ECE 110 Cramming Carnival Review

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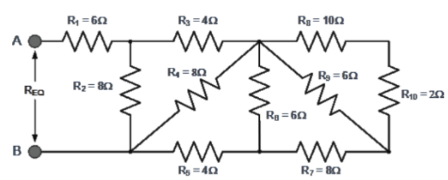
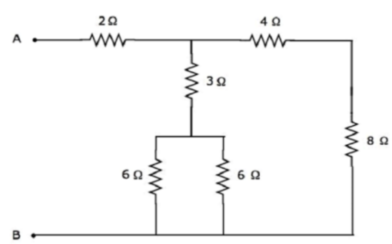
Introduction

This worksheet does not cover content in lectures after November 25th and is not meant to be a replacement for any practice exams or section reviews. Use this worksheet as a quick refresher for various topics throughout the semester and for slightly different questions than the homeworks.

Formulas not on the help sheet

Note: All formulas required for the questions are assumed to be known, as they are not provided in this sheet.





Power Efficiency & Capacitors

Question 1: Consider a car that has 400 kJ of energy at a specific speed. The car's regenerative brakes are 40% efficient at converting kinetic energy to energy stored in a battery. What is the energy added when the car brakes to half speed?

Solution:

$$\begin{aligned}
KE_o &= \frac{1}{2}mv^2 \\
KE_{\text{half}} &= \frac{1}{2}m\left(\frac{v}{2}\right)^2 = \frac{1}{2}m \cdot \frac{v^2}{4} = \frac{1}{4} \cdot \frac{1}{2}mv^2 = \frac{1}{4} \times KE_o \\
\Delta KE &= KE_o - KE_{half} = KE_o - \frac{1}{4}KE_o = \frac{3}{4}KE_o \\
&= \frac{3}{4} \cdot 400kJ = 300kJ \\
E_{added} &= \%_{eff} \cdot \Delta KE = 0.40 \cdot 300kJ \\
&= \boxed{120kJ}
\end{aligned}$$

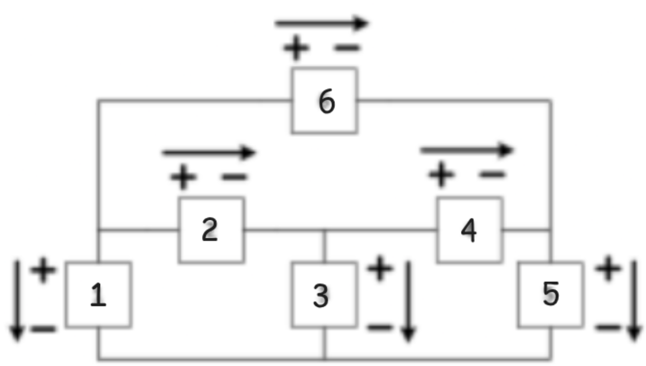
Question 2: If a 15 kWh battery has to be recharged using a 60% efficient generator with peak power of 500 W, how long does the generator need to run to fully charge the battery?

Question 3: What is the energy stored in a 4 nF capacitor charged to 9V?

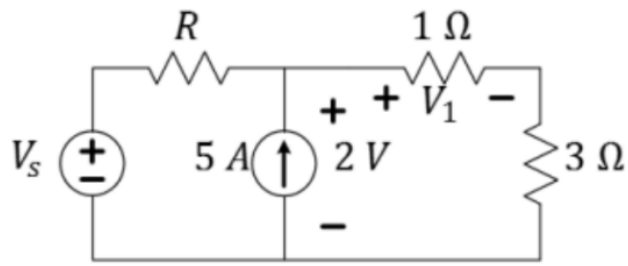
Question 4: What voltage is needed to charge the capacitor from the above question enough to lift a 2-gram mass 15 cm?

