

Artificial Intelligence

Module 4 Reasoning

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- Propositional Logic
- Reasoning Using First Order Logic
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- Unification
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Propositional Logic

- A simple logic
- Boolean Logic
 - Represent any **statement**, or **proposition**, that can take on a truth value
 - Truth value - True – true proposition
 - Truth value - False – false proposition
- Propositional symbol – The Atomic sentence
 - The indivisible syntactic elements
 - $P_{1,3}$ means Player in [1,3]. It is atomic and cannot be divided in meaningful part as P, 1, 3

		P _{1,3}	

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Propositional Logic

- The **syntax** of propositional logic defines the sentences that are permitted
- The **atomic sentences** consist of a single **proposition symbol**.
 - Each such symbol stands for a proposition that can be True proposition or False proposition.
 - $P, Q, R, W_{1,3}$ and North
- **Complex sentences** are constructed
 - From simpler sentences,
 - Using parentheses and
 - **Logical connectives**.

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Connectives

- \neg (not) \rightarrow A sentence such as $\neg W_{1,3}$ is called the **negation** of $W_{1,3}$
- \wedge (and) $\rightarrow W_{1,3} \wedge P_{3,1}$, is called a **conjunction**,
 - Parts are **conjuncts**
- \vee (or) $\rightarrow (W_{1,3} \wedge P_{3,1}) \vee W_{2,2}$ is **disjunction**
 - Parts are **disjuncts**
- \Rightarrow (implies). $(W_{1,3} \wedge P_{3,1}) \Rightarrow \neg W_{2,2}$ is called an **implication**
 - Its **premise** or **antecedent** is $(W_{1,3} \wedge P_{3,1})$
 - **conclusion** or **consequent** is $\neg W_{2,2}$
- \Leftrightarrow **biconditional** The sentence $W_{1,3} \Leftrightarrow \neg W_{2,2}$
 - if and only if \equiv

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Semantics of Propositional logic

- $\neg P$ is true iff P is false in m.
- $P \wedge Q$ is true iff both P and Q are true in m.
- $P \vee Q$ is true iff either P or Q is true in m.
- $P \Rightarrow Q$ is true except when **P is true and Q is false** in m.
- $P \Leftrightarrow Q$ is true iff P and Q are **both true or both are false** in m.

P	Q	$\neg P$	$P \wedge Q$	$P \vee Q$	$P \Rightarrow Q$	$P \Leftrightarrow Q$
false	false	true	false	false	true	true
false	true	true	false	true	true	false
true	false	false	false	true	false	false
true	true	false	true	true	true	true

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Propositional Logic

- It is raining RAINING
- It is Sunny SUNNY
- It is Windy WINDY
- If it is raining it is windy
RAINING \rightarrow WINDY
- If it is raining it is not sunny
RAINING $\rightarrow \neg$ SUNNY

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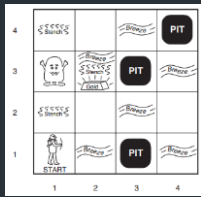
Wumpus World

Entailment \Rightarrow Logical inference
The relation between a sentence and another sentence that follow from it

- The semantics for propositional logic must
 - Specify how to compute the truth value of *any* sentence, given a model.
 - All sentences are constructed from atomic sentences and the five connectives
 - This is done recursively.
 - How to compute the truth of sentences formed with each of the five connectives?

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Wumpus World

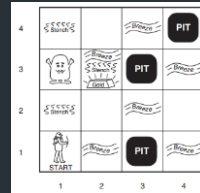


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- A cave consisting of rooms connected by passageways.
- Somewhere in the cave is
 - The terrible wumpus, a beast that eats anyone who enters its room
- The wumpus can be shot by an agent,
 - But the agent has only one arrow
- Possibility of finding
 - A heap of gold

Wumpus World



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- Some rooms contain bottomless pits
 - Can trap anyone who wanders into these rooms
 - Except for the wumpus, which is too big to fall in
- Breeze in a room indicates
 - PIT in adjacent room

Wumpus World

- Environment
 - A 4×4 grid of rooms. The agent always starts in the square labeled [1,1], facing to the right.
 - Random, Uniform distribution of room for Wumpus and Gold
 - Each square other than the start can be a pit, with probability 0.2

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- Performance measure:
 - +1000 for climbing out of the cave with the gold,
 - 1000 for falling into a pit or eaten by wumpus,
 - 1 for each action taken and
 - 10 for using up the arrow.
 - The game ends either when the agent dies or when the agent climbs out of the cave.

