ECE210 Final Review - Cramming Carnival

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Note: Problems with three stars are more difficult than what you'd see on a final exam. They will teach you a lot if you do them, but don't be worried if you get stuck on them. Everything else is either at exam-level or easier.

1 Fabled Fourier

Determine the Fourier transform of the following:

1. $f(t) = \text{sinc}(4t - 8) * -\text{rect}(t)$
2. $g(t) = \frac{1}{(4+jt)^2}$
3. $h(t) = \text{sinc}^2(3t - 3)$
4. $h'(t)$, using $h(t)$ defined above

2 Scintillating Simplfications

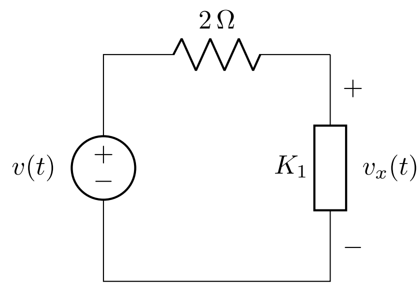
Simplify the following expressions:
a. $f(t) = (t^2 + 1)\delta(t - 1)\delta(t + 1)$
b. $g(t) = (t^2 + 1)(\delta(t - 1) + \delta(t + 1))$
c. $h(t) = (t^2 + 1) * (\delta(t - 1) + \delta(t + 1))$

3 Clever Components

The component represented as a box in the following problem has the following time-domain I-V relationship.

$$V(t) = K\frac{\mathrm{d}^2 I(t)}{\mathrm{d}t^2}$$

Let $v(t) = \sin(t)$, and let $K_1 = 2$. Assume all initial conditions are 0 in the circuit, aside from the driving force $v(t)$. Given the circuit below, answer the following questions.



a) What are the units of K? Use at most two other units in your answer.

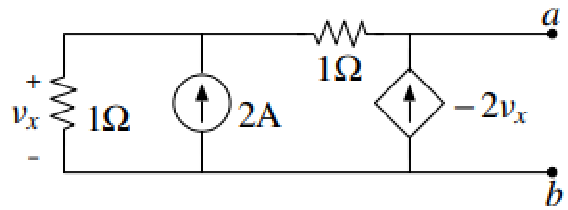
b) Find the current $i(t)$ and voltage $v_x(t)$, in the time domain, using only real functions.

4 Illuminating Impulse Response

Consider an LTI system where the input $f(t) = 5u(t)$ results in a zero-state response $y(t) = e^{5t}u(t)$. Find the impulse response of the system. Also state if the system is causal and/or BIBO-stable. If it isn't BIBO-stable, name a bounded input that will cause an unbounded output.

5 Tremendous Transformations

Determine the Thevenin equivalent of the following network between nodes a and b, and then determine the available power of the network:



6 Legendary Laplace

- Find the Laplace Transform of $x(t) = e^t u(-t)$.
- Find the Laplace Transform of $x(t) = rect(t - \frac{1}{2})$.

7 Iconic Inverses

- Find the inverse Laplace Transform of $\hat{H}(s) = \frac{1}{s(s^2+2s+2)}$.
- Find the inverse Laplace Transform of $\hat{H}(s) = e^{-4s}\frac{6s-1}{(s+1)^2(s-2)}$.

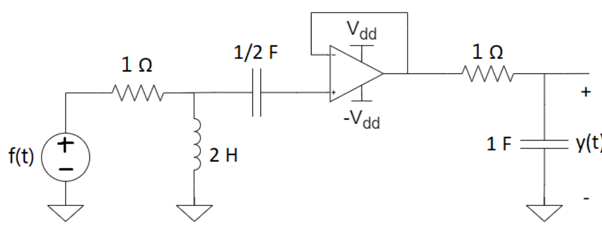
8 Perplexing Poles

Determine the poles and ROC of the following transfer function, and determine if it represents a BIBO-stable LTIC system.

$$\hat{H}(s) = \frac{s^4(s+3)}{(s^2+5s+6)}$$

9 Exquisite Electronics

Let $f(t)$ be the input to the following circuit, with $y(t)$ denoted as:



Find the circuit's transfer function in the time domain.