Kirchoff's Laws/Dividers

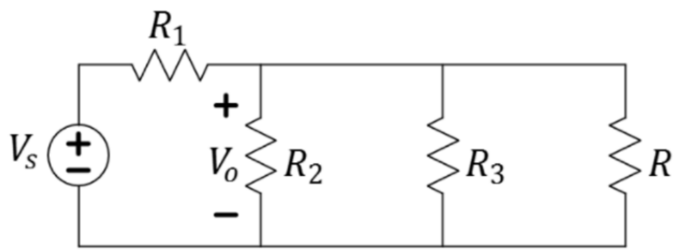
Question 5: Given the circuit below, find V1, V3, V6, I2, I4, and I5. ($V_2 = 16V, V_4 = 14V, V_5 = 20V, I_1 = 2A, I_3 = 3A, I_6 = 0A$)



Question 6: Find V1 in the circuit below.



Question 7: What value of R will result in Vo = 1V for the following circuit? (given: $R_1 = 9\Omega, R_2 = 10\Omega, R_3 = 15\Omega, V_s = 4V$)



1. We know that the voltage across parallel circuits is the same. As this is the case, if $V_0 = 1\,\mathrm{V}$, the voltage at R_1 is $3\,\mathrm{V}$, since $V_s = 4\,\mathrm{V}$.

2. By the voltage divider rule, for R_1 to be $3\,\mathrm{V}$, the resistance of the three resistors in parallel must be $\frac{1}{3}$ of that of R_1. Since $R_1 = 9\,\Omega$, the combination of R_2, R_3, and R in parallel must be $3\,\Omega$.

3. Thus:

$$3 = \left(\frac{1}{10} + \frac{1}{15} + \frac{1}{R}\right)^{-1}.$$

Solving for R, we find:

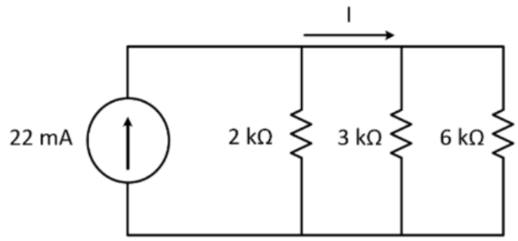
$$\frac{1}{R} = \frac{1}{3} - \left(\frac{1}{10} + \frac{1}{15}\right),$$

$$\frac{1}{R} = \frac{1}{3} - \frac{1}{6},$$

$$\frac{1}{R} = \frac{2}{6} - \frac{1}{6} = \frac{1}{6}.$$

$$R = 6\,\Omega.$$

Question 8: Find I in the circuit below.



1. Combine the right two resistors into a single resistor. The resistances are 6 kΩ and 3 kΩ in parallel:

$$R_{\text{eq}} = \left(\frac{1}{6} + \frac{1}{3}\right)^{-1} \text{k}\Omega = \left(\frac{1}{6} + \frac{2}{6}\right)^{-1} \text{k}\Omega = \left(\frac{3}{6}\right)^{-1} \text{k}\Omega = 2\text{k}\Omega.$$

2. Use the current divider rule for the two 2 kΩ resistors in parallel to obtain the current:

$$I_1 = I_{\text{total}} \cdot \frac{R_2}{R_1 + R_2},$$

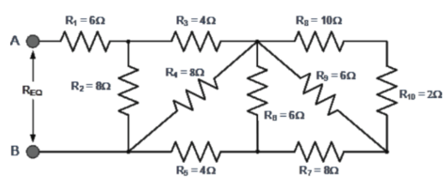
where $R_1 = R_2 = 2\text{k}\Omega$.

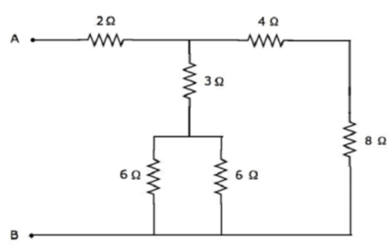
Substituting values:

$$I_1 = 22 \text{ mA} \cdot \frac{2}{2+2} = 22 \text{ mA} \cdot \frac{1}{2} = 11 \text{ mA}.$$

Equivalent Resistance / Power

Question 9: Find equivalent resistance for the circuits below.