Wumpus World

- Actuators: The agent can move
 - Forward, TurnLeft by 90°, or TurnRight by 90°
 - A miserable death if it enters a square containing a pit or a live wumpus
 - It is safe, albeit smelly, to enter a square with a dead wumpus.)
 - If an agent tries to move forward and bumps into a wall, then the agent does not move
 - Grab gold
 - Shoot
 - Climb

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Wumpus World

- The agent has five sensors, each of which gives a single bit of information
 - In the square containing the wumpus and in the directly adjacent squares – not diagonal
 - The agent will perceive a *Stench*.
 - In the squares directly adjacent to a pit,
 - The agent will perceive a *Breeze*.
 - In the square where the gold is,
 - The agent will perceive a *Glitter*.
- [Stench, Breeze, None, None, None]

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Wumpus World

- Sensors
 - When an agent walks into a wall,
 - It will perceive a *Bump*.
 - When the wumpus is killed,
 - It emits a woeful *Scream* that can be perceived anywhere in the cave
- Wumpus environment
 - Discrete, Static, and Single-agent, Sequential,
 - Partially observable (Arrow, health, location)

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Wumpus World

- For an agent the main challenge is
 - Its initial ignorance of the configuration of the environment
 - Described by rules of environment
 - Overcoming this ignorance require logical reasoning.
- The agent must choose between
 - Going home empty-handed or
 - Risking death to find the gold.

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Wumpus World

Starts with [1,1], [1,1] is a safe square:

Sensors [None, None, None, None, None].

Sensors [None, Breeze, None, None, None].

1,4	2,4	3,4	4,4
1,3	2,3	3,3	4,3
1,2	2,2	3,2	4,2
OK			
1,1	2,1	3,1	4,1
OK	OK		

A = Agent
 B = Breeze
 G = Glitter, Gold
 OK = Safe square
 P = Pit
 S = Stench
 V = Visited
 W = Wumpus

1,4	2,4	3,4	4,4
1,3	2,3	3,3	4,3
1,2	2,2	3,2	4,2
OK			
1,1	2,1	3,1	4,1
V	OK	B	pt

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Wumpus World

A cautious agent will move only into a square that it knows to be OK

[Stench, None, None, None, None]

[Stench, Breeze, Glitter, None, None]

1,4	2,4	3,4	4,4
1,3	W	2,3	3,3
1,2	S	2,2	3,2
OK	OK	OK	4,2
1,1	V	2,1	3,1
OK	OK	OK	pt

A = Agent
 B = Breeze
 G = Glitter, Gold
 OK = Safe square
 P = Pit
 S = Stench
 V = Visited
 W = Wumpus

1,4	2,4	3,4	4,4
1,3	W	2,3	3,3
1,2	S	2,2	3,2
OK	OK	OK	4,2
1,1	V	2,1	3,1
OK	OK	OK	pt

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Logical reasoning and logical agent

- A fundamental property of logical reasoning.
 - In each case for which the agent draws a conclusion from the available information,
 - Conclusion is *guaranteed* to be correct if the available information is correct.
- How to build logical agents that
 - Can represent information and
 - Draw conclusions

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Logic to wumpus world

- Logical entailment
 - $\alpha \models \beta$ means sentence α entails the sentence β .
 - if and only if, in every model in which α is true, β is also true
 - α is a **stronger** assertion than β
 - β may be true at more places, α is false
 - Sentence $x = 0$ entails the sentence $xy = 0$.

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Logic to wumpus world

- Apply this to Wumpus world
 - The agent has detected nothing in [1,1] and a breeze in [2,1].
 - These percepts, combined with the agent's knowledge of the rules of the wumpus world, constitute the KB.
 - The agent is interested in knowing
 - Whether the adjacent squares [1,2], [2,2], and [3,1] contain pits.

