MF 796: Computational Methods of Mathematical Finance

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Problem set # 4

Due: Wednesday, February 19, by 8am

Problem 1: Covariance Matrix Decomposition: Download historical data from your favorite source for 5 years and at least 100 companies or ETFs. In this problem we will look at the covariance matrix for these assets and its properties.

1. Clean the data so that your input pricing matrix is as full as possible. Fill in any gaps using a reasonable method of your choice. Explain why you chose that particular method.
2. Generate a sequence of daily returns for each asset for each date.
3. Calculate the covariance matrix of daily returns and perform an eigenvalue decomposition on the matrix. How many positive eigenvalues are there? How many were negative? If any are negative, what happened?
4. How many eigenvalues are required to account for 50% of the variance? How about 90%? Does this make sense to you?
5. Using the number of eigenvalues in the 90% threshold above, create a return stream that represents the residual return after the principal components that correspond to these eigenvalues have been removed. Plot this return stream and comment on its properties.

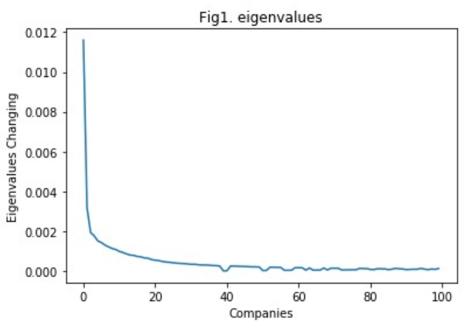
Problem 1.

1. In here, I download the data by Python, and the code at the attachment. When I get the data, I fill the gap use the latest available entry. This method can help the code to satefy running, and the use of latest data can reduce the generation of caculated error.

2. $$\text{Daily return} = \ln\left(\frac{P_t}{P_{t-1}}\right)$$

The code at the attachment.

3.



By the python code, I get the eigenvalues and the eigenvectors. By this figure, there are no negative eigenvalues exit in here.

4. 7 eigenvalues are required to account for 50% of the variance.

49 eigenvalues are required to account for 90% of the variance.

By CAPM model, a rational investor will chose larger portfolio to avoid the invest risk, and smaller portfolio will explain large proportion of risk fluctuations.

5. By the return stream, the return rate will generate relatively stable condition. Large proportion return will up to a stable condition, and it can be observed by the Fig2.

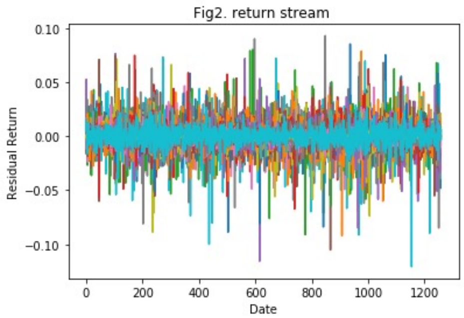


Fig2. return stream
0.10
0.05
0.00
−0.05
−0.10
Residual Return
0
200
400
600
800
1000
1200
Date

Problem 2: Portfolio Construction: In Lecture 7, we defined a Lagrangian for portfolio with constraints in matrix form by

$$L(w, \lambda) = \langle R, w \rangle - a \langle w, Cw \rangle - \langle \lambda, Gw - c \rangle \tag{1}$$

1. Form the matrix G by imposing the budget constraint, which is $\langle 1, w \rangle = 1$, and another constraint that allocates 10% of the portfolio to the first 17 securities (to simulate sector allocation). Using C from Problem 1, use your favorite method and the software package of your choice to invert $GC^{-1}G^T$ in a nice, stable way. (Hint: consider my favorite method).
2. What does the resulting portfolio look like? Would it be acceptable to most mutual funds? If not, what would you to do to fix that?

Problem 2:

$$L(w,\lambda) = \langle R, w\rangle - a\langle w, Cw\rangle - \langle \lambda, Gw - c\rangle$$
$$= \langle R, w\rangle - a\langle w, Cw\rangle - \langle \lambda, Gw\rangle + \langle \lambda, c\rangle$$
$$= \langle R, w\rangle - a\langle w, Cw\rangle - \langle G^T\lambda, w\rangle + \langle \lambda, c\rangle$$
$$\nabla_w L = R - 2aCw - G^T\lambda = 0$$
$$\nabla_\lambda L = c - Gw = 0$$
$$\Rightarrow w = \frac{1}{2a} C^{-1}(R - G^T\lambda)$$
$$2ac = GC^{-1}R - GC^{-1}G^T\lambda$$
$$\Rightarrow \lambda = (GC^{-1}G^T)^{-1} \cdot (GC^{-1}R - 2ac)$$

In this question, $\langle 1, w\rangle = 1$ and another constraint that allocates 10% of the portfolio to the first 17 securities.

(1) In this question, I use SVD to decomposition the return. Then get C by question 1, and it is covariance matrix. R represent the aveage return for the 100 companies. And a is risk aversion factor, I will make $a=1$. Then the λ and w can be calculated by the code.

(b) By the calculation, the portfolio weight mainly stabilized between -2 and 2. However, few special companies have a larger volatility, the condition due to some companies have a smaller adjusted price or the companies have just come into the market. This portfolio doesn't consider bonds and it can not be accepted by mutual funds because they don't usually short stock. Furthermore, I think that we can add some high volatility and flexibility securities to accepted mutual funds.