Part 1:

We calculate the equivalent resistance step by step:

1. R_8 and R_{10} are in series: $R_{s1} = R_8 + R_{10} = 12\,\Omega$.
2. R_{s1} and R_9 are in parallel: $R_{p1} = \left(\frac{1}{R_{s1}} + \frac{1}{R_9}\right)^{-1} = 4\,\Omega$.
3. R_{p1} and R_7 are in series: $R_{s2} = R_{p1} + R_7 = 12\,\Omega$.
4. R_{s2} and R_6 are in parallel: $R_{p2} = \left(\frac{1}{R_{s2}} + \frac{1}{R_6}\right)^{-1} = 4\,\Omega$.
5. R_{p2} and R_5 are in series: $R_{s3} = R_{p2} + R_5 = 8\,\Omega$.
6. R_{s3} and R_4 are in parallel: $R_{p3} = \left(\frac{1}{R_{s3}} + \frac{1}{R_4}\right)^{-1} = 4\,\Omega$.
7. R_{p3} and R_3 are in series: $R_{s4} = R_{p3} + R_3 = 8\,\Omega$.
8. R_{s4} and R_2 are in parallel: $R_{p4} = \left(\frac{1}{R_{s4}} + \frac{1}{R_2}\right)^{-1} = 4\,\Omega$.
9. R_{p4} and R_1 are in series: $R_{s5} = R_{p4} + R_1 = 10\,\Omega$.

Thus, the total equivalent resistance is $R_{\text{eq}} = 10\,\Omega$.

Part 2:
For the second circuit:
1. The 4Ω and 8Ω resistors are in series: $R_1 = 4\,\Omega + 8\,\Omega = 12\,\Omega$.
2. Two 6Ω resistors are in parallel: $R_2 = \left(\frac{1}{6} + \frac{1}{6}\right)^{-1} = 3\,\Omega$.
3. The 3Ω from Step 2 and another 3Ω resistor are in series: $R_3 = 3\,\Omega + 3\,\Omega = 6\,\Omega$.
4. The 6Ω from Step 3 and the 12Ω from Step 1 are in parallel: $R_4 = \left(\frac{1}{6} + \frac{1}{12}\right)^{-1} = 4\,\Omega$.
5. The 4Ω from Step 4 and the 2Ω resistor are in series: $R_{\text{eq}} = 4\,\Omega + 2\,\Omega = 6\,\Omega$.

Thus, the total equivalent resistance is $R_{\text{eq}} = 6\,\Omega$.

Question 10: If the voltage between nodes A and B in the second circuit is 9V...

1. What is the current through the 3 ohm resistor?

   We know from Question 9 that the equivalent resistance $R_{\text{eq}} = 6\,\Omega$. Using Ohm's law, the total current flowing through the circuit is:

   $$I_{\text{total}} = \frac{V}{R_{\text{eq}}} = \frac{9V}{6\,\Omega} = 1.5\,\text{A}.$$

   Using the current divider rule, we can find the current through the middle branch, which has a resistance of 6Ω (the combination of 3Ω and two 6Ω resistors in parallel):

   $$R_{\text{middle}} = 3\,\Omega + \left(\frac{1}{6} + \frac{1}{6}\right)^{-1} = 3\,\Omega + 3\,\Omega = 6\,\Omega.$$

   The current divides between the middle branch and the right branch with resistance 12Ω (the 4Ω and 8Ω resistors in series). By the current divider rule:

   $$I_{\text{middle}} = I_{\text{total}} \times \frac{R_{\text{right}}}{R_{\text{middle}} + R_{\text{right}}} = 1.5\,\text{A} \times \frac{12\,\Omega}{6\,\Omega + 12\,\Omega} = 1\,\text{A}.$$

Thus, the current through the 3Ω resistor is $1\,\mathrm{A}$.

