch
cavity	.108	.012	.072	.008
¬ cavity	.016	.064	.144	.576

- Denominator can be viewed as a normalization constant α

P(*Cavity* | *toothache*) = α* **P**(*Cavity,toothache*)

= α*[**P**(*Cavity,toothache,catch*) + **P**(*Cavity,toothache,¬ catch*)]

= α * [<0.108> + <0.012>] = α, [<.12>]

P(¬ *Cavity* | *toothache*) = α* **P**(¬ *Cavity,toothache*)

= α*[**P**(¬ *Cavity,toothache,catch*) + **P**(¬ *Cavity,toothache,¬ catch*)]

= α * [<0.016> + <0.064>] = α, [<0.08>]

= α * <0.12,0.08>

Normaliztion = <0.6,0.4>

Bayes' Rule

- Product rule P(a∧b) = P(a | b) P(b) = P(b | a) P(a)
-
  ⇒ Bayes' rule: P(a | b) = P(b | a) P(a) / P(b)

  P (cause | *effect*) = P (*effect* | cause)P (cause)/P (*effect*)
- We perceive as evidence
  - the *effect* of some unknown *cause*
  - We would like to determine that cause.

Bayes' Rule

- Useful for assessing diagnostic probability from causal probability:
- The doctor knows
  - P (symptoms | disease)) and
    - ✓ meningitis causes the patient to have a stiff neck, say, 70% of the time.
    - ✓ P(Effect|Cause)
  - want to derive a diagnosis,
    - ✓ P (disease | symptoms)
    - ✓ Given stiff neck, what may be probability of meningitis
    - ✓ P(Cause|Effect)