MA4.101 Real Analysis

Monsoon 2022

Assignment-1

1. For two sets A and B show that the following statements are equivalent:
   (a) $A \subseteq B$
   (b) $A \cup B = B$
   (c) $A \cap B = A$
2. Establish the following set theoretic relations:
   (a) $A \cup B = B \cup A$ and $A \cap B = B \cap A$ (Commutativity)
   (b) $A \cup (B \cup C) = (A \cup B) \cup C$ and $A \cap (B \cap C) = (A \cap B) \cap C$ (Associativity)
   (c) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$ and $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$ (Distributivity)
   (d) $A \subseteq B \iff B^c \subseteq A^c$
   (e) $A \setminus B = A \cap B^c$
   (f) $(A \cup B)^c = A^c \cap B^c$ and $(A \cap B)^c = A^c \cup B^c$ (De Morgan's laws)
3. Give a necessary and sufficient condition for
$$A \times B = B \times A$$
4. If A, B, C are sets, show that
   (a) $A \times B = \emptyset \iff A = \emptyset$ or $B = \emptyset$.
   (b) $(A \cup B) \times C = (A \times C) \cup (B \times C)$
   (c) $(A \cap B) \times C = (A \times C) \cap (B \times C)$
5. Suppose $f : A \to B$ and $g : B \to C$ are functions, show that
   (a) If both f and g are one to one, then $g \circ f$ is one-to-one.
   (b) If both f and g are onto, then $g \circ f$ is onto.
   (c) If both f and g are bijective, then $g \circ f$ is bijective.
6. For a function $f : X \to Y$, show that the following statements are equivalent:
   (a) f is one-to-one.
   (b) $f(A \cap B) = f(A) \cap f(B)$ holds for all $A, B \in \mathcal{P}(X)$.
7. For an arbitrary function $f : X \to Y$, prove the following identities:
   (a) $f^{-1}\left(\bigcup_{i \in I} B_i\right) = \bigcup_{i \in I} f^{-1}(B_i)$
   (b) $f^{-1}\left(\bigcap_{i \in I} B_i\right) = \bigcap_{i \in I} f^{-1}(B_i)$
   (c) $f^{-1}(B^c) = [f^{-1}(B)]^c$