Celeste Spinner Analysis - the SPIN theory

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The name "SPIN theory" is derived from the fact that this theory can be visualized as a recursive set of sliding windows "spinning" / looping around in a residual number field

Introduction

Celeste's spinners can "load" and "unload" (= turn collision on or off by setting Collidable) only on certain frames where Scene.OnInterval(float interval, float offset) returns true. interval is always 0.05f for loading and 0.25f for unloading, while offset is a random float in [0;1) chosen when the room is loaded. The following code is a decompilation of Scene.OnInterval:

```
public bool OnInterval(float interval, float offset) {
   return Math.Floor((TimeActive - offset - Engine.DeltaTime) /
        interval) < Math.Floor((TimeActive - offset) / interval);
}</pre>
```

Scene.OnInterval is intended to only return true for only one frame every interval seconds. As such the game uses this additional requirement to limit the number of collidable checks happening every frame.

This behavior leads to spinners being grouped into so called "spinner groups": the number of spinner groups is determined through the following formula:

$$n_g = \left\lfloor \frac{interval}{dt} \right\rceil = \left\lfloor interval * 60 \right\rceil$$

Spinners will check whether they should load on every frame $t=i*n_g+g$ for some integer i, where $g\in[0;n_g)$ is called the spinner's $group\ (index)$. A "group cycle" consists of n_g frames, where every spinner checks if it should load exactly once at some frame offset corresponding to its group number g. The following formula can be used to determine a spinner's group from its offset (the later

analysis will contain a proof of a more general version of this statement):

$$g = \left\lceil \frac{off_B}{dt} \right\rceil \mod n_g$$
 where $off_B = \begin{cases} (off \mod intv) - intv & \text{if } (off \mod intv) > intv - dt \\ off \mod intv & \text{else} \end{cases}$

For the scope of this document, frame indices are defined as the number of frames which have ellapsed since the scene (=Level) has become active.

As briefly mentioned above, spinners don't strictily adhere to this group pattern, can actually change which group they belong to at certain points in time - the rest of the document will be an analysis of why and when these spinner group changes occur.

Why spinner offsets drift

NOTE: This analysis assumes that all operations are performed with perfect accuracy. While real-world floating point operations are subject to imprecision, for reasons beyond the scope of this document, the 32 bit floating point operations are actually executed with 80 bits of precision, which means that in practice floating point imprecision can be neglected for OnInterval.

Let dt be the game's delta time (Engine.DeltaTime in the code), T the time since the scene / level has been instantiated (TimeActive in the code), intv and off the interval and offset parameters respectively, and n_g be the number of spinner groups same as above (note that even though intv is fixed for spinner load groups, this analysis will be a generic analysis for every possible intv and off). Note that because T is incremented by dt, we can set T = t * dt, where t is the current frame index.

"Translating" the C# code into mathmatical notation yields

$$\left| \frac{T - off - dt}{intv} \right| < \left| \frac{T - off}{intv} \right|$$

which is equivalent to

$$\frac{T-\mathit{off}-\mathit{dt}}{\mathit{intv}} < \left\lfloor \frac{T-\mathit{off}}{\mathit{intv}} \right\rfloor$$

By multiplying both sides by *intv* and substituting $\lfloor \frac{a}{b} \rfloor * b = a - (a \mod b)$ we can reformulate this to obtain:

$$T - off - dt < T - off - ((T - off) \mod intv)$$

Subtracting T - off - dt from both sides yields

$$-\mathit{dt} < -((\mathit{T}-\mathit{off}) \bmod \mathit{intv}) \iff (\mathit{T}-\mathit{off}) \bmod \mathit{intv} < \mathit{dt}$$

and because $intv > dt \iff dt \mod intv = dt$ we get

$$T - off = t * dt - off < dt \mod intv$$

For purposes which will become apparent later, let us relax our assumptions slightly from our implict dt>0 to |dt|>0. This will allow dt to be negative, which essentially means that time is flowing "backwards" through the cycle, and as such we are stepping through the cycle backwards. Additionally, we will decouple the right hand side of the condition from |dt| by instead introducing a new symbol thr, which in this case equals thr=|dt|. The relaxed statement we will continue to further analyse is:

$$t * dt - off < thr \mod intv$$

Now, because of spinner groups, we expect this expression to evaluate to the same value for every $t = c * n_g + i$, where c is the index of the group cycle and i is the offset into the cycle. Inserting this value of t into the expression yields:

$$t*\mathit{dt}-\mathit{off} = (c*n_g+i)*\mathit{dt}-\mathit{off} = c*n_g*\mathit{dt}+i*\mathit{dt}-\mathit{off} < \mathit{thr} \mod \mathit{intv}$$

