

HKN ECE 342 Final Worksheet - Cramming Carnival - Key

DC Analysis

Transistor Parameters

MOSFETs:

$$\mu_n C_{ox} = 100 \mu\text{A}/\text{V}^2; V_{TN} = 1 \text{ V}$$

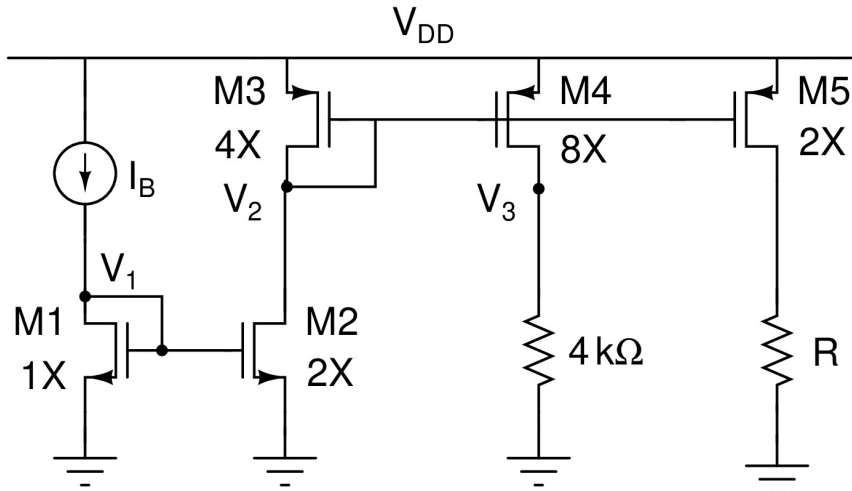
$$\mu_p C_{ox} = 50 \mu\text{A}/\text{V}^2; |V_{TP}| = 1 \text{ V}$$

BJTs:

$$\beta = 99; V_{BE, \text{on}} = 0.7 \text{ V}$$

Problem 1

For this problem, refer to the circuit below. Use $V_{DD} = 5 \text{ V}$, $I_B = 200 \mu\text{A}$, $1X = \frac{100}{1}$.



(a) Determine the DC voltages V_1 , V_2 , and V_3 .

$$V_{OV1} = \sqrt{\frac{2I_B}{\mu_n C_{ox}(100/1)}} = 0.2 \text{ V}$$

$$V_1 = V_{OV1} + V_{TN}$$

$$\boxed{V_1 = 1.2 \text{ V}}$$

$$I_{D2} = 2I_B = 2(200) \mu\text{A}$$

$$V_{OV3} = 0.2 \text{ V}$$

$$V_2 = V_{DD} - V_{OV3} - |V_{TP}|$$

$$\boxed{V_2 = 3.8 \text{ V}}$$

$$I_{D4} = 4I_B = 800 \mu\text{A}$$

$$V_3 = I_{D4}(4\text{k})$$

$$\boxed{V_3 = 3.2 \text{ V}}$$

(b) Determine the value of R such that M5 is biased at the edge of saturation.

$$I_{D5} = I_B = 200 \mu\text{A}$$

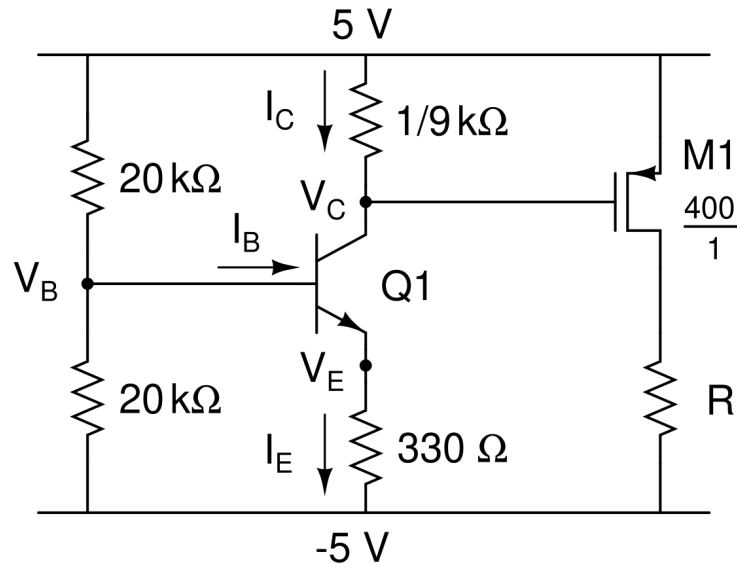
$$V_{OV5} = 5 - I_{D5}R = 0.2 \text{ V}$$

$$R = \frac{4.8}{200\mu}$$

$$\boxed{R = 24 \text{ k}\Omega}$$

Problem 2

For this problem, refer to the circuit below.



- (a) Determine the DC currents and voltages V_B , V_E , V_C , I_B , I_E , and I_C of Q1. What is its region of operation?

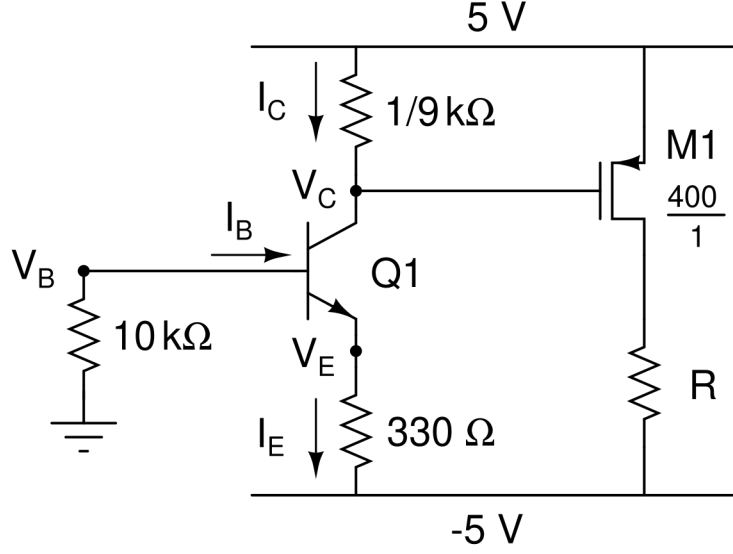
Solution:

Convert the biasing network at the base of Q1 to its Thevenin equivalent:

$$V_{TH} = (5 - (-5)) \left(\frac{20k}{20k + 20k} \right) - 5 = 0 \text{ V}$$

$$R_{TH} = 20k || 20k = 10 \text{ k}\Omega$$

The circuit can then be simplified to be:



$$I_E = \frac{-I_B(10k) - 0.7 - (-5)}{330}$$

$$330(\beta + 1)I_B = -I_B(10k) - 0.7 - (-5)$$

$$I_B(33k + 10k) = 4.3$$

$$I_B = 100 \mu A$$

$$I_E = (\beta + 1)I_B = 10 \text{ mA}$$

$$I_C = \beta I_B = 9.9 \text{ mA}$$

$$V_B = -(10k)I_B = -1 \text{ V}$$

$$V_E = -5 + (330)I_E = -1.7 \text{ V}$$

$$V_C = 5 - (1/9k)I_C = 3.9 \text{ V}$$

$$V_{CE} = 5.6 \text{ V} > V_{BE} = 0.7 \text{ V}$$

Q1 is operating in forward active mode.