10 Delightful Differential

Consider an LTIC system described by the ODE

$$\frac{d^2y}{dt^2} + \frac{dy}{dt} - 2y(t) = f(t)$$

with initial conditions $y(0^-) = 3$ and $y'(0^-) = 8$.

Determine the system's transfer function $\hat{H}(s)$, its characteristic polynomial, characteristic poles, characteristic modes, and zero-input solutions in both the s domain and the time domain. Also note if the system is BIBO stable or not and say why.

11 Insidious Inputs

Let a system be defined by its input-output relation $y(t) = x(102841) + x(t)$. Is the system Linear? Time-Invariant? BIBO-Stable? Causal? If it is BIBO-Unstable, name a bounded input that will cause an unbounded output.

12 Outrageous Outputs

Let a system be defined by the following input-output relation:

$$y(t) = \begin{cases} 9^t x(t-4) & , -\infty < t \leq 10^6 \\ sin^3(t+4)cos(t+2)x(t-1) & , 10^6 < t < \infty \end{cases}$$

Is the system Linear? Time-Invariant? BIBO-Stable? Causal? If it is BIBO-Unstable provide a bounded input that causes an unbounded input.

13 Unpopular Unit-step

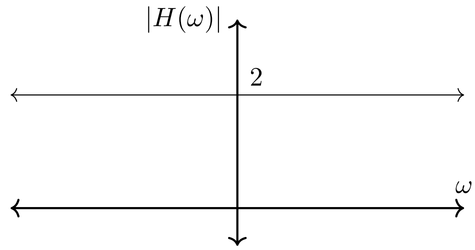
Let a system be defined by its impulse response $h(t) = u(t)$. Is the system Linear? Time-Invariant? Causal? BIBO-Stable? If it is BIBO-Unstable, name a bounded input that will cause an unbounded output.

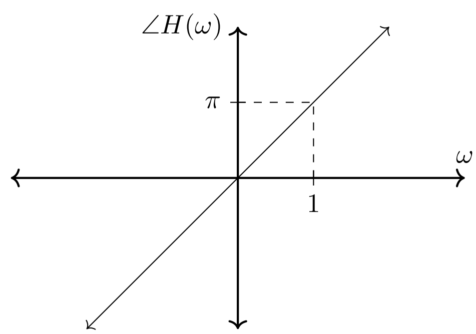
14 Disgusting Derivative

Let a system be defined by the input-output relation $y(t) = \frac{dx}{dt}$. Is the system Linear? Time-Invariant? BIBO-Stable? Causal? If it is BIBO-Unstable provide a bounded input that causes an unbounded input.

15 Peculiar Plots

Let $f(t) = 7e^{j5t} sinc^2(\frac{5t}{2})$ be the input into a system with the following impulse response:





What is the output?

16 Cataclysmic Convolution

Given:

$$f(t) = \frac{\cos(t)}{t},\ g(t) = \frac{\sin(t)}{t^2}, h(t) = \mathrm{rect}\left(\frac{t}{2}\right)$$

Find $\big(f(t) * h(t)\big) - \big(h(t) * g(t)\big)$. **Hint:** there's a derivative somewhere...

17 Rambunctious Reality

Given the Fourier transform $F(\omega) = \omega^2 cos(2\omega) sin^2(\pi\omega) + j2sin(\tau\omega)$, prove that the corresponding $f(t)$ is a real signal. You don't need to solve for $f(t)$.

18 Perceptive Proofs

18.1 Meritorious Modulation

Without using the modulation property, show that $f(t)\cos(\omega_c t)$ transforms to $\frac{1}{2}\big[F(\omega-\omega_c)+F(\omega+\omega_c)\big]$.

18.2 Immutable Invariance***

Prove an impulse train transforms into another impulse train, without explicitly using transforms 24 or 25 in your tables.

Bonus: What condition must be true for the impulse train's period to be invariant under the Fourier Transform? Recall that the impulse train has the following form, where T is the period:

$$\sum_{n=-\infty}^{\infty} \delta(t-nT)$$

18.3 Fantastic Four

Prove that applying the Fourier transform four times to a function returns the original function, scaled by some positive scaling factor $K > 1$. Also, find K.

19 Fatal Feedback***

Find the transfer function of the following circuit in the s-domain.