Introduction to signal processing tutorial

Energy and power of signals - Correlation and convolution

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Energy and power of signals

Question 1 What is the relationship between $\mathcal{L}^2(\mathbb{R})$ and $\mathcal{L}^{pm}(\mathbb{R})$? In other words, what can we say about the energy of a signal with finite (non-zero) mean power, and the mean power of a signal with finite (non-zero) energy?

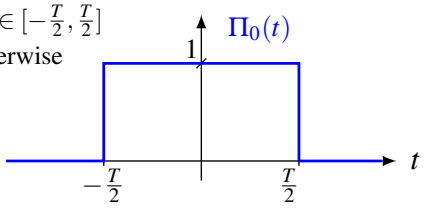
Question 2 What is the mean power of the signal $x : t \mapsto A\cos(\frac{2\pi}{T}t)$ with $A > 0$ and T the period of x? (You can start with $T = 1$ or $T = 2\pi$ to simplify if necessary).
Does the previous result change if we add a phase ϕ $\left(x(t) = A\cos(\frac{2\pi}{T}t + \phi)\right)$?

Reminder: $\cos^2(a) = \frac{1}{2}(1 + \cos(2a))$

Question 3 What is the mean power $P_{x_1+x_2}$ of the sum of two signals $x_1 + x_2$ with $x_1, x_2 \in \mathcal{L}^{pm}(\mathbb{R})$? Same question for signals with finite energy?

Correlation and convolution

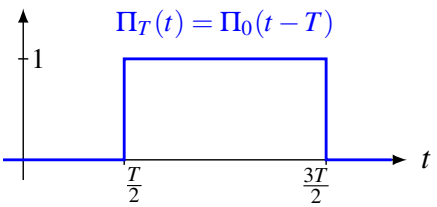
We consider the following window function: $\Pi_0(t) = \begin{cases} 1 & \forall t \in [-\frac{T}{2}, \frac{T}{2}] \\ 0 & \text{otherwise} \end{cases}$



Question 1 Calculate and plot the autocorrelation of the window function Π_0 (with itself). Is the result different for the convolution product?

Question 2 We now consider a delayed version of Π_0 by a factor of T: $\Pi_T(t) = \Pi_0(t - T)$. Calculate the cross-correlation $\Gamma_{\Pi_0\Pi_T}$ and the convolution $(\Pi_0 * \Pi_T)$ of Π_0 and Π_T. Where is located the maximum of the cross-correlation and the convolution?

$\Pi_T(t) = \begin{cases} 1 & \forall t \in [\frac{T}{2}, \frac{3T}{2}] \\ 0 & \text{otherwise} \end{cases}$



Question 3 We now consider the function Δ, which is a kind of double window function. Same question as before, but now between Π_0 and Δ.

$$\Delta(t) = \begin{cases} 1 & \forall t \in [-\frac{T}{2}, 0] \\ 2 & \forall t \in [0, \frac{T}{2}] \\ 0 & \text{otherwise} \end{cases}$$