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80	81	82	83	84	85	86	87	88	89	90	91	92	93	94	95	96	97	98	99	100	101	102	103	104	105	106	107	108	109	110	111	112	113	114	115	116	117	118	119	120	121	122	123	124	125	126	127	128	129	130	131	132	133	134	135	136	137	138	139	140	141	142	143	144	145	146	147	148	149	150	151	152	153	154	155	156	157	158	159	160	161	162	163	164	165	166	167	168	169	170	171	172	173	174	175	176	177	178	179	180	181	182	183	184	185	186	187	188	189	190	191	192	193	194	195	196	197	198	199	200	201	202	203	204	205	206	207	208	209	210	211	212	213	214	215	216	217	218	219	220	221	222	223	224	225	226	227	228	229	230	231	232	233	234	235	236	237	238	239	240	241	242	243	244	245	246	247	248	249	250	251	252	253	254	255	256	257	258	259	260	261	262	263	264	265	266	267	268	269	270	271	272	273	274	275	276	277	278	279	280	281	282	283	284	285	286	287	288	289	290	291	292	293	294	295	296	297	298	299	300	301	302	303	304	305	306	307	308	309	310	311	312	313	314	315	316	317	318	319	320	321	322	323	324	325	326	327	328	329	330	331	332	333	334	335	336	337	338	339	340	341	342	343	344	345	346	347	348	349	350	351	352	353	354	355	356	357	358	359	360	361	362	363	364	365	366	367	368	369	370	371	372	373	374	375	376	377	378	379	380	381	382	383	384	385	386	387	388	389	390	391	392	393	394	395	396	397	398	399	400	401	402	403	404	405	406	407	408	409	410	411	412	413	414	415	416	417	418	419	420	421	422	423	424	425	426	427	428	429	430	431	432	433	434	435	436	437	438	439	440	441	442	443	444	445	446	447	448	449	450	451	452	453	454	455	456	457	458	459	460	461	462	463	464	465	466	467	468	469	470	471	472	473	474	475	476	477	478	479	480	481	482	483	484	485	486	487	488	489	490	491	492	493	494	495	496	497	498	499	500	501	502	503	504	505	506	507	508	509	510	511	512	513	514	515	516	517	518	519	520	521	522	523	524	525	526	527	528	529	530	531	532	533	534	535	536	537	538	539	540	541	542	543	544	545	546	547	548	549	550	551	552	553	554	555	556	557	558	559	560	561	562	563	564	565	566	567	568	569	570	571	572	573	574	575	576	577	578	579	580	581	582	583	584	585	586	587	588	589	590	591	592	593	594	595	596	597	598	599	600	601	602	603	604	605	606	607	608	609	610	611	612	613	614	615	616	617	618	619	620	621	622	623	624	625	626	627	628	629	630	631	632	633	634	635	636	637	638	639	640	641	642	643	644	645	646	647	648	649	650	651	652	653	654	655	656	657	658	659	660	661	662	663	664	665	666	667	668	669	670	671	672	673	674	675	676	677	678	679	680	681	682	683	684	685	