Let us now define the residual drift r as $r = intv - |dt| * n_g$, where $|r| \leq \frac{|dt|}{2}$. Inserting this new symbol into our condition we get:

$$\begin{array}{c} c*n_g*dt+i*dt-o\!f\!f\!< t\!hr \mod i\!ntv\\ \iff c*\operatorname{sgn} dt*(i\!nt\!v-r)+i*dt-o\!f\!f\!< t\!hr \mod i\!ntv\\ \iff i*dt-o\!f\!f\!-\operatorname{sgn} dt*r*c< t\!hr \mod i\!ntv\\ \iff \mathbf{i}*dt-(o\!f\!f\!+\operatorname{sgn} dt*\mathbf{r}*\mathbf{c})< t\!hr \mod i\!ntv\\ \end{array}$$

Note that the final expression differs from what we would expect if all spinner cycles were exactly equal! Instead of $i*dt-of\!f < thr \mod intv$, which would mean that the values of our expression would loop every group cycle (as it is only dependent on the group offset i), we got $i*dt-(of\!f+\operatorname{sgn} dt*r*c) < thr \mod intv$. This means that the effective offset of each spinner shifts by $\operatorname{sgn} dt*r$ every n_q frames!

When group changes happen - the check range

Let us start by finding all values $i \in [0; n_g]$ for which $i * dt - off < thr \mod intv$ is true. We can differentiate between two cases on the sign of dt:

• When dt > 0, then we define

$$\mathit{off}_B = \begin{cases} (\mathit{off} \bmod \mathit{intv}) - \mathit{intv} & \text{if } (\mathit{off} \bmod \mathit{intv}) > \mathit{intv} - \mathit{dt} \\ \mathit{off} \bmod \mathit{intv} & \text{else} \end{cases}$$

(so that $\mathit{off} = \mathit{off}_B \mod \mathit{intv} \land -\mathit{dt} < \mathit{off}_B \leq \mathit{intv} - \mathit{dt}$). Solving for i yields:

$$\left\lceil \frac{\mathit{off}_B}{\mathit{dt}} \right\rceil = \left| \left\lceil \frac{\mathit{off}_B}{|\mathit{dt}|} \right\rceil \right| \leq i < \left| \left\lceil \frac{\mathit{off}_B}{|\mathit{dt}|} \right\rceil \right| + \left\lceil \frac{\mathit{thr} - (-\mathit{off}_B \bmod |\mathit{dt}|)}{|\mathit{dt}|} \right\rceil$$

• When dt < 0, then we define

$$\mathit{off}_B = \begin{cases} (\mathit{off} \bmod \mathit{intv}) - \mathit{intv} & \text{if } \mathit{off} \neq 0 \\ 0 & \text{else} \end{cases}$$

(so that $\mathit{off} = \mathit{off}_B \mod \mathit{intv} \land -\mathit{intv} < \mathit{off}_B \leq 0).$ We then reformulate the statement as:

$$-\mathit{off}_B - i * |\mathit{dt}| < \mathit{thr} \mod \mathit{intv}$$

Solving for i yields:

$$\left\lfloor \frac{-\mathit{off}_B}{|\mathit{dt}|} \right\rfloor - \left\lceil \frac{\mathit{thr} - (-\mathit{off}_B \bmod |\mathit{dt}|)}{|\mathit{dt}|} \right\rceil < i \leq \left\lfloor \frac{-\mathit{off}_B}{|\mathit{dt}|} \right\rfloor = \left\lceil \left\lceil \frac{\mathit{off}_B}{|\mathit{dt}|} \right\rceil \right\rceil$$

(Note that these ranges are not perfectly accurate - there can be group cycles where two or no values of $i \in [0; n_g)$ will fulfill the condition (namely $i = 0 \land i = n_g - 1$ and $i = n_g$). These are results of drifts which cross cycle boundaries, and these drift-affected solutions of the equation will not be considered when determining spinner's group)

This means that spinner groups are not actually singluar discrete values, but ranges of values (called the check range). We will continue to call $g = \left|\left\lceil\frac{off_B}{|dt|}\right\rceil\right| \mod n_g$ the group of the spinner, however keep in mind that in practice, the actual check results can differ from what we expect using the simple definition of spinner groups.