2. What is the power through the 3 ohm resistor?

   The power dissipated through a resistor is given by $P = I^2R$. For the 3Ω resistor:
   $$P_3 = I_{3\Omega}^2 \times 3\,\Omega = (1\,\mathrm{A})^2 \times 3\,\Omega = 3\,\mathrm{W}.$$

3. What is the power through the 8 ohm resistor?

   The current through the 8Ω resistor is the same as the current through the right branch, which is:
   $$I_{8\Omega} = I_{\text{total}} - I_{\text{middle}} = 1.5\,\mathrm{A} - 1\,\mathrm{A} = 0.5\,\mathrm{A}.$$
   The power dissipated through the 8Ω resistor is:
   $$P_8 = I_{8\Omega}^2 \times 8\,\Omega = (0.5\,\mathrm{A})^2 \times 8\,\Omega = 2\,\mathrm{W}.$$

4. What resistor has the highest power output?

   Since power is given by $P = I^2R$, we can analyze the power dissipated in each resistor.

   - The 4Ω resistor and the 8Ω resistor have the same current flowing through them. Since the current is the same, we know the 4Ω resistor does not have the highest power output.
   - The 3Ω resistor dissipates more power than the 8Ω resistor, as we computed earlier, and the power dissipated in each of the 6Ω resistors in parallel is $P = 0.5^2 \times 6 = \frac{3}{2}\,\mathrm{W}$.
   - The power through the 2Ω resistor is $P = 1.5^2 \times 2 = 4.5\,\mathrm{W}$.

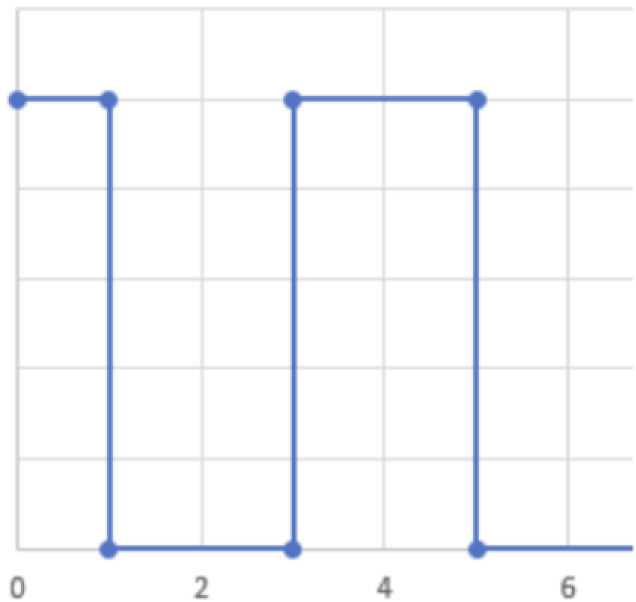
   Thus, the 2Ω resistor has the highest power output.

PWM

Question 11: Imagine a square wave that outputs 15W from 0 to 12 seconds and 5W from 12 to 20 seconds. This square wave corresponds to a 10 ohm resistor.

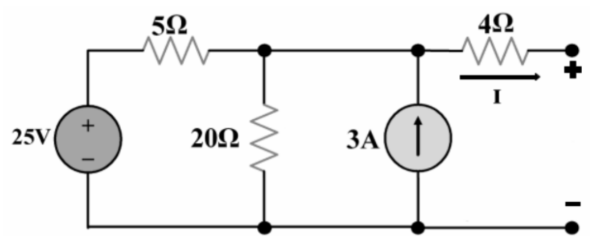
1. What is the Average Power of this waveform?
2. What is the RMS Voltage of this waveform?

Question 12: Given a limited portion of this graphed waveform, what is its duty cycle?


0
2
4
6

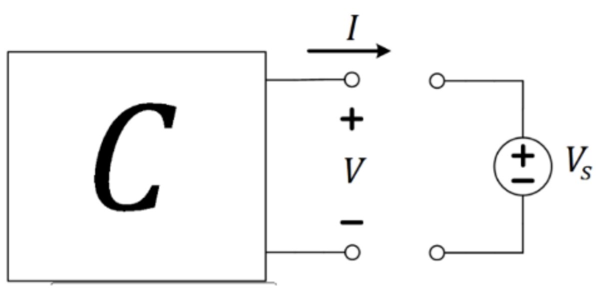
I-V Equations

Question 13: What is the short circuit current and the open circuit voltage for the circuit below?



Question 14: If this circuit were to be placed in series with another circuit with an IV equation of $I = 0.005V - 0.025$, assuming the same polarities given above, what would be the operating current and voltage?