(b) Determine the value of R such that M1 is biased at the edge of saturation.

$$V_{OV1} = 5 - V_C - |V_{TP}| = 0.1$$

$$I_{D1} = \frac{\mu_p C_{ox}}{2} \left(\frac{400}{1} \right) V_{OV1}^2 = \frac{5 - V_{OV1} - (-5)}{R}$$

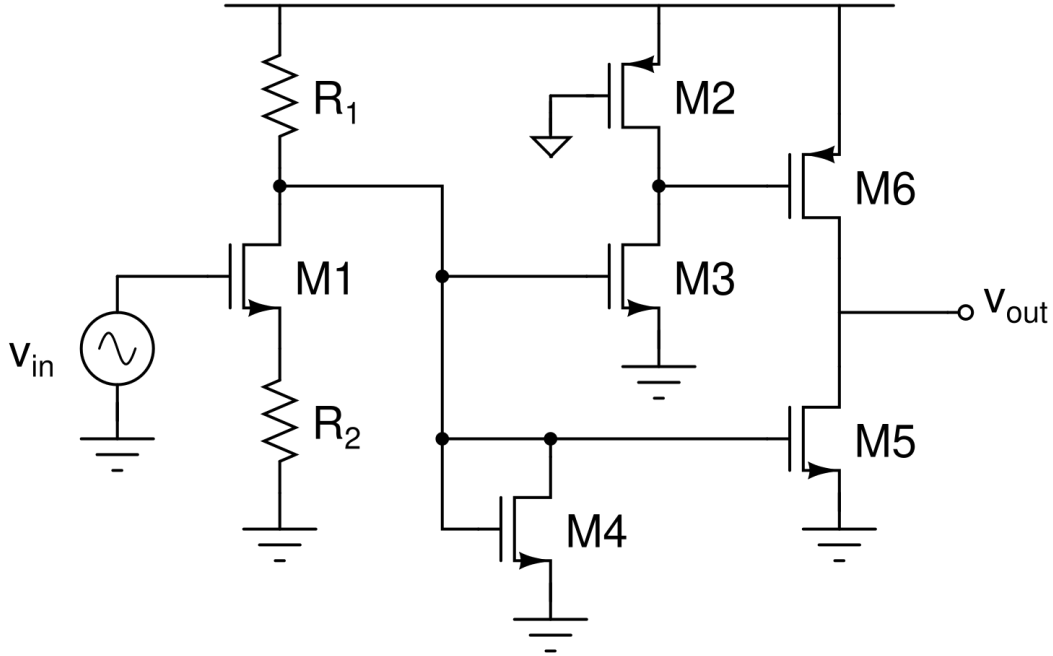
$$100\mu R = 9.9$$

$$R = 99 \text{ k}\Omega$$

Small-Signal Analysis

Problem 1

Determine the values of G_M , R_{OUT} , and $A_v = v_{out}/v_{in}$ of this amplifier. Assume all MOSFETs are biased in saturation. Do not assume $r_{ds} = \infty$, though you can assume $g_m r_{ds} \gg 1$.



Solution: For such problems, it is often advantageous to break down the problem into multiple stages, find the gain of that particular part, and multiply with gain of subsequent state, etc. Let A_{v1} be defined as gain at the drain of M1. Then,

$$G_{M1} = \frac{g_{m1}}{(1 + g_{m1}R_2)}$$

$$R_{OUT1} = (1/g_{m4}) || (R_1) || (r_{ds1} + R_2 + g_{m1}r_{ds1}R_2)$$

$$A_{v1} = -G_{M1}R_{OUT1}$$

Let A_{v3} be defined as gain at the drain of M3:

$$G_{M3} = g_{m3}$$

$$R_{OUT3} = r_{ds2} || r_{ds3}$$

$$A_{v3} = -G_{M3}R_{OUT3}$$

Then by $i_d = g_m v_{gs}$:

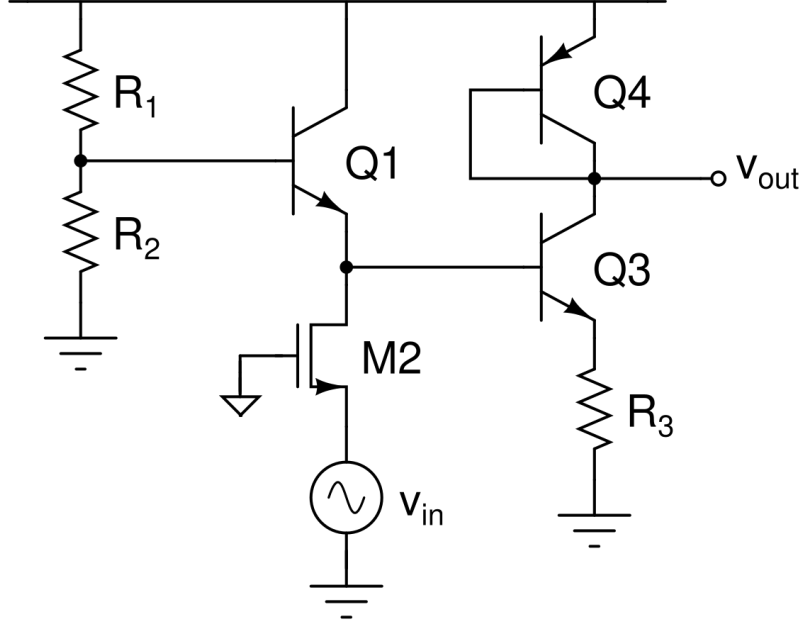
$$G_M = A_{v1}A_{v3}g_{m6} + A_{v1}g_{m5}$$

$$R_{OUT} = r_{ds5} || r_{ds6}$$

$$A_v = -G_MR_{OUT}$$

Problem 2

Determine the values of G_M , R_{OUT} , and $A_v = v_{out}/v_{in}$ of this amplifier. Assume all MOSFETs are biased in saturation, all BJTs are biased in forward active mode, $r_{ds} \neq \infty$, $r_0 = \infty$, and $g_m r_{ds} \gg 1$.