Additionaly, this means that spinner check behavior is affected to two types of drift: drifts of the spinner group g, called $group \ drifts$, and temporary anomalies of the length $L = \left\lceil \frac{thr - (-off_B \ \text{mod} \ | dt|)}{|dt|} \right\rceil$ of the check range, called $length \ drifts$.

From now on, we will call off_B the bounded offset of the spinner. It will become useful later as $0 \le \left|\left\lceil \frac{off_B}{|dt|}\right\rceil\right| * dt - off_B < |dt|$ is true.

Group drifts

We will disregard thr by assuming that it always is |dt| for the rest of the group drift analysis - this elliminates length drift, which could otherwise affect things as well

Let $r' = \operatorname{sgn} dt * r$ and $\operatorname{off}_c = \operatorname{off}_B + c * r'$. Assuming our ideal definition of spinner groups, we expect that the following holds for all integer values of c, where $0 \leq i < n_g$ and $0 \leq i * dt - \operatorname{off}_{c-1} < |dt|$:

$$i*\mathit{dt}-\mathit{off}_{c-1}<|\mathit{dt}|\mod\mathit{intv}\iff i*\mathit{dt}-\mathit{off}_{c}<|\mathit{dt}|\mod\mathit{intv}$$

We now want to find all values of c for which this is not true. Because of $|r'| \leq intv - |dt|$, the right hand side can not be true only because of the modulo term - as such we can restate our earlier condition as:

$$0 \le v_c < |dt| \iff 0 \le v_{c-1} = v_c + r' < |dt|$$

(where $v_c = i * dt - off_c$)

This is obviously not always true - we can determine the values of c for which it won't be by distinguishing between two cases depending on the sign of r':

• When r' < 0, then $v_c > v_{c-1}$. As such we want to find all c for which the following holds:

$$\begin{split} 0 &\leq v_{c-1} < |\mathit{dt}| \land \neg \left(0 \leq v_c \lor v_c < |\mathit{dt}|\right) \\ &\iff 0 \leq v_{c-1} = v_c + r' < |\mathit{dt}| \leq v_c \\ &\iff |\mathit{dt}| \leq v_c < |\mathit{dt}| - r' \\ &\implies v_c = \left(g * \mathit{dt} - \left(\mathit{off}_B + c * r'\right)\right) < -r' \mod |\mathit{dt}| \\ &\iff -\mathit{off}_B - c * r' < -r' \mod |\mathit{dt}| \\ &\iff \mathbf{c} * \left(-\mathbf{r}'\right) - \mathit{off}_{\mathbf{B}} < |\mathbf{r}'| \mod |\mathit{dt}| \end{split}$$

- As $|dt| \leq v_c < 2*|dt|$ holds, the condition is true for $i_c = i_{c-1} - {\rm sgn} \; dt$ because

$$(i_{c-1} - \operatorname{sgn} dt) * dt - \mathit{off}_c = i_{c-1} * dt - |dt| - \mathit{off}_c = v_c - |dt| \wedge 0 \leq v_c - |dt| < |dt|$$

- * This means that spinner drifts shift the spinner group by $-\operatorname{sgn} dt = \operatorname{sgn} r$!
- Note that $0 \le i_c * dt off_c < |dt|$ still holds.
- When r' > 0, then $v_c < v_{c-1}$. As such we want to find all c for which the following holds:

$$\begin{split} 0 \leq v_{c-1} < |\mathit{dt}| \land \neg \left(0 \leq v_c \lor v_c < |\mathit{dt}|\right) \\ \iff v_c < 0 \leq v_{c-1} = v_c + r' < |\mathit{dt}| \\ \iff -r' \leq v_c < 0 \\ \iff 0 \leq v_c + r' < r' \\ \implies v_c + r' = \left(g * \mathit{dt} - \left(\mathit{off}_B + c * r'\right)\right) + r' < r' \mod |\mathit{dt}| \\ \iff -\mathit{off}_B - c * r' + r' < r' \mod |\mathit{dt}| \\ \iff \mathbf{c} * (-\mathbf{r'}) - \left(\mathit{off}_{\mathbf{B}} - \mathbf{r'}\right) < |\mathbf{r'}| \mod |\mathit{dt}| \end{split}$$

– As $-|\mathit{dt}| \leq v_c < 0$ holds, the condition is true for $i_c = i_{c-1} + \operatorname{sgn} \mathit{dt}$ because

$$(i_{c-1} + \operatorname{sgn} dt) * dt - off_c = i_{c-1} * dt + |dt| - off_c = v_c + |dt| \land 0 \le v_c + |dt| < |dt|$$

* This means that spinner drifts shift the spinner group by $\operatorname{sgn} dt = \operatorname{sgn} r$!

