Sirindhorn International Institute of Technology
Thammasat University

MAS116: MATHEMATICS I
1/2024 FINAL EXAMINATION

Name .... **ID** .... **Section** .... **Seat No.** ....

Instructor: Dr. Adisak Seesanea

Date and Time: Monday, December 2, 2024, 13:30-16:30 (3 hours)

General Instructions:

- This exam paper has 11 pages, including this page. It consists of 7 main problems with a total score of 100 points. Some optional bonus problems are provided.
- This is a closed-book exam. No calculators allowed.
- You are not allowed to be out of the examination room during the examination. Going to the restroom may result in a score deduction.
- Write your name, student ID, section, and seat number clearly on each exam sheet.
- You are supposed to complete the exam independently. Academic honesty is fundamental in this course.
- The examination paper cannot be taken out of the examination room. A violation will result in a zero score for the examination.
- Your solutions will be carefully graded based on the written methods rather than the final answers. Please make sure that your solutions are logically presented and readable.
- Your total score will be weighted to 30% of your final grade as indicated in the course syllabus.

Problem	Score
1	
2	
3	
4	
5	
6	
7	
Bonus	
Total	

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Problem 1. [**14 points**]

Let $\alpha \neq -1$ be an arbitrary real number. Evaluate the indefinite integral

$$\int x^{\alpha} \ln x \, dx.$$

Hint: Apply the integration by parts formula

$$\int u \, dv = u \, v - \int v \, du.$$

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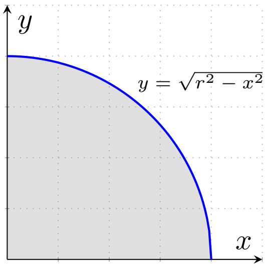
Problem 2. [14 points]

Prove that the area of a circle of radius $r > 0$ is πr^2.

Hint: Use the Fundamental Theorem of Calculus and a trigonometric substitution, for example, $x = r\sin\theta$, to evaluate definite integral $\int_0^r \sqrt{r^2 - x^2}\, dx$, which gives a quarter of the area. Recall that $\sin^2\theta + \cos^2\theta = 1$, $\cos^2\theta = \frac{1}{2}(1 + \cos 2\theta)$, and $\sin 2\theta = 2\sin\theta\cos\theta$.



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Problem 3. [14 points]

Determine an antiderivative $F(x)$ of the function

$$x^{2023}\tan^{2024}(\pi x^{2024})\,\sec^4(\pi x^{2024})$$

such that $F(0) = \pi$.

<u>Hint</u>: Make a substitution $u = \tan(\pi x^{2024})$. Observe that

$$\frac{d}{dx}\tan x = \sec^2 x \qquad \text{and} \qquad \sec^2 x - \tan^2 x = 1.$$

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Problem 4. [**14 points**]

Employ the method of partial fractions to evaluate the indefinite integral

$$\int \frac{x^4}{x^2-1}\,dx.$$

Hint: Perform long division on the integrand, write the proper fraction as a sum of partial fractions:

$$\frac{x^4}{x^2-1} = Q(x) + \frac{R(x)}{x^2-1} = Q(x) + \frac{A}{x-1} + \frac{B}{x+1},$$

where A and B are real numbers.

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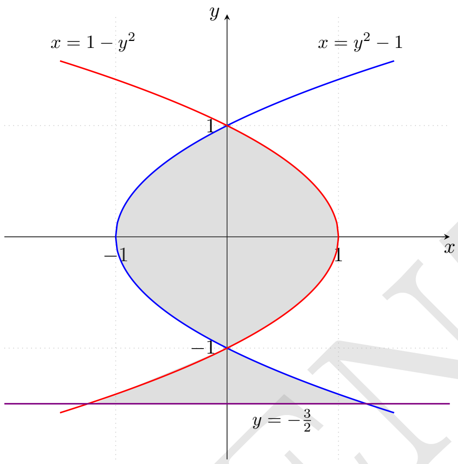
Problem 5. [**15 points**]: Solve only ONE problem, either **(5a)** or **(5b)**.

