Workshop session 4 (September 2024)

Triple integrals

Exercice 20.

(1) For the solid region $0 \le x \le y \le z \le 1$, find the boundaries in $\iiint dx\,dy\,dz$ and calculate the volume.

(2) Reverse the order of integration in the previous question to $\iiint dz\,dy\,dx$ and find the limits of integration. The four faces of this tetrahedron are the planes $x = 0$ and $y = x$ and _____.

Exercice 21. We want to find the limits in $\iiint dx\,dy\,dz$ or $\iiint dz\,dy\,dx$. Calculate the volume.

(1) A circular cylinder of height 6 and base $x^2 + y^2 \le 1$.

(2) The part of this same cylinder located under the plane $z = x$. Look at the base. Draw a picture.

Cylindrical and spherical coordinates

Exercice 27. Convert coordinates xyz to cylindrical coordinates $r\theta z$ and spherical $\rho\phi\theta$ (pay attention to θ in the third case).

(1) $(D, 0, 0)$ **(2)** $(0, -D, 0)$ **(3)** $(0, 0, D)$ **(4)** $(3, 4, 5)$

Exercice 28. From the limits of integration, describe each region and find its volume. The inner integral has the inner limits.

(1) $\displaystyle\int_{\theta=0}^{2\pi}\int_{r=0}^{1/\sqrt{2}}\int_{z=r}^{\sqrt{1-r^2}} r\,dz\,dr\,d\theta$

(2) $\displaystyle\int_{\theta=0}^{\pi}\int_{0}^{1}\int_{0}^{1+r^2} r\,dz\,dr\,d\theta$

(3) $\displaystyle\int_{\theta=0}^{2\pi}\int_{z=0}^{1}\int_{r=0}^{2-z} r\,dr\,dz\,d\theta$

(4) $\displaystyle\int_{\theta=0}^{\pi}\int_{0}^{\pi}\int_{0}^{\pi} r\,d\theta\,dr\,dz$

(5) $\displaystyle\int_{0}^{\pi/2}\int_{0}^{\pi/2}\int_{0}^{1} \rho^2\sin\phi\,d\rho\,d\phi\,d\theta$

(6) $\displaystyle\int_{0}^{2\pi}\int_{0}^{\pi/3}\int_{\sec\phi}^{2} \rho^2\sin\phi\,d\rho\,d\phi\,d\theta$

(7) $\displaystyle\int_{0}^{\pi}\int_{0}^{\pi}\int_{0}^{\sin\phi} \rho^2\sin\phi\,d\rho\,d\phi\,d\theta$

(8) $\displaystyle\int_{0}^{2\pi}\int_{0}^{\pi/4}\int_{1}^{3} \rho^2\sin\phi\,d\rho\,d\phi\,d\theta$

(9) $\displaystyle\int_{0}^{\pi}\int_{0}^{\pi}\int_{0}^{\pi} \rho^2\sin\phi\,d\rho\,d\phi\,d\theta$

(10) $\displaystyle\int_{0}^{1}\int_{0}^{1}\int_{0}^{1} \rho^2\sin\phi\,d\rho\,d\phi\,d\theta$