- Note that $0 \le i_c * dt off_c < |dt|$ still holds.
- When r' = 0, then there is no group drifting, and as such group changes will never occur

(our initial assumption of $0 \le i_c * dt - off_c < |dt|$ can be shown to inductively hold for all values of c when $i_0 = g$, because we know that $0 \le g * dt - off_B < |dt|$ holds as our base case, and that when a tick is not affected by drift, then by definition $0 \le v_{c-1} < |dt| \iff 0 \le v_c < |dt|$ holds, or if it is, we have shown that the condition still holds)

Note how both non-trivial cases match the exact structure of our original statement derived from OnInterval! This means that the problem is recursive, and the spinner cycle indices where group drift occurs behave just like a recursive instance of the original problem with the following parameters:

$$\begin{split} dt_R &= -r' = -\operatorname{sgn} dt * r \\ t_R &= c \\ intv_R &= |dt| \\ off_R &= \begin{cases} off_B & \text{for } r' < 0 \iff \operatorname{sgn} dt \neq \operatorname{sgn} r \\ off_B - r' &= off_B - \operatorname{sgn} dt * r \end{cases} &\text{for } r' > 0 \iff \operatorname{sgn} dt = \operatorname{sgn} r \\ thr_R &= |r'| \\ (\text{where } r = intv - |dt| * n_a) \end{split}$$

(note that because $thr_R=|dt_R|$, recursive cycles are not affected by length drift - their check range will always have a length of 1)

All of this implies that the cycle indices c during which spinners change groups are also cyclic, with spinners changing groups ever group cycle index c where $c=j*n_{g_R}+g_R$, where $n_{g_R}=\left\lfloor\frac{intv_R}{|dt_R|}\right\rceil$ is the period of these recursive "group drift cycles", and g_R is the recursive "group drift cycle group". When r<0, group drifts decrement the group index (= load checks happen one frame earlier than expected), and when r>0, they increment it (= load checks happen one frame later than expected).

Length drifts

As we derived above, the actual group of a spinner is not a discrete value, but instead a range of values, namely:

$$\begin{split} i \in \begin{cases} [g; g + L) & \text{for } dt > 0 \\ (g - L; g] & \text{for } dt < 0 \end{cases} \\ \text{(where } L = \left\lceil \frac{thr - (-off_B \bmod |dt|)}{|dt|} \right\rceil) \end{split}$$

(from now on, we will consider L over off_c instead of off_B) TODO: Can this be safely done?

L is the length of this check range, and by setting thr = k * |dt| + o, where $k = \left\lfloor \frac{thr}{|dt|} \right\rfloor$ (called the *base length*) and $o = thr \mod |dt|$ (called the *length offset*), we can reformulate L as

$$L = \left\lceil \frac{k * |\mathit{dt}| + o - (-\mathit{off}_c \bmod |\mathit{dt}|)}{|\mathit{dt}|} \right\rceil = k + \left\lceil \frac{o - (-\mathit{off}_c \bmod |\mathit{dt}|)}{|\mathit{dt}|} \right\rceil = k + l$$

Because of $0 \le o < |dt|$, l can only ever be 0 or 1, as $-|dt| < o - (-off \mod |dt|) < |dt|$. l is 1 (which is called a length drift) only if

$$\begin{split} o - (-\mathit{off}_c \bmod |\mathit{dt}|) > 0 \\ \iff o > (-\mathit{off}_c \bmod |\mathit{dt}|) \\ \iff -\mathit{off}_c < o \mod |\mathit{dt}| \\ \iff -(\mathit{off}_B + r' * c) < o \mod |\mathit{dt}| \\ \iff \mathbf{c} * (-\mathbf{r}') - \mathit{off}_\mathbf{B} < \mathbf{o} \mod |\mathit{dt}| \end{split}$$

