Computer Vision
Assignment №12

Theoretical Questions
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1 Compactness, Solidity and Eccentricity

We calculate these 3 feature descriptors for the following shapes:

$$Compactness = \frac{4\pi Area}{Perimeter^2}$$
$$Solidity = \frac{Area}{ConvexArea}$$
$$Eccentricity = \sqrt{1 - (\frac{MinorAxisLength}{MajorAxisLength})^2}$$

1. **Rhombus with a and $2a$ diagonal length:**

   - Compactness: Using Pythagorean theorem, we calculate the length of the rhombus edge (E is the length of edge):

     $$E^2 = (\frac{LongerDiagonal}{2})^2 + (\frac{ShorterDiagonal}{2})^2$$

     $$\Rightarrow E^2 = \sqrt{a^2 + \frac{a^2}{4}} = \sqrt{\frac{5a^2}{4}} = \frac{a\sqrt{5}}{2}$$

     So the perimeter of the rhombus is calculated:

     $$P = 4E \Rightarrow P = 4 * \frac{a\sqrt{5}}{2} = 2a\sqrt{5}$$

Now we calculate the compactness:

$$\frac{1}{2}\frac{4\pi * a * 2a}{(2a\sqrt{5})^2} = \frac{\pi}{5}$$

- Solidity:

  A rhombus is a convex area, so area equals to convex area, so the solidity is 1.

- Eccentricity:

  In a rhombus, the major axis is the longer diagonal and the minor axis is the shorter diagonal, so:

$$\sqrt{1-(\frac{a}{2a})^2} = \sqrt{1-(\frac{1}{2})^2} = \sqrt{\frac{3}{4}} = \frac{\sqrt{3}}{2}$$

2. **Rectangle with a and $2a$ edges length:**

- Compactness:

$$\frac{4\pi * a * 2a}{(2*(a+2a))^2} = \frac{8\pi a^2}{36a^2} = \frac{2\pi}{9}$$

- Solidity:

  A rectangle is a convex area, so area equals to convex area, so the solidity is 1.

- Eccentricity:

  In a rectangle, the major axis is the diagonal and the minor axis is the line perpendicular to the diagonal and passing the center of the rectangle, so:

  The diagonal is calculated using Pythagorean theorem:

$$D^2 = (LongerEdge)^2 + (ShorterEdge)^2$$

$$\Rightarrow D^2 = \sqrt{a^2+4a^2} = \sqrt{5a^2} = a\sqrt{5}$$

  The minor axis also is calculated with Triangle Similarity theorem (Two similar angles):

$$\frac{Half\ of\ Minor\ Axis}{Shorter\ Edge} = \frac{Half\ of\ Diagonal}{Longer\ Edge} \Rightarrow$$

$$\frac{x}{a} = \frac{\frac{a\sqrt{5}}{2}}{2a} \Rightarrow$$

$$x = \frac{a\sqrt{5}}{4} \Rightarrow$$

$$Minor\ Axis = 2x = \frac{a\sqrt{5}}{2}$$

Now for eccentricity:

$$\sqrt{1 - (\frac{\frac{a\sqrt{5}}{2}}{a\sqrt{5}})^2} = \sqrt{1 - (\frac{1}{2})^2} = \sqrt{\frac{3}{4}} = \frac{\sqrt{3}}{2}$$

2 $\mathbf{LBP_8^1}$ Histogram

We assume that the LBP code is rotation variant.

1. First we apply the 90° clockwise rotation. For this purpose, the decimal values of histogram are converted to binary each one of them shift-rotate to right and for 2 places.

   - $(7)_{10} = (00000111)_2 \Rightarrow$ binary shift and rotation(2) $\Rightarrow (11000001)_2 = (193)_{10}$
   - $(62)_{10} = (00111110)_2 \Rightarrow$ binary shift and rotation(2) $\Rightarrow (10001111)_2 = (143)_{10}$
   - $(112)_{10} = (01110000)_2 \Rightarrow$ binary shift and rotation(2) $\Rightarrow (00011100)_2 = (28)_{10}$
   - $(255)_{10} = (11111111)_2 \Rightarrow$ binary shift and rotation(2) $\Rightarrow (11111111)_2 = (255)_{10}$

2. Multiplying the intensity values by a factor doesn't change the LBP patterns, so nothing changes when image values are multiplied by 2.

3. Final Histogram of LBP code:

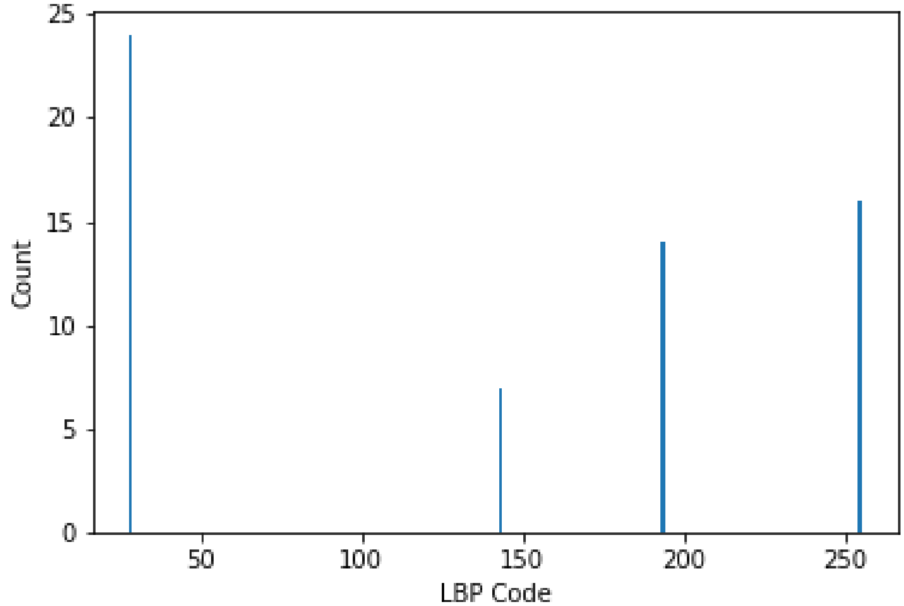

Figure 1: Histogram of LBP code