<u>Workshop session 3</u> (September 2024)

Double integrals

Exercice 29. Compute the following double integrals by two integrations

(1) $\displaystyle\int_{y=0}^{1}\int_{x=0}^{2} x^2\,dx\,dy$ and $\displaystyle\int_{y=0}^{1}\int_{x=0}^{2} y^2\,dx\,dy$ **(3)** $\displaystyle\int_{0}^{\pi/2}\int_{0}^{\pi/4} \sin(x+y)\,dx\,dy$ and $\displaystyle\int_{1}^{2}\int_{0}^{2} \frac{dx\,dy}{(x+y)^2}$

(2) $\displaystyle\int_{y=2}^{2e}\int_{x=1}^{e} 2xy\,dx\,dy$ and $\displaystyle\int_{y=2}^{2e}\int_{x=1}^{e} \frac{dx\,dy}{xy}$ **(4)** $\displaystyle\int_{0}^{1}\int_{1}^{2} y\,e^{xy}\,dx\,dy$ and $\displaystyle\int_{-1}^{1}\int_{0}^{3} \frac{dy\,dx}{\sqrt{3+2x+y}}$

Exercice 30. Draw the region and compute the area. Then invert the order of integration (mind the boundaries) and compute again the resulting area.

(1) $\displaystyle\int_{x=1}^{2}\int_{y=1}^{2x} dy\,dx$ **(3)** $\displaystyle\int_{0}^{\infty}\int_{e^{-2x}}^{e^{-x}} dy\,dx$ **(5)** $\displaystyle\int_{-1}^{1}\int_{y^2}^{1} dx\,dy$

(2) $\displaystyle\int_{0}^{1}\int_{x^3}^{x} dy\,dx$ **(4)** $\displaystyle\int_{-1}^{1}\int_{x^2-1}^{1-x^2} dy\,dx$ **(6)** $\displaystyle\int_{-1}^{1}\int_{x=y}^{|y|} dx\,dy$

Exercice 31. Compute the following integrals

(1) $\displaystyle\int_{0}^{b}\int_{0}^{a} \frac{\partial^2 f}{\partial x\partial y}\,dx\,dy$ **(2)** $\displaystyle\int_{0}^{b}\int_{0}^{a} \frac{\partial f}{\partial x}\,dx\,dy$

Change of variables

Exercice 11. The domain R is a disk share with: $0 \le r \le 1$ and $\pi/4 \le \theta \le 3\pi/4$.

(1) What is the area of R? Verify by integration in polar coordinates.

(2) Find bounds on $\iint dy\,dx$ to get area of ??R, and integrate. Extra: Find the limits on $\iint dx\,dy$.

(3) The equation (2.1) with $\alpha = \pi/4$ rotates R in the region uv. We then have $S =$ ______. Find bounds on $\iint du\,dv$.

(4) Calculate the height of the centroid $\overline{y}$ of R by changing $\iint y\,dx\,dy$ to polar coordinates. Divide by the area of R.

(5) The region R is characterized by $\overline{x} = 0$ because ______. After the rotation of $\alpha = \pi/4$, the center of gravity $(\overline{x}, \overline{y})$ of R becomes the center of gravity located in ______ of S.

(6) Find the center of any corner $0 \le r \le A$, $0 \le \theta \le b$.

Exercice 13. Using polar coordinates, find the volume below the surface of equation $z = x^2 + y^2$ above the unit disk $x^2 + y^2 \le 1$.