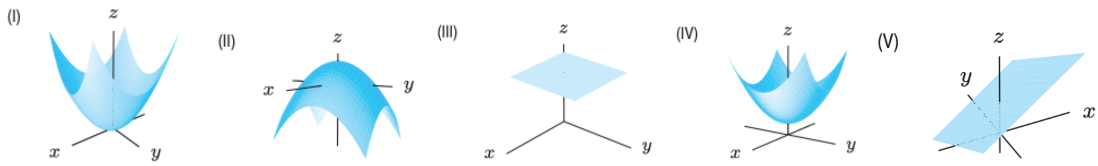
# Workshop session 1 (September 2024)

## Surfaces and levels sets

**Exercice 2.** Without a calculator, indicate which function corresponds to which figure below.

**(a)** $z = 2 + x^2 + y^2$ **(c)** $z = 2$ **(e)** $z = 2 + 2x - y$

**(b)** $z = 2(x^2 + y^2)$ **(d)** $z = 2 - x^2 - y^2$




**Exercice 5.** We consider the functions

$$f_1(x_1, x_2) = x_1^2 - x_2^2 \qquad \text{et} \qquad f_2(x_1, x_2) = 2x_1x_2$$

Sketch the level sets associated with $f_1(x_1, x_2) = 12$ and $f_2(x_1, x_2) = 16$ on the same diagram. Indicate on the diagram the values of $\boldsymbol{x} = [x_1, x_2]^\intercal$ for which $\boldsymbol{f}(\boldsymbol{x}) = [f_1(x_1, x_2), f_2(x_1, x_2)]^\intercal = [12, 16]^\intercal$ where the exponent $^\intercal$ indicate a column vector.

## Limits and continuity

**Exercice 6.** In the following cases, use algebraic techniques to evaluate the limit.

**(1)** $\displaystyle\lim_{(x,y)\to(0,0)} \frac{x^4 - 4y^4}{x^2 + 2y^2}$ **(2)** $\displaystyle\lim_{(x,y)\to(0,0)} \frac{x^2 - xy}{\sqrt{x} - \sqrt{y}}$ **(3)** $\displaystyle\lim_{(x,y,z)\to(0,0,0)} \frac{x^2 - y^2 - z^2}{x^2 + y^2 - z^2}$

**Exercice 11.** We consider the function $\langle\cdot|\cdot\rangle_2 : \mathbb{R}^2 \times \mathbb{R}^2 \to \mathbb{R}$, defined by

$$\langle \boldsymbol{x}|\boldsymbol{y}\rangle_2 = 2x_1y_1 + 3x_2y_1 + 3x_1y_2 + 5x_2y_2$$

where $\boldsymbol{x} = (x_1, x_2)^\intercal$ et $\boldsymbol{y} = (y_1, y_2)^\intercal$ are column vectors. After determining the matrix $\boldsymbol{Q}$ such that $\langle \boldsymbol{x}|\boldsymbol{y}\rangle_2 = \boldsymbol{x}^\intercal \boldsymbol{Q}^2 \boldsymbol{y}$, show the following results ($\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{z} \in \mathbb{R}^2$ and $\lambda \in \mathbb{R}$) :

**(1)** $\langle \boldsymbol{x}|\boldsymbol{x}\rangle_2 \geq 0$ **(3)** $\langle \boldsymbol{x} + \boldsymbol{y}|\boldsymbol{z}\rangle_2 = \langle \boldsymbol{x}|\boldsymbol{z}\rangle_2 + \langle \boldsymbol{y}|\boldsymbol{z}\rangle_2$

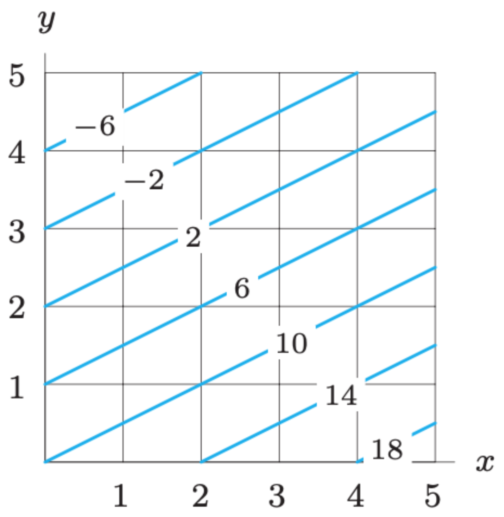
**(2)** $\langle \boldsymbol{x}|\boldsymbol{y}\rangle_2 = \langle \boldsymbol{y}|\boldsymbol{x}\rangle_2$ **(4)** $\langle \boldsymbol{x}|\lambda\boldsymbol{y}\rangle_2 = \lambda\langle \boldsymbol{x}|\boldsymbol{y}\rangle_2$

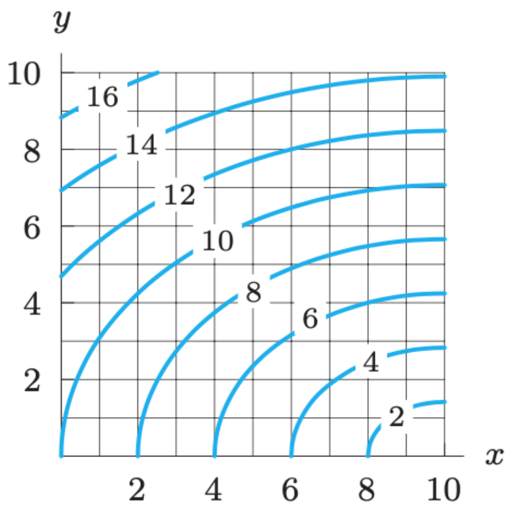
## The partial derivative

**Exercice 13.**

**(1)** The figure below on the left is a contour diagram for a function $z = f(x, y)$. $f_x$ is positive or negative? Same question for $f_y$. Estimate $f(2, 1)$, $f_x(2, 1)$ and $f_y(2, 1)$.

**(2)** Give an approximated value of $f_x(3, 5)$ using the contour diagram of $f(x, y)$ below on the right.




**Exercice 14.** Compute $f_{xx}, f_{xy} = f_{yx}$ and $f_{yy}$ for the following functions.

**(1)** $x^2 + 3xy + 2y^2$
**(2)** $(x + 3y)^2$
**(3)** $(x + iy)^3$
**(4)** $e^{ax+by}$
**(5)** $1/\sqrt{x^2 + y^2}$
**(6)** $(x + y)^n$
**(7)** $\cos ax \sin by$
**(8)** $1/(x + iy)$

## Tangent planes and linear approximation

**Exercice 17.** Find the tangent plane and the normal vector at $P$.

**(1)** $z = \sqrt{x^2 + y^2},\ P(0, 1, 1)$
**(2)** $x + y + z = 17,\ P(3, 4, 10)$
**(3)** $z = x/y,\ P(6, 3, 2)$
**(4)** $z = e^{x+2y},\ P(0, 0, 1)$
**(5)** $x^2 + y^2 + z^2 = 6,\ P(1, 2, 1)$
**(6)** $x^2 + y^2 + 2z^2 = 7,\ P(1, 2, 1)$
**(7)** $z = x^y,\ P(1, 1, 1)$
**(8)** $V = \pi r^2 h,\ P(2, 2, 8\pi)$