Question 15: If the open circuit voltage of a circuit containing ideal sources and resistors is measured at Voc = 8 V, while the current through the short circuit across the circuit is Isc = 200 mA, what would be the power in watts absorbed by an ideal voltage source, Vs = 4, placed across the terminals?

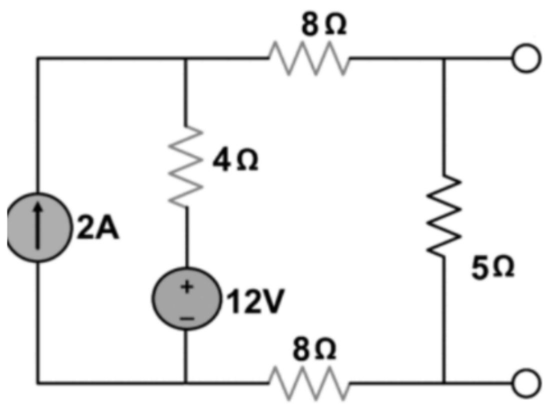


Norton and Thevenin

Question 16: Give Norton and Thevenin Forms for the subcircuit shown on the previous page.

Question 17: What is the Norton resistance of the circuit below? What is the Thevenin Resistance?

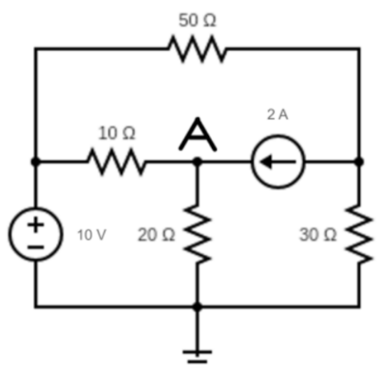
Question 18: Draw the Thevenin and Norton Equivalents.



Nodal Analysis

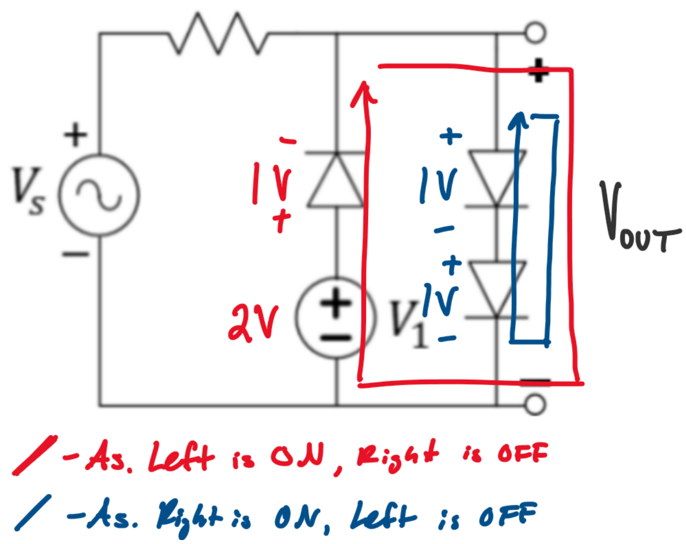
Question 19: Find the voltage at node A for this circuit.

Question 20: Find the voltage drop across the 10, 30, and 50 ohm resistors.



Diodes

Question 21: Assume an ideal-offset model and Von = 1 volt. If Vs = 5 cos(ωt) volts and V1 = 2 volts, what are the maximum and minimum voltages across the open nodes?



Solution: Both branches of the diode cannot be ON at the same time, as if positive current flows through one, negative current would flow through the other (which is impossible). We start by solving for V_{out} assuming one diode configuration is ON (below, we start with the left branch) while the other is OFF.