686	687	688	689	690	691	692	693	694	695	696	697	698	699	700	701	702	703	704	705	706	707	708	709	710	711	712	713	714	715	716	717	718	719	720	721	722	723	724	725	726	727	728	729	730	731	732	733	734	735	736	737	738	739	740	741	742	743	744	745	746	747	748	749	750	751	752	753	754	755	756	757	758	759	760	761	762	763	764	765	766	767	768	769	770	771	772	773	774	775	776	777	778	779	780	781	782	783	784	785	786	787	788	789	790	791	792	793	794	795	796	797	798	799	800	801	802	803	804	805	806	807	808	809	810	811	812	813	814	815	816	817	818	819	820	821	822	823	824	825	826	827	828	829	830	831	832	833	834	835	836	837	838	839	840	841	842	843	844	845	846	847	848	849	850	851	852	853	854	855	856	857	858	859	860	861	862	863	864	865	866	867	868	869	870	871	872	873	874	875	876	877	878	879	880	881	882	883	884	885	886	887	888	889	890	891	892	893	894	895	896	897	898	899	900	901	902	903	904	905	906	907	908	909	910	911	912	913	914	915	916	917	918	919	920	921	922	923	924	925	926	927	928	929	930	931	932	933	934	935	936	937	938	939	940	941	942	943	944	945	946	947	948	949	950	951	952	953	954	955	956	957	958	959	960	961	962	963	964	965	966	967	968	969	970	971	972	973	974	975	976	977	978	979	980	981	982	983	984	985	986	987	988	989	990	991	992	993	994	995	996	997	998	999	1000	1001	1002	1003	1004	1005	1006	1007	1008	1009	1010	1011	1012	1013	1014	1015	1016	1017	1018	1019	1020	1021	1022	1023	1024	1025	1026	1027	1028	1029	1030	1031	1032	1033	1034	1035	1036	1037	1038	1039	1040	1041	1042	1043	1044	1045	1046	1047	1048	1049	1050	1051	1052	1053	1054	1055	1056	1057	1058	1059	1060	1061	1062	1063	1064	1065	1066	1067	1068	1069	1070	1071	1072	1073	1074	1075	1076	1077	1078	1079	1080	1081	1082	1083	1084	1085	1086	1087	1088	1089	1090	1091	1092	1093	1094	1095	1096	1097	1098	1099	1100	1101	1102	1103	1104	1105	1106	1107	1108	1109	1110	1111	1112	1113	1114	1115	1116	1117	1118	1119	1120	1121	1122	1123	1124	1125	1126	1127	1128	1129	1130	1131	1132	1133	1134	1135	1136	1137	1138	1139	1140	1141	1142	1143	1144	1145	1146	1147	1148	1149	1150	1151	1152	1153	1154	1155	1156	1157	1158	1159	1160	1161	1162	1163	1164	1165	1166	1167	1168	1169	1170	1171	1172	1173	1174	1175	1176	1177	1178	1179	1180	1181	1182	1183	1184	1185	1186	1187	1188	1189	1190	1191	1192	1193	1194	1195	1196	1197	1198	1199	1200	1201	1202	1203	1204	1205	1206	1207	1208	1209	1210	1211	1212	1213	1214	1215	1216	1217	1218	1219	1220	1221	12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Wumpus world in Propositional logic

