

ECE 210 Midterm 1 Worksheet

Note: This worksheet is not guaranteed to be entirely representative of the midterm's contents. Material may appear on this worksheet which will not appear on the midterm, and vice versa.

Complex Number Review

1) Find the roots of the following polynomials:

a) $x^2 + x + 1$

b) $x^3 + 1$

c) $x^2 + 4x + 2$

d) $x^2 + 2jx + 1$

2) Evaluate the following expressions:

a) $(1 + 3j)(4 - j)$

b) $(1 - j)(2 + j)$

c) $(1 + 3j)(4 - j)^{-1}$

d) $(1 - j)(2 + j)^{-1}$

3) Prove the following:

Triangle Inequality: Show that $|z_1| + |z_2| \geq |z_1 + z_2|$.

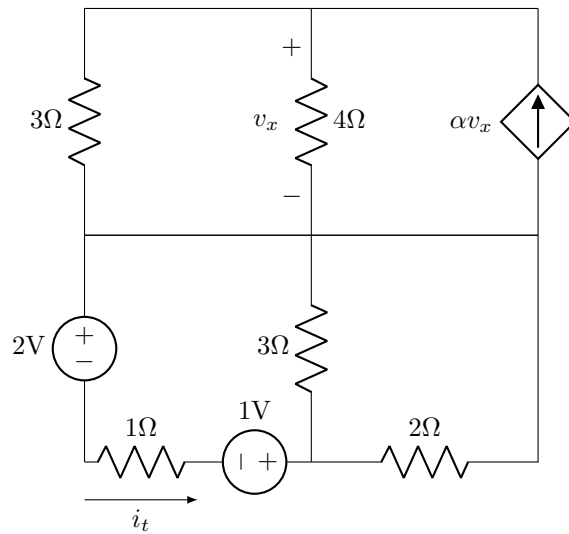
Hyperbolic Functions: Show that $\sin(jx) = j \sinh(x)$.

Hyperbolic Functions II: Show that $\sinh(x) = \tan(y) \implies \sin(y) = \pm \tanh(x)$.

Note: $\sinh(x) = [\exp(x) - \exp(-x)]/2$ $\tanh(x) = [\exp(x) - \exp(-x)]/[\exp(x) + \exp(-x)]$.

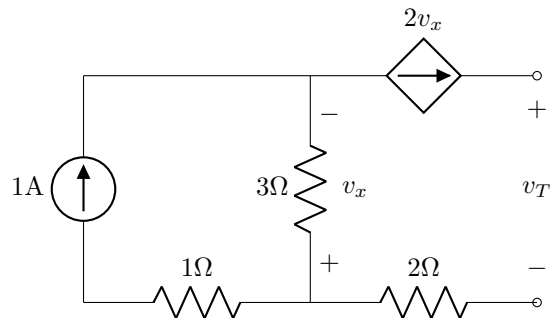
Resistive Circuit Analysis

4) Consider the circuit below.



- There is only one value of α that makes this circuit a valid circuit. Find it.
- Find i_t , the current across the $1\ \Omega$ resistor, as indicated.

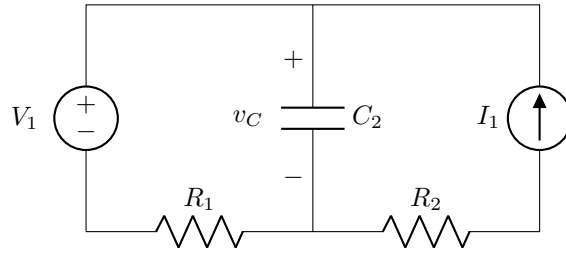
5) Given the circuit below;



- Find the Thevenin equivalent voltage.
- Find the Norton equivalent current.
- If a $5\ \Omega$ load is connected across the terminals of this circuit, how much power is dissipated across the load?

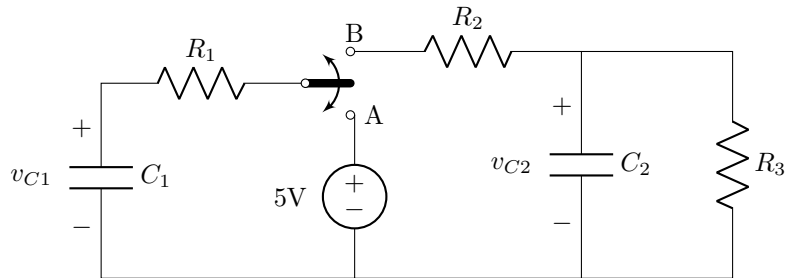
N-Order Circuits

6) Initially, the capacitor holds some charge Q_0 , and both sources are off, with a value of 0 volts and amps respectively. At time $t = 0$, both the voltage and current source are turned on.



- Find the zero-input solution for $v_C(t)$, the voltage across the capacitor.
- Find the zero-state solution for $v_C(t)$, the voltage across the capacitor.
- Find $v_C(t)$.

7) Initially, both capacitors C_1 and C_2 are entirely discharged.



- At time $t = t_1$, the switch is thrown to position A, connecting the left half to the voltage source. Find a symbolic expression for $v_{C1}(t)$, the voltage across the capacitor C_1 .
- Enough time passes such that C_1 is entirely charged. The switch is then thrown at time $t = t_2$ from position A to position B, disconnecting the left half from the voltage source and connecting it instead to the right half of the circuit, with the voltage source left entirely disconnected. Find the new expression for $v_{C1}(t)$, the voltage across the capacitor C_1 .

Note: The solution of a second order differential equation $a\ddot{x} + b\dot{x} + c = 0$ with initial conditions $x(t_0) = m$, $\dot{x}(t_0) = n$, is given as:

$$x = C_1 \exp(r_1 x) + C_2 \exp(r_2 x)$$

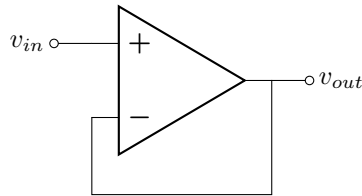
where r_1, r_2 are the roots of $ax^2 + bx + c$, and C_1, C_2 are found from initial conditions.

Operational Amplifiers

8) In ECE 210, operational amplifiers are treated as magic triangles with specific input output relations. A more realistic op-amp model would look something like this;

$$i_+ = i_- = 0$$
$$v_{out} = 10^6 \times (v_+ - v_-)$$

a) Using the above equations, show that the ideal op-amp rule $v_+ = v_-$ is approximately true when the output of the chip is connected to the inverting input (v_-) of the chip, as shown in the figure below.



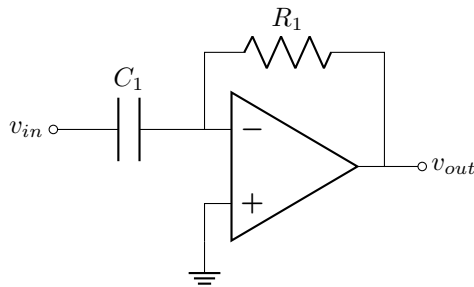
b) What happens when the output of the chip is instead connected back to the non-inverting terminal (v_+)?

c) Find $v_{out}(t)$ in terms of $v_{in}(t)$, assuming both voltages are in reference to a common ground, using:

i) ideal op-amp approximations.

ii) the given op-amp model.

Assume the capacitor has some non-zero initial charge q_0 .



iii) Do your results from c.i) and c.ii) match?