Assingment 3, task 3
Group: 3

Angel Ontiveros, Lassi Laaksosaari

12.12.2022

Task 3: RANSAC properties (pen & paper)
The fraction of inliers is ϵ, and m points are required to define a single model hypothesis. Prove that:

$$k \geq \frac{\log(1-p)}{\log(1-\epsilon^m)}$$

model hypothesis iterations are required for RANSAC to succeed with probability at least p (success: at least one model hypothesis has only inlier points)

Solution

1. Prob. that a single point is correct (inlier) $= \epsilon$

2. By the multiplication rule, prob. that sample of m points is correct $= \epsilon^m$

3. By the negation rule, prob. of a sample of m points fails $= 1 - \epsilon^m$

4. By multiplication rule, the probability of k samples of m points failing $= (1 - \epsilon^m)^k$

5. By negation rule, prob. not all k samples fail (at least one correct) $= 1 - (1 - \epsilon^m)^k$

We want the probability of success (at least one model hypothesis is correct) to be at least p, then

$$p \geq 1 - (1-\epsilon^m)^k$$
$$\implies p - 1 \geq -(1-\epsilon^m)^k$$
$$\implies 1 - p \leq (1-\epsilon^m)^k$$
$$\implies \log(1-p) \leq \log[(1-\epsilon^m)^k]$$
$$\implies \log(1-p) \leq k\log(1-\epsilon^m)$$
$$\therefore k \geq \frac{\log(1-p)}{\log(1-\epsilon^m)}$$