This defines another recursive cycle with

$$\begin{split} dt_L &= -r' \\ t_L &= c \\ intv_L &= |dt| \\ of\!f_L &= of\!f_B \\ thr_L &= o = thr \operatorname{mod} |dt| \end{split}$$

Note that this almost exactly matches the definition of the regular recursive cycle! The only two differences are:

- $|r'| = thr_R \neq thr_L = o$
 - This difference does not affect the group indicies $g_R = g_L$, however a further recursive cycle is required to predict the recursive length drift of L_L .
- when r' > 0: $\mathit{off} r' = \mathit{off}_R \neq \mathit{off}_L = \mathit{off}$
 - In this case, the recursive group drift cycle is one tick behind of the length drift cycle. The regular recursive cycle can still be used to predict the length drift, but one must look at the values from the next tick instead of the current value. The recursive length drift cycle of L_L must then also be adjusted to use off_R instead of off_L , as this cancels out the fact that values are taken from the next tick.
 - This also means that length drifts either happen in the same group cycle, or in the group cycle before an actual group drift happens.

TODO: Length drift inversion for potentially cleaner results?

Why this even happens

Careful readers might have noticed that all of the above analysis implicitly assumed that $r = intv \mod dt \neq 0$, meaning that spinner offsets drift at all.

However, as $dt = \frac{1}{60}$ and intv = 0.05, r should be $r = 0.05 \mod \frac{1}{60} = 0$. Then why is the above analysis relevant at all?

Simply put, in practice, the imprecisions of floating point numbers mean that neither dt nor intv are their ideal, non-drifting values. The following effects cause them to be ever so slightly different:

- 0.05 is not exactly representable as a floating point number the values used by the game's calculations is actually 0.0500000007450580596923828125 (this also affects dt)
- TimeActive is incremented every frame by 0.016666699200868606567382812500 (closest float value of 0.0166667). However, as it only is a 32 bit float, depending on the current magnitude of TimeActive, the actual effective dt will vary slightly because of float cancellation, and become less and less precise as time goes on. This results in "ranges" of different effective dt values, which will be examined later.
 - Note that thr is unaffected this effective delta time imprecision - it will always remain the at the same precision and always equal exactly 0.016666699200868606567382812500

The effective deltatime ranges

When adding two floating point numbers (like TimeActive and DeltaTime) with different exponents, a phenomenon called "float cancellation" occurs. This means that the lowest bits of the mantissa, which can now no longer be stored in the result are cut off. Additionally, the mantissa is rounded either up or down, depending on the highest trimmed of bit.

All of this effectively causes the effective deltatime to loose precision as the exponent of TimeActive gets bigger and bigger. Note that the effective deltatime is the same between two values of TimeActive with the same exponent, which allows for entire ranges of TimeActive values to be assigned the same effective deltatime. One edge case has to be taken into account though, which is when the next TimeActive value is already in the next range. In this case, the effective deltatime will be unique in some cases for one frame.

Included in the appendix are tables with various bits of information regarding the effective deltatime ranges.

Appendix: Some data

Effective Deltatime - Frame Ranges

| TimeActive Exponent | Start Frame | End Frame | | | | |
|---------------------|-------------|-----------|--|--|--|--|
| 121 | 00000000 | 00000001 | | | | |
| 122 | 00000001 | 00000003 | | | | |
| 123 | 00000003 | 00000007 | | | | |
| 124 | 00000007 | 00000014 | | | | |
| 125 | 00000014 | 00000029 | | | | |
| 126 | 00000029 | 00000059 | | | | |
| 127 | 00000059 | 00000119 | | | | |
| 128 | 00000119 | 00000240 | | | | |
| 129 | 00000240 | 00000479 | | | | |
| 130 | 00000479 | 00000960 | | | | |
| 131 | 00000960 | 00001920 | | | | |
| 132 | 00001920 | 00003840 | | | | |
| 133 | 00003840 | 00007679 | | | | |
| 134 | 00007679 | 00015361 | | | | |
| 135 | 00015361 | 00030724 | | | | |
| 136 | 00030724 | 00061452 | | | | |
| 137 | 00061452 | 00122683 | | | | |
| 138 | 00122683 | 00246044 | | | | |
| 139 | 00246044 | 00492768 | | | | |
| 140 | 00492768 | 00986216 | | | | |
| 141 | 00986216 | 01918283 | | | | |
| 142 | 01918283 | 04015435 | | | | |
| 143 | 04015435 | 08209739 | | | | |
| 144 | 08209739 | 16598346 | | | | |
| 145 | 16598346 | 24986954 | | | | |
| 146 | 24986954 | | | | | |