KVL Loops are shown above with assumptions the Voltage source has extremes high or low enough that they will conduct current.

$$V_{out} = 2V - 1V = 1V.$$

For current to flow in the correct direction, the voltage on the right of the resistor must have a larger value than the voltage on the left. We have determined $V_{out} = 1V$, we now check whether V_S can be smaller than that number.

$$V_{S\,\min} = -5V < 1V \Rightarrow \boxed{V_{out\,\min} = 1V}$$

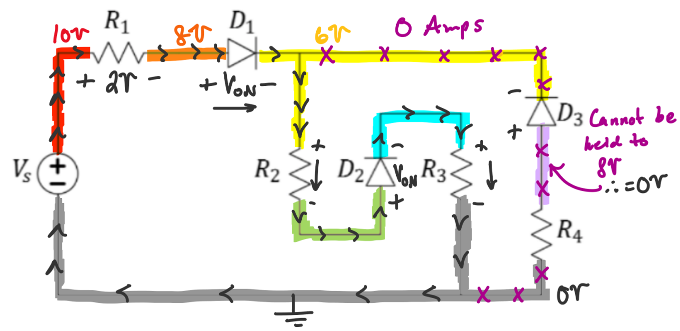
Restart this process with the KVL loop with the right branch of diodes.

$$V_{out} = 1V + 1V = 2V$$

Current must flow in the resistor from left to right to satisfy the diode assumption. Find a value of V_S that is greater than our calculated V_{out}

$$V_{S\,\max} = 5 > 2V \Rightarrow \boxed{V_{out\,\max} = 2V}$$

Question 22: In the circuit below, which diodes are on? Furthermore, if Vs = 10 V, all the diodes have Von = 2V under the offset ideal model, and the voltage drop over R1 is also 2V, what is the voltage drop across the other resistors, assuming they have an equal resistance?



Solution:
When looking at D_3, we notice two things:

- The **positive** terminal of D_3 is connected to the **negative** terminal of V_S through a resistor.
- There is only one source of power in the circuit (V_S). (Because of this, the smallest nodal voltage that any node could be held at is 0V (the negative terminal of the V_S))

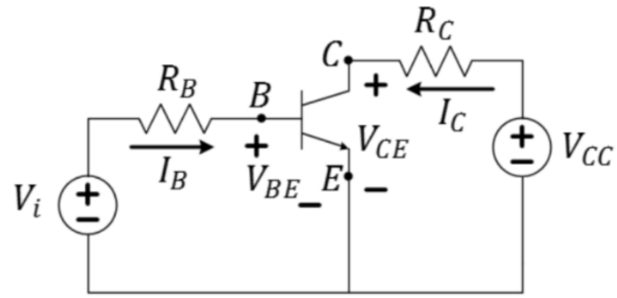
Thus, it is impossible for the voltage difference across the diode, V_{D_3}, to be greater than $0V$. In fact, if we assume D_1 to be on, we can calculate that $V_{yellow} = V_S - V_{R_1} - V_{ON} = 6V$. V_{pink} would need to be $2V$ greater than V_{yellow} in order for D_3 to turn on. Since a resistor is a passive component (adds no power to the circuit), it is impossible to raise the pink node to above $6V + 2V = 8V$, which would allow D_3 to turn on

Since D_3 is not conducting any current, $I_{R_1} = I_{R_2} = I_{R_3} = I_{D_1} = I_{D_2} > 0A$. We know the currents will be positive since R_1's voltage drop is positive and moving away from the voltage source (no other supply of power in the circuit). Because the currents are all positive, we can conlcude that $\boxed{D_1 \text{ and } D2 \text{ are ON}}$. For any two resistors of equal resistance and equal current, we know that their voltage drops must also be equal. Algebraically, we can show this as follows:

$$I_{R_1} = I_{R_2} = I_{R_3}$$
$$\text{Using Ohm's Law: } \frac{2V}{R} = \frac{V_{R_2}}{R} = \frac{V_{R_3}}{R}$$
$$2V = V_{R_2} = V_{R_3}$$
$$V_{R_2} = \boxed{2V}$$
$$V_{R_3} = \boxed{2V}$$

BJTs

Question 23: The properties of the transistor are that VBE on is 1V, β is 120, and VCE sat is 0.2 V. In this circuit, VCC is 9V, RC is 150Ω, and RB is 30000Ω. What are the maximum and minimum values for VCE if Vi's output is variable between 0V and 9V?