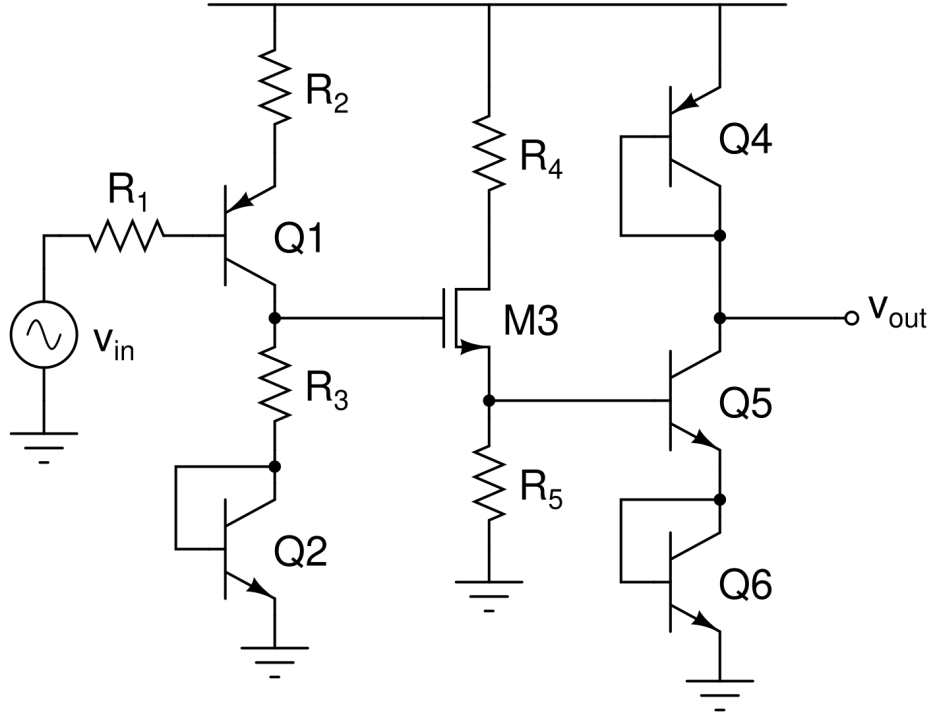


Solution:

$$\begin{aligned}
 G_{M1} &= -g_{m2} \\
 R_{OUT1} &= r_{ds2} \parallel \left(\frac{R_1 \parallel R_2 + r_{\pi 1}}{\beta + 1} \right) \\
 A_{v1} &= -G_{M1} R_{OUT1} \\
 G_{M2} &= \frac{1}{\frac{R_{OUT1} + r_{\pi 3}}{\beta} + \frac{\beta + 1}{\beta} R_3} \\
 R_{OUT2} &= \frac{r_{\pi 4}}{\beta + 1} \\
 A_{v2} &= -G_{M2} R_{OUT2} \\
 A_v &= A_{v1} A_{v2}
 \end{aligned}$$

Problem 3

Determine the values of G_M , R_{OUT} , and $A_v = v_{out}/v_{in}$ of this amplifier. Assume all MOSFETs are biased in saturation, all BJTs are biased in forward active mode, $r_{ds} \neq \infty$, $r_0 = \infty$, and $g_m r_{ds} \gg 1$.



Solution: At the gate of M3,

$$G_{M1} = \frac{1}{\frac{R_1 + r_{\pi 1}}{\beta_1} + \frac{\beta_1 + 1}{\beta_1} R_2}$$

$$R_{OUT1} = \frac{r_{\pi 2}}{\beta_2 + 1} + R_3$$

$$A_{v1} = -G_{M1} R_{OUT1}$$

At the source of M3,

$$G_{M2} = -\frac{g_{m3}}{1 + \frac{R_4}{r_{ds3}}}$$

$$R_{OUT2} = \frac{1}{g_{m3}} \left(1 + \frac{R_4}{r_{ds3}} \right) || R_5$$

$$A_{v2} = -G_{M2} R_{OUT2}$$

The last branch is basically repeat of first branch.

$$G_{M3} = \frac{1}{\frac{r_{\pi 5} + R_{OUT2}}{\beta_5} + \frac{\beta_5 + 1}{\beta_5} R_y}$$

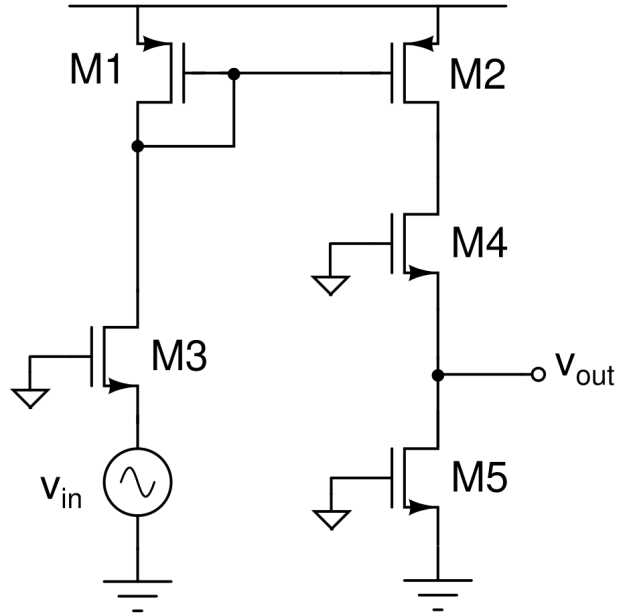
$$R_y = \frac{r_{\pi 6}}{\beta_6 + 1}$$

$$A_{v3} = -G_{m3} R_{out3}$$

$$A_v = A_{v1} A_{v2} A_{v3}$$

Problem 4

Determine the values of G_M , R_{OUT} , and $A_v = v_{out}/v_{in}$ of this amplifier. Assume all MOSFETs are biased in saturation. Do not assume $r_{ds} = \infty$, though you can assume $g_m r_{ds} \gg 1$.



$$G_{M1} = -g_{m3}$$

$$R_{OUT1} = r_{ds3} \parallel \left(\frac{1}{g_{m1}} \right)$$

$$A_{v1} = -G_{M1} R_{OUT1}$$

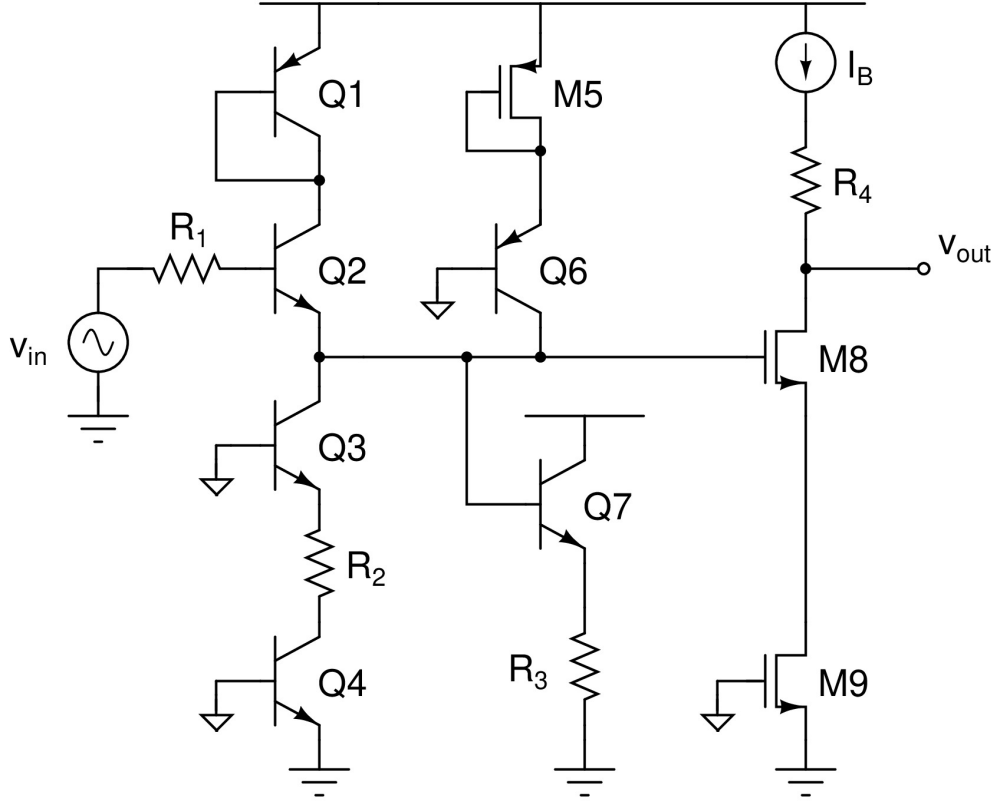
$$G_M = A_{v1} g_{m2}$$

$$R_{OUT} = r_{ds5} \parallel \left(\frac{1}{g_{m4}} \left(1 + \frac{r_{ds2}}{r_{ds4}} \right) \right)$$