Effective Deltatime - First ${\tt TimeActive}$ value in range

| TimeActive Exponent | First TimeActive value in range |
|---------------------|-------------------------------------|
| 121 | 0.016666699200868606567382812500 |
| 122 | 0.033333398401737213134765625000 |
| 123 | 0.066666796803474426269531250000 |
| 124 | 0.133333608508110046386718750000 |
| 125 | 0.250000476837158203125000000000 |
| 126 | 0.500001132488250732421875000000 |
| 127 | 1.000002503395080566406250000000 |
| 128 | 2.000001668930053710937500000000 |
| 129 | 4.0166664123535156250000000000000 |
| 130 | 8.0000534057617187500000000000000 |
| 131 | 16.0165977478027343750000000000000 |
| 132 | 32.01635360717773437500000000000000 |

| TimeActive Exponent | First TimeActive value in range |
|---------------------|---|
| 133 | 64.0158691406250000000000000000000 |
| 134 | 128.01287841796875000000000000000000 |
| 135 | 256.01495361328125000000000000000000 |
| 136 | 512.00244140625000000000000000000000 |
| 137 | 1024.010742187500000000000000000000000 |
| 138 | 2048.0156250000000000000000000000000000000000 |
| 139 | 4096.0009765625000000000000000000000 |
| 140 | 8192.00488281250000000000000000000000 |
| 141 | 16384.01367187500000000000000000000000 |
| 142 | 32768.0039062500000000000000000000000000000000000 |
| 143 | 65536.0078125000000000000000000000000000000000000 |
| 144 | 131072.0156250000000000000000000000000000000000 |
| 145 | 262144.00000000000000000000000000000000000 |
| 146 | 524288.0000000000000000000000000000000000 |

Effective Deltatime - Effective dt values

| TimeActive Exponent | Effective dt value |
|---------------------|--|
| 121 | - |
| 122 | 0.0166666992008686065673828125 |
| 123 | 0.0166666954755783081054687500 |
| 124 | 0.0166666954755783081054687500 |
| 125 | 0.0166667103767395019531250000 |
| 126 | 0.0166667103767395019531250000 |
| 127 | 0.0166666507720947265625000000 |
| 128 | 0.0166666507720947265625000000 |
| 129 | 0.0166668891906738281250000000 |
| 130 | 0.01666641235351562500000000000 |
| 131 | 0.01666641235351562500000000000 |
| 132 | 0.01666641235351562500000000000 |
| 133 | 0.01667022705078125000000000000 |
| 134 | 0.016662597656250000000000000000 |
| 135 | 0.01666259765625000000000000000 |
| 136 | 0.016662597656250000000000000000 |
| 137 | 0.016723632812500000000000000000 |
| 138 | 0.016601562500000000000000000000 |
| 139 | 0.016601562500000000000000000000 |
| 140 | 0.016601562500000000000000000000 |
| 141 | 0.017578125000000000000000000000 |
| 142 | 0.0156250000000000000000000000000000000000 |
| 143 | 0.0156250000000000000000000000000000000000 |
| 144 | 0.0156250000000000000000000000000000000000 |
| | |

| TimeActive Exponent | Effective dt value |
|---------------------|--|
| 145 146 | $\begin{array}{c} 0.031250000000000000000000000000\\ 0.0000000000$ |