- There is no pit in [1,1]:
 - R1 : $\neg P_{1,1}$
 - R4 : $\neg B_{1,1}$
 - R5 : $B_{2,1}$
- A square is breezy if and only if there is a pit in a neighboring square.
 - R2 : $B_{1,1} \Leftrightarrow (P_{1,2} \vee P_{2,1})$
 - R3 : $B_{2,1} \Leftrightarrow (P_{1,1} \vee P_{2,2} \vee P_{3,1})$

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Pros and cons of propositional logic

- Propositional logic is **declarative**
- Propositional logic allows partial/disjunctive/negated information
 - Unlike most data structures and databases
- Propositional logic is **compositional**:
 - Meaning of $B_{1,1} \wedge P_{1,2}$ is derived from meaning of $B_{1,1}$ and of $P_{1,2}$

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Pros and cons of propositional logic

- Propositional logic has very limited expressive power
 - $B_{1,1} \Leftrightarrow (P_{1,2} \vee P_{2,1})$
 - Unlike natural Language
 - E.g., cannot say "pits cause breezes in adjacent squares" except by writing one sentence for each square

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Standard logical equivalences

$(\alpha \wedge \beta) \equiv (\beta \wedge \alpha)$	commutativity of \wedge
$(\alpha \vee \beta) \equiv (\beta \vee \alpha)$	commutativity of \vee
$((\alpha \wedge \beta) \wedge \gamma) \equiv (\alpha \wedge (\beta \wedge \gamma))$	associativity of \wedge
$((\alpha \vee \beta) \vee \gamma) \equiv (\alpha \vee (\beta \vee \gamma))$	associativity of \vee
$\neg(\neg\alpha) \equiv \alpha$	double-negation elimination
$(\alpha \Rightarrow \beta) \equiv (\neg\beta \Rightarrow \neg\alpha)$	contraposition
$(\alpha \Rightarrow \beta) \equiv (\neg\alpha \vee \beta)$	implication elimination
$(\alpha \Leftrightarrow \beta) \equiv ((\alpha \Rightarrow \beta) \wedge (\beta \Rightarrow \alpha))$	biconditional elimination
$\neg(\alpha \wedge \beta) \equiv (\neg\alpha \vee \neg\beta)$	De Morgan
$\neg(\alpha \vee \beta) \equiv (\neg\alpha \wedge \neg\beta)$	De Morgan
$(\alpha \wedge (\beta \vee \gamma)) \equiv ((\alpha \wedge \beta) \vee (\alpha \wedge \gamma))$	distributivity of \wedge over \vee
$(\alpha \vee (\beta \wedge \gamma)) \equiv ((\alpha \vee \beta) \wedge (\alpha \vee \gamma))$	distributivity of \vee over \wedge

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First-order logic

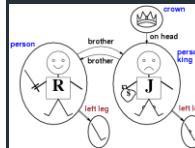
- Whereas propositional logic assumes the world contains **facts**
- First-order logic (like natural language) assumes the world contains
 - Objects**: people, houses, numbers, colors, baseball games, wars, ...
 - Relations**: red, round, prime, brother of, bigger than, part of, comes between, ...
 - Functions**: father of, best friend, one more than, plus, ...

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First-order logic

- A relation
 - The set of **tuples** of objects that are related.
 - A tuple is a collection of objects
 - Arranged in a fixed order
 - Written with angle brackets surrounding the objects
 - {<Richard the Lionheart, King John>, <King John, Richard the Lionheart>}
 - Picture may be used in place of name

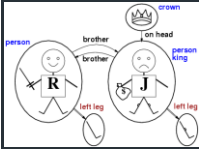


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First-order logic

- A function
 - A object is related to **one** object in function way
 - Left leg of, Mother of,
- Total function in FOL
 - Value for each tuple
 - Each object



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Syntax of FOL: First-order logic

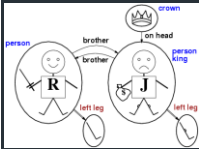
- Symbols – Begin with upper case latter
 - Constant Symbols
 - Objects
 - Richard, John, 2, VIT
 - Predicate symbols
 - Relation
 - Brother, OnHead, Person, >, King,
 - Function Symbols
 - LeftLegOf, SqrtOf

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Syntax of FOL: First-order logic

- Interpretation (Intended Interpretation)
 - Constant, Predicate, and Function symbols refer to
 - Specific objects, relations and functions
 - Richard refers to Richard the Lionheart
 - John refers to the evil King John
 - Brother refers to the brotherhood relation
 - OnHead refers to the "on head" relation
 - Between Crown and John
 - Person, King, and Crown refer to Set of objects
 - Person, King, and Crown

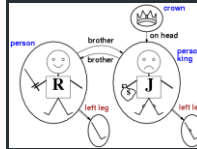


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Syntax of FOL: First-order logic

- Interpretation also mean
 - Richard to Crown
 - John to John's left leg
 - All possible combination of Symbol and objects
 - All objects may not have name
 - Some objects are without name, may have relation, function
 - crown, leg
 - An objects may have multiple name
 - Richard, John → crown



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Syntax of FOL: First-order logic

- Variables x, y, a, b, \dots
 - Small letters
- Connectives $\neg, \Rightarrow, \wedge, \vee, \Leftrightarrow$
- Equality $=$
- Quantifiers \forall, \exists
- Predicate and Function symbol have an **arity**
 - Fixes the number of arguments

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Term and Atomic sentences

- Term is logical expression that refers to a object
 - Constant Symbol is a term
 - Not all objects need have distinct symbols
 - Leftleg (John)
 - Left leg of King John is an object
- Atomic sentence = *predicate* ($term_1, \dots, term_n$) or $term_1 = term_2$
 - E.g., $Brother(KingJohn, RichardTheLionheart)$
 - $> (Length(LeftLegOf(Richard)), Length(LeftLegOf(KingJohn)))$
 - $Married(Father(Richard), Mother(John))$