$$A_v = -G_M R_{OUT}$$

Problem 5

Determine the values of G_M , R_{OUT} , and $A_v = v_{out}/v_{in}$ of this amplifier. Assume all MOSFETs are biased in saturation, all BJTs are biased in forward active mode, $r_{ds} \neq \infty$, $r_0 = \infty$, and $g_m r_{ds} \gg 1$.



$$G_{M1} = -\frac{1}{\frac{R_1 + r_{\pi 2}}{\beta + 1}}$$

$$R_{OUT1} = \left(\frac{R_1 + r_{\pi 2}}{\beta + 1} \right) \parallel [r_{\pi 7} + (\beta + 1)R_3]$$

$$A_{v1} = -G_{M1}R_{OUT1}$$

$$G_M = A_{v1} \frac{g_{m8}}{1 + g_{m8}r_{ds9}}$$

$$R_{OUT} = r_{ds8} + (1 + g_{m8}r_{ds8})r_{ds9}$$

$$A_v = G_MR_{OUT}$$

Bode Plots

Problem 1

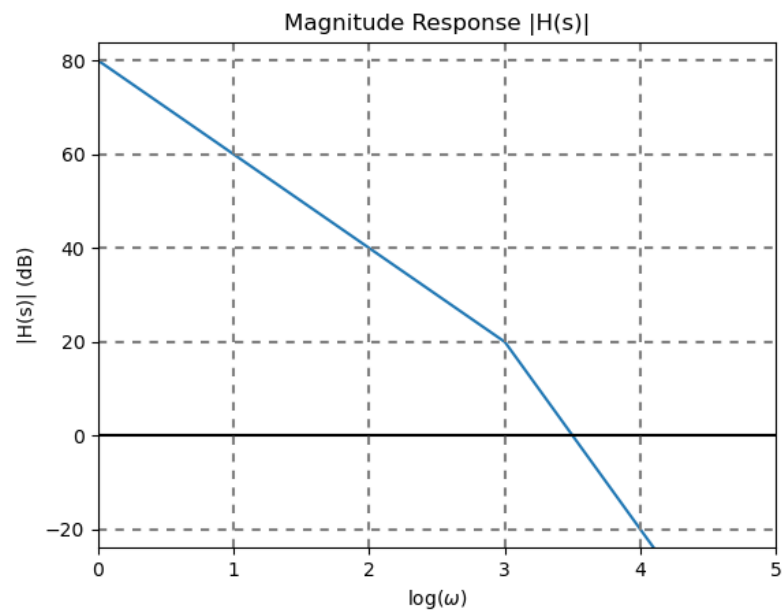
For the following amplifier transfer functions, **(i)** plot the magnitude response, **(ii)** determine the unity gain frequency, and **(iii)** plot the phase response:

(a)

$$H(s) = \frac{10^4}{s(1 + s/10^3)}$$

Solution:

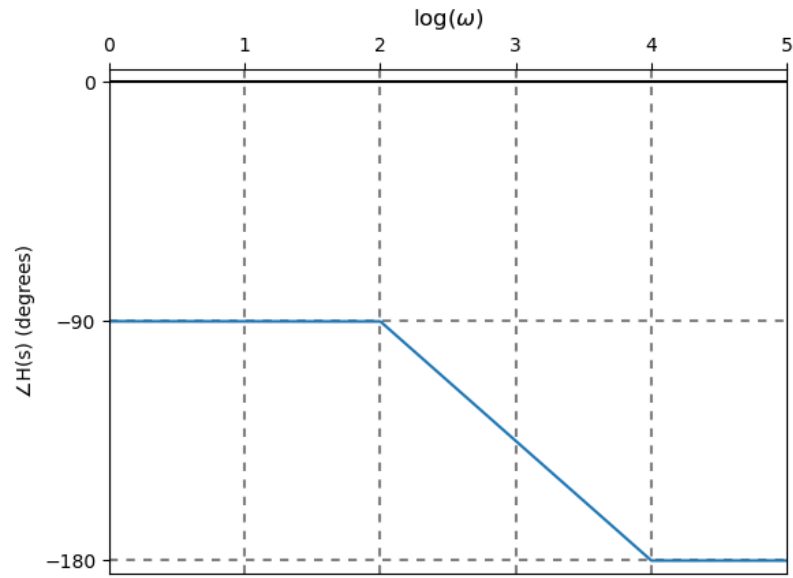
(i) The magnitude response is plotted below:



(ii)

$$\omega_{ugf} = \sqrt{10} \cdot 10^3 \text{ rad/s}$$

(iii) The phase response is plotted below:

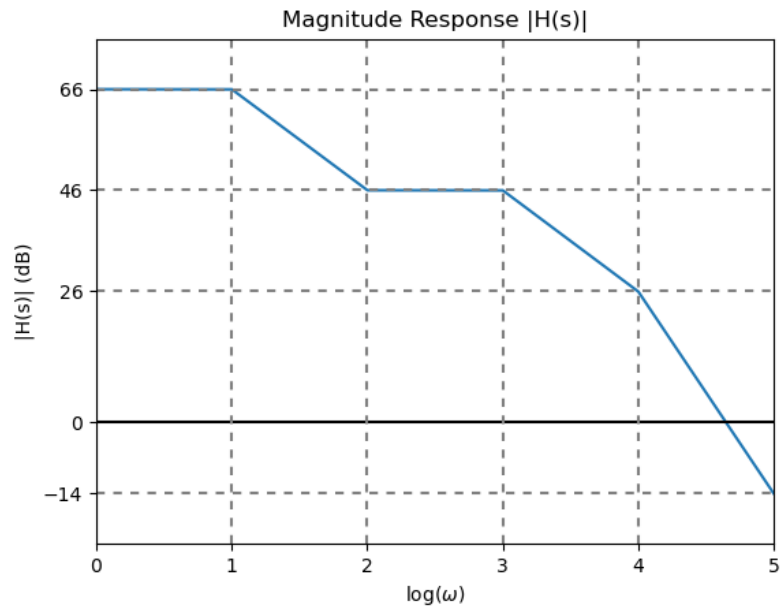


(b)

$$H(s) = \frac{2000 \left(1 + \frac{s}{10^2}\right)}{\left(1 + \frac{s}{10}\right) \left(1 + \frac{s}{10^3}\right) \left(1 + \frac{s}{10^4}\right)}$$

