Computer Vision
Assignment №4

Theoretical Questions

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1 Frequency Domain Magnitude

1. **Vertical Noise**

   The periodic noise is periodic in a vertical manner; The stripes of black and white are horizontal so the change of values of color intensity is vertical. Hence, The noise frequency is vertical in Fourier Transform magnitude.

2. **Stripe width: 4**

   If the stripe width is increased to 4, the number of frequencies to make the image increases, so the magnitude image will have more dots in the vertical central line.

3. **Stripe width: 1**

   If the stripe width is decreased to 1, the number of frequencies to make the image is less diverse and is around the center of the magnitude, so the magnitude image will have less dots in the vertical central line and mainly a high intensity value point in the center of the magnitude image. Also there is a slightly bright point in the highest part of the magnitude indicating the existence of a highest frequency component.s

2 2D Fourier Transform

1. **(0, 0) Frequency Component**

   Without signal normalization, the (0, 0) component of Fourier Transform is equal to the sum of all color intensities. With signal normalization it is equal to the mean.

I used the formula from lectures which does not have any normalization term, hence the real value of (0, 0) component of the transform is: $8 * 8 * 20 = 1280$

There is no imaginary part so: $(0, 0)comp. = 1280.0 + 0j$

2. **Computing Fourier Transform**

   I am using the formula without the $\frac{1}{MN}$ normalization term.

   Image: $\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$

   Fourier Transform is a linear transform, so the above image can be written in the form of summation of twp matrices:

   $$\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} + \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

   So, the Fourier Transform is calculated as below: (First matrix is A and the second is B respectively.)

   $$\mathcal{F}(Image) = \mathcal{F}(A) + \mathcal{F}(B)$$

   A is a matrix filled with 1s, since there is no change in neither of x or y dimensions, the Fourier Transform of this matrix only has a value in the (0, 0) component and it's value is the sum of all values in the matrix $\approx 1 * 4 = 4$ with no imaginary values (Coefficient of j is zero)

   $$\mathcal{F}(A) = \begin{bmatrix} 4.0 + 0j & 0 \\ 0 & 0 \end{bmatrix}$$

   now for matrix B we compute the transform using formula:

   $$F(u, v) = \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} f(x, y) e^{-j2\pi(ux/M + vy/N)}, M = 2, N = 2$$

We start from top left indices:

- For $u = 0$ and $v = 0$:

$$F(0,0) = \sum_{x=0}^{1}\sum_{y=0}^{1} f(x,y) * 1 = 2.0 + 0j$$

- For $u = 0$ and $v = 1$:

$$F(0,1) = \sum_{x=0}^{1}\sum_{y=0}^{1} f(x,y)e^{-j2\pi(y/2)} = 0.0 + 0j$$

- For $u = 1$ and $v = 0$:

$$F(1,0) = \sum_{x=0}^{1}\sum_{y=0}^{1} f(x,y)e^{-j2\pi(x/2)} = 0.0 + 0j$$

- For $u = 1$ and $v = 1$:

$$F(1,1) = \sum_{x=0}^{1}\sum_{y=0}^{1} f(x,y)e^{-j2\pi(x/M+y/N)} = -2.0 + 0j$$

So the Fourier Transform of matrix B is:

$$\mathcal{F}(B) = \begin{bmatrix} 2.0 + 0j & 0 \\ 0 & -2.0 + 0j \end{bmatrix}$$

In conclusion, Fourier Transform of image is calculated as following:

$$\mathcal{F}(Image) = \begin{bmatrix} 4.0 + 0j & 0 \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} 2.0 + 0j & 0 \\ 0 & -2.0 + 0j \end{bmatrix} = \begin{bmatrix} 6.0 + 0j & 0 \\ 0 & -2.0 + 0j \end{bmatrix}$$