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Complex sentences

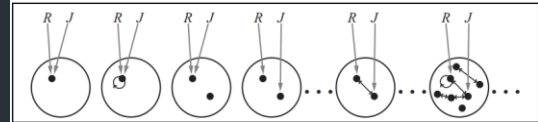
- $\neg S_1$
- $S_1 \wedge S_2$
- $S_1 \vee S_2$
- $S_1 \Rightarrow S_2$
- $S_1 \Leftrightarrow S_2$
- Complex sentences are made from atomic sentences using connectives
 - $Sibling(KingJohn, Richard) \Rightarrow Sibling(Richard, KingJohn)$
 - $>(1,2) \vee \leq(1,2)$
 - $>(1,2) \wedge \neg >(1,2)$

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First-order logic

- Multiple Models
 - Relation with arrow referring to objects and Symbols
 - Same object two names(1)
 - One object with some relation – Two names(2)
- One object with no names(3)
- Separate objects name, with relation(4)(5)
- Many combinations(6)



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Truth in first-order logic

- An atomic sentence $predicate(term_1, \dots, term_n)$ is true
 - iff the objects referred to by $term_1, \dots, term_n$ are in the relation referred to by predicate
- Model contains objects (domain elements) and relations among them
- Sentences are true with respect to a model and an interpretation
- Interpretation specifies referents for constant symbols \rightarrow objects
 - predicate symbols \rightarrow relations
 - function symbols \rightarrow functional relations

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Universal quantification

- $\forall \langle \text{variables} \rangle \langle \text{sentence} \rangle$
- Everyone at VIT is smart:
 - $\forall x \text{ At}(x, \text{VIT}) \Rightarrow \text{Smart}(x)$
- $\forall x P$ is true in a model m
 - iff P is true with x being each possible object in the model
- Roughly speaking, equivalent to the conjunction of instantiations of P
 - $\text{At}(\text{KingJohn}, \text{VIT}) \Rightarrow \text{Smart}(\text{KingJohn})$
 $\wedge \text{At}(\text{Richard}, \text{VIT}) \Rightarrow \text{Smart}(\text{Richard})$
 $\wedge \text{At}(\text{VIT}, \text{VIT}) \Rightarrow \text{Smart}(\text{VIT})$
 $\wedge \dots$
 - [one sentence equal to many of propositional logic and relation to all the objects]

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Universal quantification

- All romans were either loyal to Caesar or hated him
- $\forall x: \text{Roman}(x) \rightarrow \text{loyalto}(x, \text{Caesar}) \vee \text{hate}(x, \text{Caesar})$
- $\forall x P$ is true, $P = \text{King}(x) \wedge \text{Person}(x)$
 - Logical expression P is true for each object in the model
 - P is true for all possible extended interpretation from m
 - x refers to each domain element in each extended interpretation
 - The universally quantified sentence,
 - Equivalent to asserting a whole list of individual implications

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A common mistake to avoid

- Typically, \Rightarrow is the main connective with \forall
- Common mistake: using \wedge as the main connective with \forall
 - $\forall x \text{ At}(x, \text{VIT}) \wedge \text{Smart}(x)$
 - means "Everyone is at VIT and everyone is smart"
 - Do not use \wedge with $\forall x$
 - for " \Rightarrow " the implication is true whenever its premise is false.

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Existential quantification

- \exists <variables> <sentence>
- Someone at VIT is smart:
- $\exists x \text{ At}(x, \text{VIT}) \wedge \text{Smart}(x)$
- $\exists x P$ is true in a model m iff P is true with x being some possible object in the model
- Equivalent to the **disjunction of instantiations of P**
 - $\text{At}(\text{KingJohn}, \text{VIT}) \wedge \text{Smart}(\text{KingJohn})$
 - $\vee \text{At}(\text{Richard}, \text{VIT}) \wedge \text{Smart}(\text{Richard})$
 - $\vee \text{At}(\text{VIT}, \text{VIT}) \wedge \text{Smart}(\text{VIT})$
 - $\vee \dots$
- one sentence equal to many of propositional logic with OR relation to all the objects