Solution:

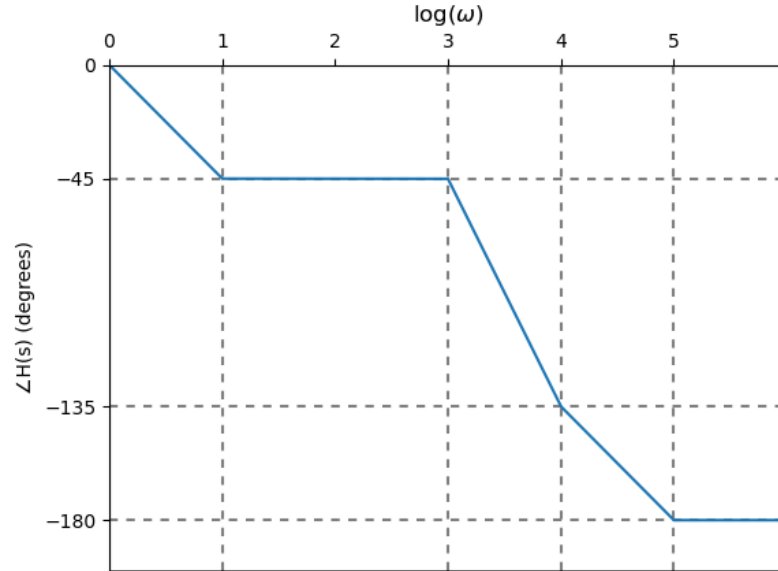
(i) The magnitude response is plotted below:



(ii) After $\omega = 10^4$ rad/s, the magnitude response decreases at 40 dB / decade, or 12 dB / octave. Thus, $|H(s)|_{dB} = 6$ dB at $\omega = \sqrt{10} \cdot 10^4$ rad/s. Since the magnitude response decreases at 12 dB / octave, $|H(s)|_{dB}$ will be 0 dB at $\omega = \sqrt{2} \cdot \sqrt{10} \cdot 10^4$ rad/s.

$$\omega_{ugf} = \sqrt{20} \cdot 10^4 \text{ rad/s}$$

(iii) The phase response is plotted below:

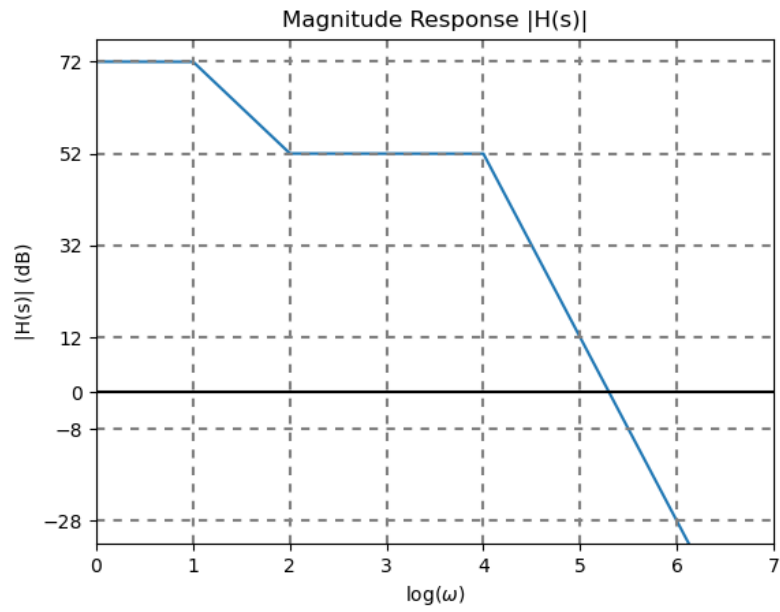


(c)

$$H(s) = \frac{4000(1 + s/10^2)}{(1 + s/10)(1 + s/10^4)^2}$$

Solution:

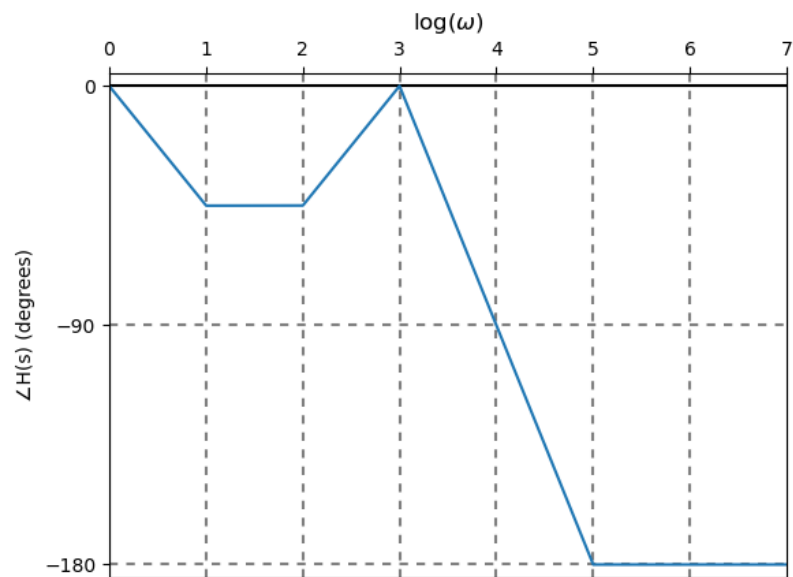
(i) The magnitude response is plotted below:



(ii)

$$\omega_{ugf} = 2 \cdot 10^5 \text{ rad/s}$$

(iii) The phase response is plotted below:



Problem 2

For each of the amplifier transfer functions in Problem 1, determine the incremental output voltage response $v_{out}(t)$ to an incremental input voltage $v_{in}(t) = 10 \cos(10^3 \cdot t)$ mV.

Solution:

(a)

$$H(s) = \frac{10^4}{s(1 + s/10^3)}$$

From the Bode plot:

$$|H(j10^3)| = 20 \text{ dB} = 10 \text{ V/V}$$

$$\angle H(j10^3) = -135^\circ$$

$$v_{out}(t) = 10 \cdot |H(j10^3)| \cos(10^3 \cdot t + \angle H(j10^3)) \text{ mV}$$

$$\boxed{v_{out}(t) = 100 \cos(10^3 \cdot t - 135^\circ) \text{ mV}}$$

(b)

$$H(s) = \frac{2000 \left(1 + \frac{s}{10^2}\right)}{\left(1 + \frac{s}{10}\right) \left(1 + \frac{s}{10^3}\right) \left(1 + \frac{s}{10^4}\right)}$$