Effective Deltatime - Transition dt values

| TimeActive Exponent | Transition dt value |
|---------------------|---|
| 121 | 0.0166666992008686065673828125000000000000 |
| 122 | 0.0166666992008686065673828125000000000000 |
| 123 | 0.01666669547557830810546875000000000000000 |
| 124 | 0.01666669547557830810546875000000000000000 |
| 125 | 0.01666668057441711425781250000000000000000 |
| 126 | 0.01666671037673950195312500000000000000000 |
| 127 | 0.01666665077209472656250000000000000000000 |
| 128 | 0.01666688919067382812500000000000000000000 |
| 129 | 0.01666641235351562500000000000000000000000 |
| 130 | 0.01666736602783203125000000000000000000000 |
| 131 | 0.01666641235351562500000000000000000000000 |
| 132 | 0.01667022705078125000000000000000000000000000000000000 |
| 133 | 0.016662597656250000000000000000000000000000000000 |
| 134 | 0.016677856445312500000000000000000000000000000000000 |
| 135 | 0.016662597656250000000000000000000000000000000000 |
| 136 | 0.016662597656250000000000000000000000000000000000 |
| 137 | 0.0166015625000000000000000000000000000000 |
| 138 | 0.01684570312500000000000000000000000000000000000 |
| 139 | 0.0166015625000000000000000000000000000000 |
| 140 | 0.017578125000000000000000000000000000000000000 |
| 141 | 0.017578125000000000000000000000000000000000000 |
| 142 | 0.0195312500000000000000000000000000000000000 |
| 143 | 0.023437500000000000000000000000000000000000 |
| 144 | 0.0156250000000000000000000000000000000000 |
| 145 | 0.0312500000000000000000000000000000000000 |
| 146 | _ |

Recursive cycle periods

Note: cycle 0 is the spinner group cycle (which always has a length of 3 for all exponents other than 145), and cycle 1 is the first recursive group change cycle.

| TimeActive Exponent | C0 | C1 | C2 | С3 | C4 | C5 | C6 | C7 | C8 |
|---------------------|----|--------|----|----|----|----|----|----|----|
| 121 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 122 | 3 | 172074 | 3 | 9 | 0 | 0 | 0 | 0 | 0 |

| TimeActive Exponent | C0 | C1 | C2 | С3 | C4 | C5 | C6 | C7 | C8 |
|---------------------|----|--------|--------|---------|-------|----|----|----|----|
| 123 | 3 | 194519 | 5 | 2 | 2 | 0 | 0 | 0 | 0 |
| 124 | 3 | 194519 | 5 | 2 | 2 | 0 | 0 | 0 | 0 |
| 125 | 3 | 127827 | 4 | 9 | 0 | 0 | 0 | 0 | 0 |
| 126 | 3 | 127827 | 4 | 9 | 0 | 0 | 0 | 0 | 0 |
| 127 | 3 | 344148 | 3 | 4 | 0 | 0 | 0 | 0 | 0 |
| 128 | 3 | 344148 | 3 | 4 | 0 | 0 | 0 | 0 | 0 |
| 129 | 3 | 24994 | 3 | 12 | 2 | 2 | 0 | 0 | 0 |
| 130 | 3 | 21824 | 3 | 5 | 13 | 0 | 0 | 0 | 0 |
| 131 | 3 | 21824 | 3 | 5 | 13 | 0 | 0 | 0 | 0 |
| 132 | 3 | 21824 | 3 | 5 | 13 | 0 | 0 | 0 | 0 |
| 133 | 3 | 1561 | 6 | 3 | 10 | 4 | 2 | 2 | 0 |
| 134 | 3 | 1365 | 12 | 273 | 0 | 0 | 0 | 0 | 0 |
| 135 | 3 | 1365 | 12 | 273 | 0 | 0 | 0 | 0 | 0 |
| 136 | 3 | 1365 | 12 | 273 | 0 | 0 | 0 | 0 | 0 |
| 137 | 3 | 98 | 7 | 48 | 3 | 4 | 6 | 2 | 0 |
| 138 | 3 | 85 | 3084 | 17 | 0 | 0 | 0 | 0 | 0 |
| 139 | 3 | 85 | 3084 | 17 | 0 | 0 | 0 | 0 | 0 |
| 140 | 3 | 85 | 3084 | 17 | 0 | 0 | 0 | 0 | 0 |
| 141 | 3 | 6 | 2 | 3 | 11651 | 2 | 4 | 0 | 0 |
| 142 | 3 | 5 | 838861 | 0 | 0 | 0 | 0 | 0 | 0 |
| 143 | 3 | 5 | 838861 | 0 | 0 | 0 | 0 | 0 | 0 |
| 144 | 3 | 5 | 838861 | 0 | 0 | 0 | 0 | 0 | 0 |
| 145 | 2 | 3 | 2 | 1677721 | 0 | 0 | 0 | 0 | 0 |
| 146 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

How this can be used to efficiently predict spinner group changes

TODO