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Existential quantification

- $\exists x P, P = \text{Crown}(x) \wedge \text{OnHead}(x, \text{John})$ The crown is a crown \wedge the crown is on John's head

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A common mistake to avoid

Summary of use of qualifiers
 \Rightarrow and $\forall x$
 \wedge and $\exists x$

- Typically, \wedge is the main connective with \forall
- Common mistake: using \Rightarrow as the main connective with \exists :
 $\exists x \text{ At}(x, \text{VIT}) \Rightarrow \text{Smart}(x)$
 - One additional meaning,
 - Smart for object x is true even if anyone who is not at VIT!
 - Even if $\text{At}(x, \text{VIT})$ is false, $\text{Smart}(x)$ is true.

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Properties of quantifiers

- $\forall x \forall y$ is the same as $\forall y \forall x$
- $\exists x \exists y$ is the same as $\exists y \exists x$
- $\exists x \forall y$ is **not** the same as $\forall y \exists x$
- $\exists x \forall y \text{ Loves}(x, y)$
 - "There is a person who loves everyone in the world"
- $\forall y \exists x \text{ Loves}(x, y)$
 - "Everyone in the world is loved by at least one person"

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Properties of quantifiers

- Some one like Broccoli
- $\exists x \text{ Likes}(x, \text{Broccoli})$
- There is no one who does not like Broccoli
- $\neg \forall x \neg \text{Likes}(x, \text{Broccoli})$
- **Quantifier duality:**
 - Each of \forall, \exists can be expressed using the other qualifier
- Every one like icecream Vs.
 $\forall x \text{ Likes}(x, \text{IceCream})$
- There is no one who does not like ice cream
 $\neg \exists x \neg \text{Likes}(x, \text{IceCream})$

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Properties of quantifiers

- Every one does not likes (dislikes) FH vs.
 - There does not exist any one who likes FH
- $\forall x \neg \text{Likes}(x, \text{FH}) \equiv \neg \exists x \text{ Likes}(x, \text{FH})$
- $\forall x \neg P \equiv \neg \exists x P$ $\neg(P \wedge Q) \equiv \neg P \vee \neg Q$
- $\neg \forall x P \equiv \exists x \neg P$ $\neg(P \vee Q) \equiv \neg P \wedge \neg Q$
- $\forall x P \equiv \neg \exists x \neg P$ $P \wedge Q \equiv \neg(\neg P \vee \neg Q)$
- $\exists x P \equiv \neg \forall x \neg P$ $P \vee Q \equiv \neg(\neg P \wedge \neg Q)$

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Using FOL

Using FOL to build the knowledge base

- **Assertion**
 - Tell the KB \rightarrow Add sentences to KB
 - $\text{TELL}(\text{KB}, \text{King}(\text{John}))$
 - $\text{TELL}(\text{KB}, \text{Person}(\text{Richard}))$
 - $\text{TELL}(\text{KB}, \forall x \text{King}(x) \Rightarrow \text{Person}(x))$
- **Ask questions, queries or goals**
 - If logically entailed by the knowledge base it will be affirmative
 - $\text{ASK}(\text{KB}, \text{King}(\text{John}))$ returns true
 - $\text{ASK}(\text{KB}, \text{Person}(\text{John}))$ returns true
 - True, False is not sufficient

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Using FOL

Using knowledge base to get facts

- If we want to know what value of x
 - Makes the sentence true
 - $\text{ASKVARS}(\text{KB}, \text{Person}(x))$
 - Stream of answers
 - $\{x/\text{John}\}$ and $\{x/\text{Richard}\}$
 - **A substitution or binding list.**
 - Every way of making the query true will bind the variables to specific values
 - \rightarrow IF KB has $\text{King}(\text{John}) \vee \text{King}(\text{Richard})$
 - \rightarrow there is no binding to x for the query $\exists x \text{King}(x)$
 - ASKVARS is not applicable

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Using FOL

"Sibling" is symmetric

$$\forall x, y \text{Sibling}(x, y) \Leftrightarrow \text{Sibling}(y, x)$$

The kinship domain:

- **Brothers are siblings**

$$\forall x, y \text{Brother}(x, y) \Leftrightarrow \text{Sibling}(x, y)$$
- **One's mother is one's female parent**

$$\forall m, c \text{Mother}(c) = m \Leftrightarrow (\text{Female}(m) \wedge \text{Parent}(m, c))$$

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FOL Applications

- $\text{Takes}(x, c, s)$: student x takes course c in semester s ;
- $\text{Passes}(x, c, s)$: student x passes course c in semester s ;
- $\text{Score}(x, c, s)$: the score obtained by student x in course c in semester s ;
- $x > y$: x is greater than y ;
- $\text{Buys}(x, y, z)$: x buys y from z
- F and G : specific French and Greek courses

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FOL Applications

- Some students took French in spring 2001.

$$\exists x \text{Student}(x) \wedge \text{Takes}(x, F, \text{Spring2001})$$

- Every student who takes French passes it.

$$\forall x, s \text{Student}(x) \wedge \text{Takes}(x, F, s) \Rightarrow \text{Passes}(x, F, s).$$

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FOL Applications

- Only one student took Greek in spring 2001.

$$\exists x \text{Student}(x) \wedge \text{Takes}(x, G, \text{Spring2001}) \\ \wedge \forall y \ y \neq x \Rightarrow \neg \text{Takes}(y, G, \text{Spring2001})$$

- No person buys an expensive policy.

$$\forall x, y, z \text{Person}(x) \wedge \text{Policy}(y) \wedge \text{Expensive}(z) \Rightarrow \\ \neg \text{Buys}(x, y, z).$$

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FOL Applications

- There is an agent who sells policies only to people who are not insured.

$$\exists x \text{ Agent}(x) \wedge \forall y, z \text{ Policy}(y) \wedge \text{Sells}(x, y, z) \Rightarrow (\text{Person}(z) \wedge \neg \text{Insured}(z))$$

- There is a barber who shaves all men in town who do not shave themselves.

$$\exists x \text{ Barber}(x) \wedge \forall y \text{ Man}(y) \wedge \neg \text{Shaves}(y, y) \Rightarrow \text{Shaves}(x, y)$$

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FOL - Wumpus world

- First-order sentence stored in the KB
 - the percept and the time at which it occurred
 - $\text{Percept}([\text{Stench}, \text{Breeze}, \text{Glitter}, \text{None}, \text{None}], 5)$
- Actions
 - $\text{Turn}(\text{Right}), \text{Turn}(\text{Left}), \text{Forward}, \text{Shoot}, \text{Grab}, \text{Climb}$
- Best action query
 - $\text{ASKVARS}(\exists a \text{ BestAction}(a, 5))$
- Returns a binding list
 - $\{a/\text{Grab}\}$

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FOL - Wumpus world

- Current state from percept data
 - $\forall t, s, g, m, c \text{ Percept}([s, \text{Breeze}, g, m, c], t) \Rightarrow \text{Breezy}(t)$
 - $\forall t, s, b, m, c \text{ Percept}([s, b, \text{Glitter}, m, c], t) \Rightarrow \text{Glitter}(t)$
- Reflex behavior
 - $\forall t \text{ Glitter}(t) \Rightarrow \text{BestAction}(\text{Grab}, t)$
- Squares $\text{Square}_{1,2}$
- Adjacency
 - $\forall x, y, a, b \text{ Adjacent}([x, y], [a, b]) \Leftrightarrow (x = a \wedge (y = b - 1 \vee y = b + 1)) \vee (y = b \wedge (x = a - 1 \vee x = a + 1))$

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FOL - Wumpus world

- Location of agent at Square s and at time t
 - $\text{At}(\text{Agent}, s, t)$
- Agent infers property of square from percept
 - $\forall s, t \text{ At}(\text{Agent}, s, t) \wedge \text{Breezy}(t) \Rightarrow \text{Breezy}(s)$
- The agent can deduce pits & Wumpus
 - $\forall s \text{ Breezy}(s) \Leftrightarrow \exists r \text{ Adjacent}(r, s) \wedge \text{Pit}(r)$
 - More expressive than Propositional logic
- Agent's location & orientation (Space & Time)
 - Successive state

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