From the Bode plot:

$$|H(j10^3)| = 46 \text{ dB} = 200 \text{ V/V}$$

$$\angle H(j10^3) = -45^\circ$$

$$v_{out}(t) = 10 \cdot |H(j10^3)| \cos(10^3 \cdot t + \angle H(j10^3)) \text{ mV}$$

$$\boxed{v_{out}(t) = 2 \cos(10^3 \cdot t - 45^\circ) \text{ V}}$$

(c)

$$H(s) = \frac{4000(1 + s/10^2)}{(1 + s/10)(1 + s/10^4)^2}$$

From the Bode plot:

$$|H(j10^3)| = 52 \text{ dB} = 400 \text{ V/V}$$

$$\angle H(j10^3) = 0^\circ$$

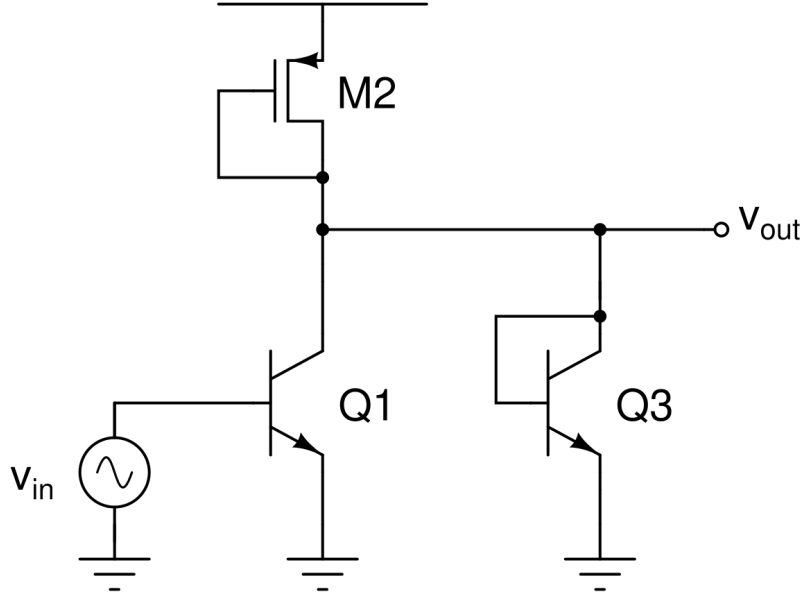
$$v_{out}(t) = 10 \cdot |H(j10^3)| \cos(10^3 \cdot t + \angle H(j10^3)) \text{ mV}$$

$$\boxed{v_{out}(t) = 4 \cos(10^3 \cdot t) \text{ V}}$$

Open Circuit Time Constants

Problem 1

Use the open-circuit time constant method to estimate the -3 dB frequency, $\omega_{-3\text{dB}}$, of this amplifier. Consider C_{gs} , C_{gd} , C_{π} , C_{μ} , and r_{ds} . Ignore r_o .



Solution:

$\tau_{C_{\pi 1}}$:

$$R_{C_{\pi 1}} = 0$$

$$\tau_{C_{\pi 1}} = 0$$

$\tau_{C_{\mu 1}}$:

$$R_{C_{\mu 1}} = \left(\frac{1}{g_{m2}} \right) \parallel \left(\frac{r_{\pi 3}}{\beta + 1} \right)$$

$$\tau_{C_{\mu 1}} = \left[\left(\frac{1}{g_{m2}} \right) \parallel \left(\frac{r_{\pi 3}}{\beta + 1} \right) \right] C_{\mu 1}$$

$\tau_{C_{gs 2}}$:

$$R_{C_{gs 2}} = \left(\frac{1}{g_{m2}} \right) \parallel \left(\frac{r_{\pi 3}}{\beta + 1} \right)$$

$$\tau_{C_{gs 2}} = \left[\left(\frac{1}{g_{m2}} \right) \parallel \left(\frac{r_{\pi 3}}{\beta + 1} \right) \right] C_{gs 2}$$

$\tau_{C_{gd 2}}$:

$$R_{C_{gd 2}} = 0$$

$$\tau_{C_{gd 2}} = 0$$

$\tau_{C_{\pi 3}}$:

$$R_{C_{\pi 3}} = \left(\frac{1}{g_{m2}} \right) \parallel \left(\frac{r_{\pi 3}}{\beta + 1} \right)$$

$$\tau_{C_{\pi 3}} = \left[\left(\frac{1}{g_{m2}} \right) \parallel \left(\frac{r_{\pi 3}}{\beta + 1} \right) \right] C_{\pi 3}$$

$$\tau_{C_{\mu 3}}:$$

$$R_{C_{\mu 3}} = 0$$

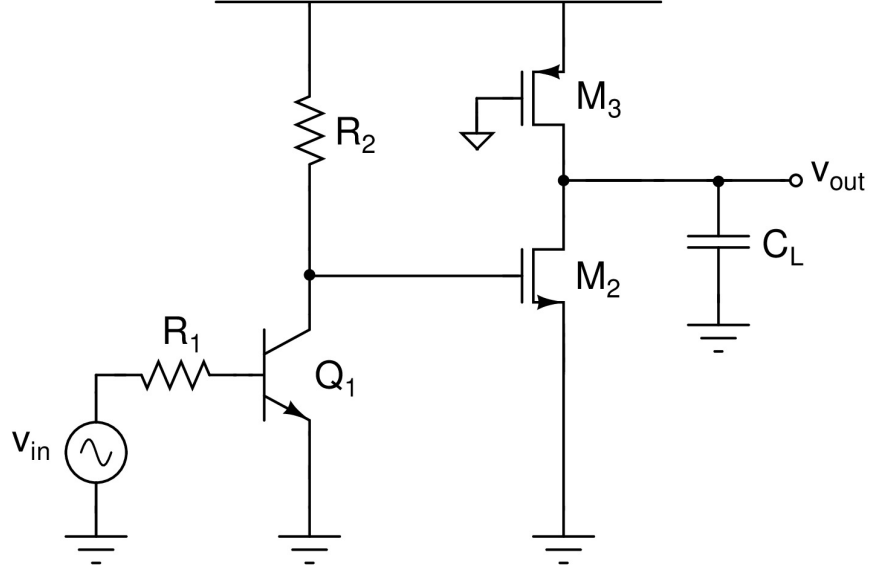
$$\tau_{C_{\mu 3}} = 0$$

$$\omega_{-3\text{dB}} = \frac{1}{\tau_{C_{\pi 1}} + \tau_{C_{\mu 1}} + \tau_{C_{gs2}} + \tau_{C_{gd2}} + \tau_{C_{\pi 3}} + \tau_{C_{\mu 3}}}$$

$$\omega_{-3\text{dB}} = \frac{1}{\left[\left(\frac{1}{g_{m2}} \right) \parallel \left(\frac{r_{\pi 3}}{\beta + 1} \right) \right] (C_{\mu 1} + C_{gs2} + C_{\pi 3})}$$

Problem 2

Use the open-circuit time constant method to estimate the -3 dB frequency, $\omega_{-3\text{dB}}$, of this amplifier. Consider C_{gs} , C_{gd} , C_π , C_μ , and r_{ds} . Ignore r_o .