Solution:

An important property of BJTs is that V_{CE} can only ever assume the values ranging from $[V_{CE,sat}, V_{CC}]$. A great graph to view that displays the behavior of a BJT is Figure 6 of the Canvas Module, "BJT Applications."

Calculate V_{CE} assuming that the BJT is in the ON, ACTIVE region for both extreme values of V_i:

- If $I_C \leq 0$ (equivalent statements: $V_{CE} > V_{CC}$, $V_i < V_{BE,ON}$), then the assumption that the BJT is ON is incorrect, and V_{CE}'s actual value is V_{CC}.
- If $V_{CE,sat} < V_{CE} < V_{CC}$, then the assumption that the BJT is in the ON, ACTIVE region is correct and the calculated value of V_{CE} is correct.
- If $V_{CE} \leq V_{CE,sat}$, then the BJT is in the ON, SATURATED region and V_{CE}'s actual value is $V_{CE,sat}$.

We see that V_I gets below the value of $V_{BE,ON}$, so we know that

$$\boxed{V_{CE,max} = V_{CC} = 9V}.$$

Assuming the BJT is in the active region:

$$\begin{aligned}
I_C &= \beta I_B \\
\frac{V_{CC} - V_{CE,min}}{R_C} &= \beta \frac{V_{i,max} - V_{BE,on}}{R_B} \\
\Rightarrow V_{CE,min} &= V_{CC} - \beta \frac{R_C(V_{i,max} - V_{BE,on})}{R_B} \\
V_{CE,min} &= 9V - (120)\frac{(150\Omega)(9V - 1V)}{30k\Omega} \\
V_{CE,min} &= \boxed{4.2V} \text{ (within bounds } V_{CE,sat} < V_{CE} < V_{CC}.)
\end{aligned}$$

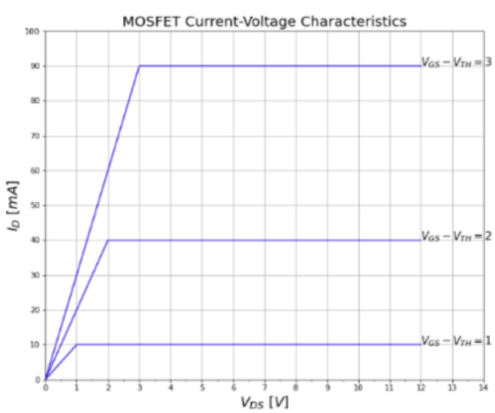
Question 24: If Vi was set to 5V, what would VCE be?
Solution:
Assume that the BJT is in the active region:

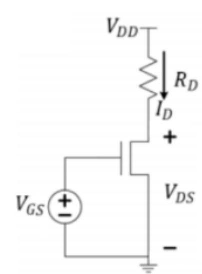
$$
\begin{aligned}
I_C &= \beta I_B \\
\frac{V_{CC} - V_{CE}}{R_C} &= \beta \frac{V_i - V_{BE,on}}{R_B} \\
\Rightarrow V_{CE} &= V_{CC} - \beta \frac{R_C(V_i - V_{BE,on})}{R_B} \\
V_{CE} &= 9V - (120)\frac{(150\Omega)(5V - 1V)}{30k\Omega} \\
&= \boxed{6.6V} \text{ (within bounds } V_{CE,sat} < V_{CE} < V_{CC}.)
\end{aligned}
$$

Since V_{CE} is in the range $V_{CE,sat} < V_{CE} < V_{CC}$, we know our assumption that the BJT was in the active region is correct, and our calculated V_{CE} is correct.

MOSFETs/cMOS logic

Question 25: An IC dissipates 110W. If the IC has a 5% activity factor α, frequency of 10GHz, and 1nF gate capacitance, what is the maximum number of transistors that can be in the IC if it can operate at up to 9V?





Question 26: The given circuit with a MOSFET in series with a voltage source of 6V and a resistor with a resistance of 120Ω Find the transistor parameter k and a value for VDS that results in I = 30 mA, given that VGS - VTH = 2.

Bonus Questions

Question 27: Give the IV equation, Norton equivalent, and Thevenin Equivalent for the circuit below.