(5a). Compute the area of the region enclosed by

$$x = y^2 - 1, \qquad x = 1 - y^2 \qquad \text{and} \qquad y = -\frac{3}{2}.$$



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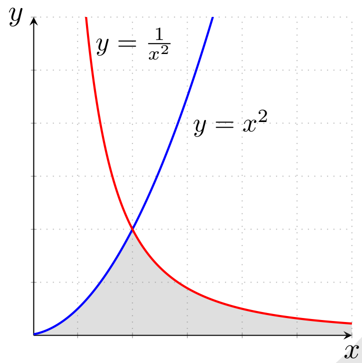
(5b). Compute the area of the region enclosed by

$$y = x^2, \qquad y = \frac{1}{x^2} \qquad \text{and} \qquad y = 0.$$



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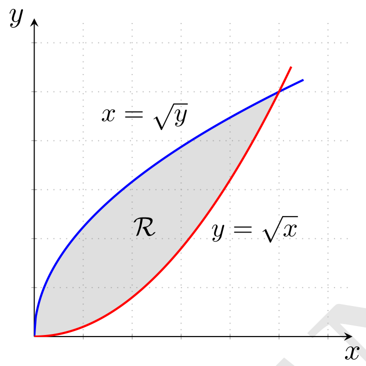
Problem 6. [**15 points**]: Solve only ONE problem, either **(6a)** or **(6b)**.

Consider a solid of revolution $\mathcal{S}$ obtained by rotating the region $\mathcal{R}$ enclosed by

$$x = \sqrt{y} \qquad \text{and} \qquad y = \sqrt{x},$$

about the y-axis.



(6a). Find the volume of $\mathcal{S}$ by using y as the variable of integration (slicing $\mathcal{S}$ horizontally).

Hint: The volume $V = \displaystyle\int_{y=c}^{y=d} A(y)\,dy$.

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(6b). Find the volume of $\mathcal{S}$ by using the cylindrical shell method.

Hint: The volume $V = \int_{x=a}^{x=b} 2\pi x f(x)\,dx$.

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Problem 7. [14 points]

Solve the initial value problem

$$(*) \qquad \begin{cases} \dfrac{dy}{dx} = e^x e^{-y} + e^x + 2e^{-y} + 2, \\ y(0) = \ln(e-1). \end{cases}$$

Hint: Write the nonlinearity $e^x e^{-y} + e^x + 2e^{-y} + 2 = (e^{-y}+1)(e^x+2)$.

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Bonus Problem. [15 points]

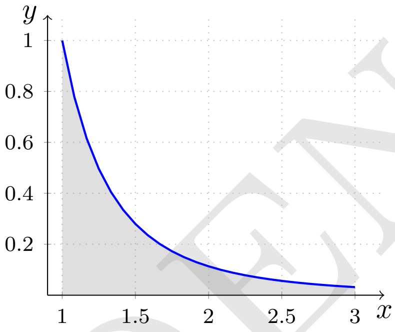
Determine whether the statement is true or false. Circle your answer for each statement. No justification is needed.

(8a). Any two antiderivatives of a function $f(x)$ differ only by a constant.

TRUE FALSE

(8b). The area of the region enclosed by $y = \dfrac{1}{x^{\pi}}$, $x = 1$, and $y = 0$ is infinite.

Hint:



TRUE FALSE

(8c). If $F(x) = \displaystyle\int_1^{e^x} \ln t \, dt$, then $\dfrac{dF}{dx} = xe^x$.

Hint: Use the Fundamental Theorem of Calculus and Chain Rule.

TRUE FALSE

(8d). If $f(x)$ and $g(x)$ are continuous functions on $[a, b]$, then

$$\int_a^b f(x)g(x)\,dx = \int_a^b f(x)\,dx \int_a^b g(x)\,dx$$

TRUE FALSE

(8e). If the function $f(x)$ is continuous on $[a, b]$, then the definite integral $\displaystyle\int_a^b f(x)\,dx$ is equal to the limit of its Riemann sums as the number of subintervals approaches infinity.

TRUE FALSE