Solution:

τ_{C_π} :

$$R_{C_\pi} = R_1 || r_{\pi 1}$$

$$\tau_{C_\pi} = (R_1 || r_{\pi 1}) C_\pi$$

τ_{C_μ} :

$$R_{left} = R_1 || r_{\pi 1}$$

$$R_{right} = R_2$$

$$G_M = \frac{1}{\frac{r_{\pi 1}}{\beta_1}} = g_{m1}$$

$$R_{C_\mu} = R_{left} + R_{right} + G_M R_{left} R_{right}$$

$$R_{C_\mu} = R_1 || r_{\pi 1} + R_2 + g_{m1} (R_1 || r_{\pi 1}) R_2$$

$$\tau_{C_\mu} = R_{C_\mu} C_\mu$$

$\tau_{C_{gs2}}$:

$$R_{C_{gs2}} = R_2$$

$$\tau_{C_{gs2}} = R_2 C_{gs2}$$

$\tau_{C_{gd2}}$:

$$R_{left} = R_2$$

$$R_{right} = r_{ds2} || r_{ds3}$$

$$G_M = g_{m2}$$

$$R_{C_{gd2}} = R_2 + R_{right} + G_M R_{left} R_{right}$$

$$R_{C_{gd2}} = R_2 + r_{ds2} || r_{ds3} + g_{m2} R_2 (r_{ds2} || r_{ds3})$$

$$\tau_{C_{gd2}} = R_{C_{gd2}} C_{gd2}$$

$\tau_{C_{gs3}}:$

$$R_{C_{gs3}} = 0$$

$$\tau_{C_{gs3}} = 0$$

$\tau_{C_{gd3}}:$

$$R_{C_{gd3}} = r_{ds2} || r_{ds3}$$

$$\tau_{C_{gd3}} = (r_{ds2} || r_{ds3}) C_{gd3}$$

$\tau_{C_L}:$

$$R_{C_L} = r_{ds2} || r_{ds3}$$

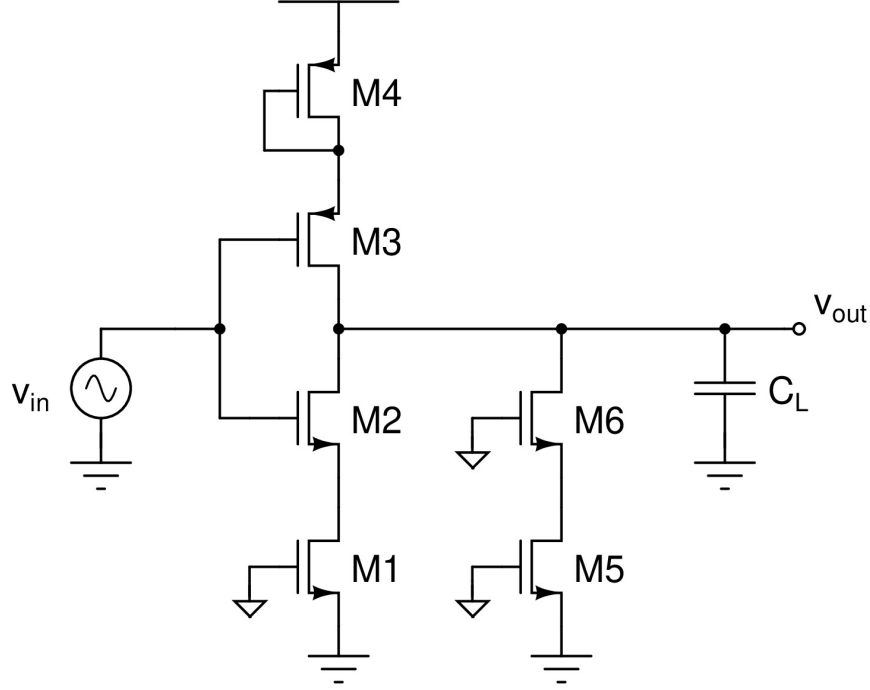
$$\tau_{C_L} = (r_{ds2} || r_{ds3}) C_L$$

Using the OCTC method:

$$\omega_{-3dB} = \frac{1}{\tau_{C_\pi} + \tau_{C_\mu} + \tau_{C_{gs2}} + \tau_{C_{gd2}} + \tau_{C_{gs3}} + \tau_{C_{gd3}} + \tau_{C_L}}$$

Problem 3

Use the open-circuit time constant method to estimate the -3 dB frequency, $\omega_{-3\text{dB}}$, of this amplifier. Consider C_{gs} , C_{gd} , C_π , C_μ , and r_{ds} .



Solution:

$\tau_{C_{gs1}}$:

$$R_{C_{gs1}} = 0$$

$$\tau_{C_{gs1}} = 0$$

$\tau_{C_{gd1}}$:

$$R_{C_{gd1}} = \left[\frac{1}{g_{m2}} \left(1 + \frac{R_x}{r_{ds2}} \right) \right] \parallel r_{ds1}$$

$$R_x = \left(r_{ds3} + \frac{1}{g_{m4}} \right) \parallel [r_{ds6} + (1 + g_{m6}r_{ds6})r_{ds5}]$$

$$\tau_{C_{gd1}} = R_{C_{gd1}} C_{gd1}$$

$\tau_{C_{gs2}}$:

$$R_{C_{gs2}} = \left[\frac{1}{g_{m2}} \left(1 + \frac{R_x}{r_{ds2}} \right) \right] \parallel r_{ds1}$$

$$R_x = \left(r_{ds3} + \frac{1}{g_{m4}} \right) \parallel [r_{ds6} + (1 + g_{m6}r_{ds6})r_{ds5}]$$

$$\tau_{C_{gs2}} = R_{C_{gs2}} C_{gs2}$$

$\tau_{C_{gd2}}$:

$$R_{C_{gd2}} = \left(r_{ds3} + \frac{1}{g_{m4}} \right) \parallel [r_{ds2} + (1 + g_{m2}r_{ds2})r_{ds1}] \parallel [r_{ds6} + (1 + g_{m6}r_{ds6})r_{ds5}]$$

$$\tau_{C_{gd2}} = R_{C_{gd2}} C_{gd2}$$

$\tau_{C_{gs3}}:$

$$R_{C_{gs3}} = \left[\frac{1}{g_{m3}} \left(1 + \frac{R_y}{r_{ds3}} \right) \right] \parallel \left(\frac{1}{g_{m4}} \right)$$

$$R_y = [r_{ds2} + (1 + g_{m2}r_{ds2})r_{ds1}] \parallel [r_{ds6} + (1 + g_{m6}r_{ds6})r_{ds5}]$$

$\tau_{C_{gs3}} = R_{C_{gs3}} C_{gs3}$

$\tau_{C_{gd3}}:$

$$R_{C_{gd3}} = \left(r_{ds3} + \frac{1}{g_{m4}} \right) \parallel [r_{ds2} + (1 + g_{m2}r_{ds2})r_{ds1}] \parallel [r_{ds6} + (1 + g_{m6}r_{ds6})r_{ds5}]$$

$\tau_{C_{gd3}} = R_{C_{gd3}} C_{gd3}$

$\tau_{C_{gs4}}:$

$$R_{C_{gs4}} = \left[\frac{1}{g_{m3}} \left(1 + \frac{R_y}{r_{ds3}} \right) \right] \parallel \left(\frac{1}{g_{m4}} \right)$$

$$R_y = [r_{ds2} + (1 + g_{m2}r_{ds2})r_{ds1}] \parallel [r_{ds6} + (1 + g_{m6}r_{ds6})r_{ds5}]$$

$\tau_{C_{gs4}} = R_{C_{gs4}} C_{gs4}$

$\tau_{C_{gd4}}:$

$$R_{C_{gd4}} = 0$$

$\tau_{C_{gd4}} = 0$

$\tau_{C_{gs5}}:$

$$R_{C_{gs5}} = 0$$

$\tau_{C_{gs5}} = 0$

$\tau_{C_{gd5}}:$

$$R_{C_{gd5}} = \left[\frac{1}{g_{m6}} \left(1 + \frac{R_z}{r_{ds6}} \right) \right] \parallel r_{ds5}$$

$$R_z = \left(r_{ds3} + \frac{1}{g_{m4}} \right) \parallel [r_{ds2} + (1 + g_{m2}r_{ds2})r_{ds1}]$$

$\tau_{C_{gd5}} = R_{C_{gd5}} C_{gd5}$

$\tau_{C_{gs6}}:$

$$R_{C_{gs6}} = \left[\frac{1}{g_{m6}} \left(1 + \frac{R_z}{r_{ds6}} \right) \right] \parallel r_{ds5}$$

$$R_z = \left(r_{ds3} + \frac{1}{g_{m4}} \right) \parallel [r_{ds2} + (1 + g_{m2}r_{ds2})r_{ds1}]$$

$\tau_{C_{gs6}} = R_{C_{gs6}} C_{gs6}$

$\tau_{C_{gd6}}:$

$$R_{C_{gd6}} = \left(r_{ds3} + \frac{1}{g_{m4}} \right) \parallel [r_{ds2} + (1 + g_{m2}r_{ds2})r_{ds1}] \parallel [r_{ds6} + (1 + g_{m6}r_{ds6})r_{ds5}]$$

$\tau_{C_{gd6}} = R_{C_{gd6}} C_{gd6}$

$\tau_{C_L}:$

$$R_{C_L} = \left(r_{ds3} + \frac{1}{g_{m4}} \right) \parallel [r_{ds2} + (1 + g_{m2}r_{ds2})r_{ds1}] \parallel [r_{ds6} + (1 + g_{m6}r_{ds6})r_{ds5}]$$

$\tau_{C_L} = R_{C_L} C_L$

Using the OCTC method:

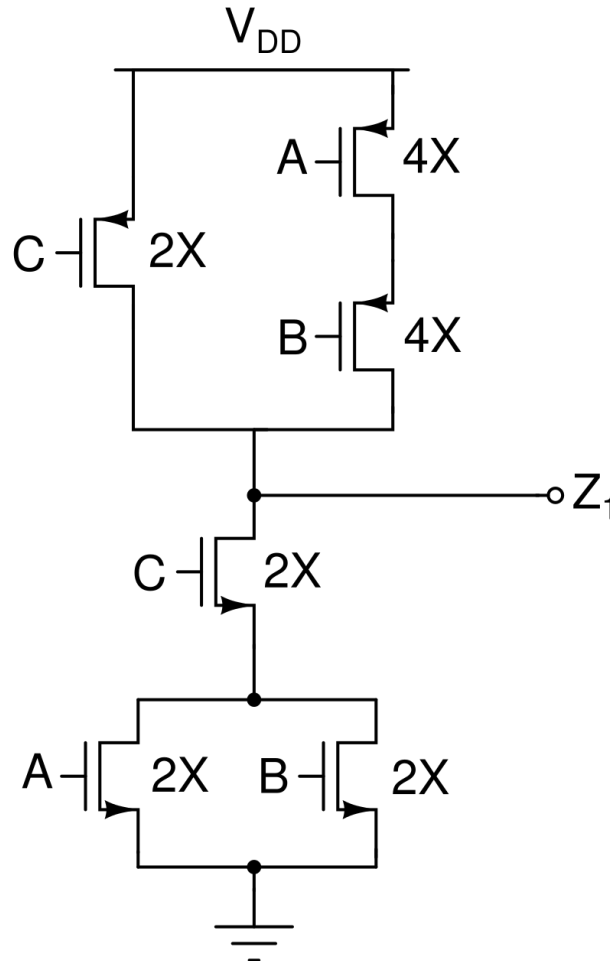
$$\omega_{-3dB} = \frac{1}{\tau_{C_{gs1}} + \tau_{C_{gd1}} + \tau_{C_{gs2}} + \tau_{C_{gd2}} + \tau_{C_{gs3}} + \tau_{C_{gd3}} + \tau_{C_{gs4}} + \tau_{C_{gd4}} + \tau_{C_{gs5}} + \tau_{C_{gd5}} + \tau_{C_{gs6}} + \tau_{C_{gd6}} + \tau_{C_L}}$$

CMOS Logic Circuits

For each part, design and draw the schematic of a CMOS logic gate that implements the Boolean expression. In each schematic, label the size of each transistor such that the worst case delays of the pull-up and pull-down networks are equivalent to those of a standard minimum-sized inverter with $(W/L)_P/(W/L)_N = 2$. Inverted inputs are not available.

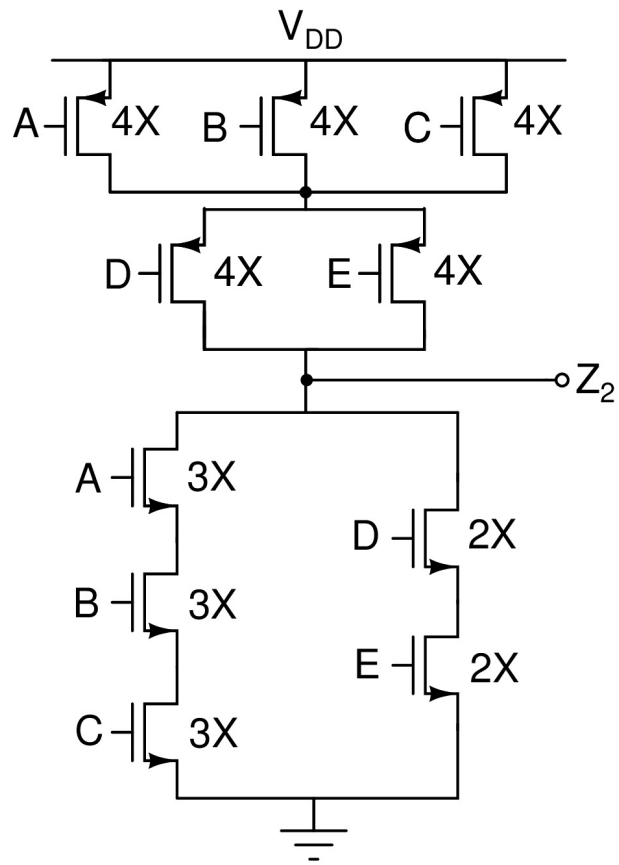
(a) $Z_1 = \overline{(A + B) \cdot C}$

Solution:



(b) $Z_2 = \overline{(A \cdot B \cdot C) + (D \cdot E)}$

Solution:



(c) $Z_3 = \overline{A} \cdot (\overline{B} + \overline{C} + \overline{D})$

Solution:

$$\overline{Z_3} = \overline{\overline{A} \cdot (\overline{B} + \overline{C} + \overline{D})}$$

$$\overline{Z_3} = A + (\overline{B} + \overline{C} + \overline{D})$$

$$\overline{Z_3} = A + (B \cdot C \cdot D)$$

$$Z_3 = \overline{A + (B \cdot C \cdot D)}$$

