

Class 2

▶ 0:00 / 46:34



Binomial Theorem

▶ 0:00 / 8:04



$$\sum_{k=3}^n \binom{n}{x} x^k y^{n-k}$$

THE PROBABILITY SET-UP

The following definition of a-field is needed to put the theory on sound foundation. In our course, we typically take F to be set of all subsets of the sample space S so that all the conditions are automatically satisfied.

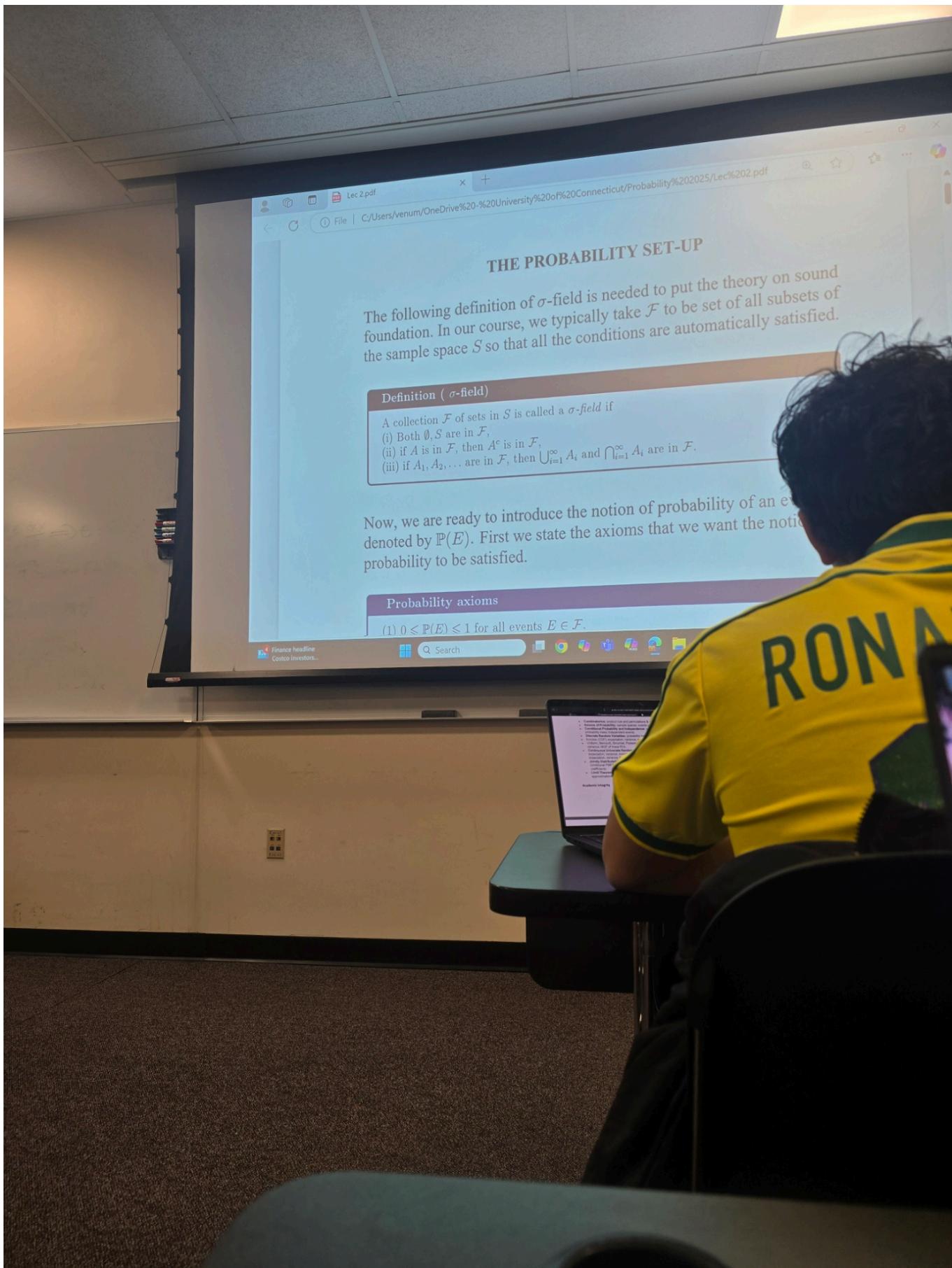
Definition (a-field) A collection F of sets in S is called a a-field if (i) Both J . S are in F ,

(iii) if

. are in F , then A , and A , are in F .

Now, we are ready to introduce the notion of probability of an event denoted by $P(E)$. First we state the axioms that we want the notion of probability to be satisfied.

Probability axioms



If two events are two events

$$P(E \cup F) = P(E) + P(F) - P(E \cap F)$$

$$\mathbb{P}(E \cup F) = \mathbb{P}(E) + \mathbb{P}(F) - \mathbb{P}(E \cap F)$$

Example 1: If $\mathbb{P}(E) = 0.4$, $\mathbb{P}(F) = 0.6$, and $\mathbb{P}(E \cup F) = 0.8$, then
find

$$(i) \mathbb{P}(E \cap F)$$

$$0.8 = 0.4 + 0.6 - \mathbb{P}(E \cap F)$$

$$(ii) \mathbb{P}(E \cap F^c)$$

$$\mathbb{P}(E \cap F^c) = 0.2$$

$$(iii) \mathbb{P}(E^c \cap F^c)$$

$$(i) \mathbb{P}(E \cup F) = \mathbb{P}(E) + \mathbb{P}(F) - \mathbb{P}(E \cap F)$$

$$0.8 = 0.4 + 0.6 - \mathbb{P}(E \cap F) \Rightarrow \mathbb{P}(E \cap F) = 0.2$$

Samples spaces in which the outcomes are equally likely are called uniform sample spaces.
The following is our main tool in computing probabilities of events in the uniform space

If S is a uniform sample space and E is any event is

$$P(E) = \frac{\text{Number of outcomes in } E}{\text{Number of Outcomes in } S}$$

Example 2 Two cards are selected at random from a standard deck of playing cards what is the probability that

- they are both aces
- neither of them is an ace
- exactly one of them is an ace.

Answers:

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$$\frac{1}{221}$$

$$\frac{1128}{1326}$$

$$\frac{4(48)}{1326}$$

A group of 5 people is selected at random. What is the probability that at least two of them have the same birthday?

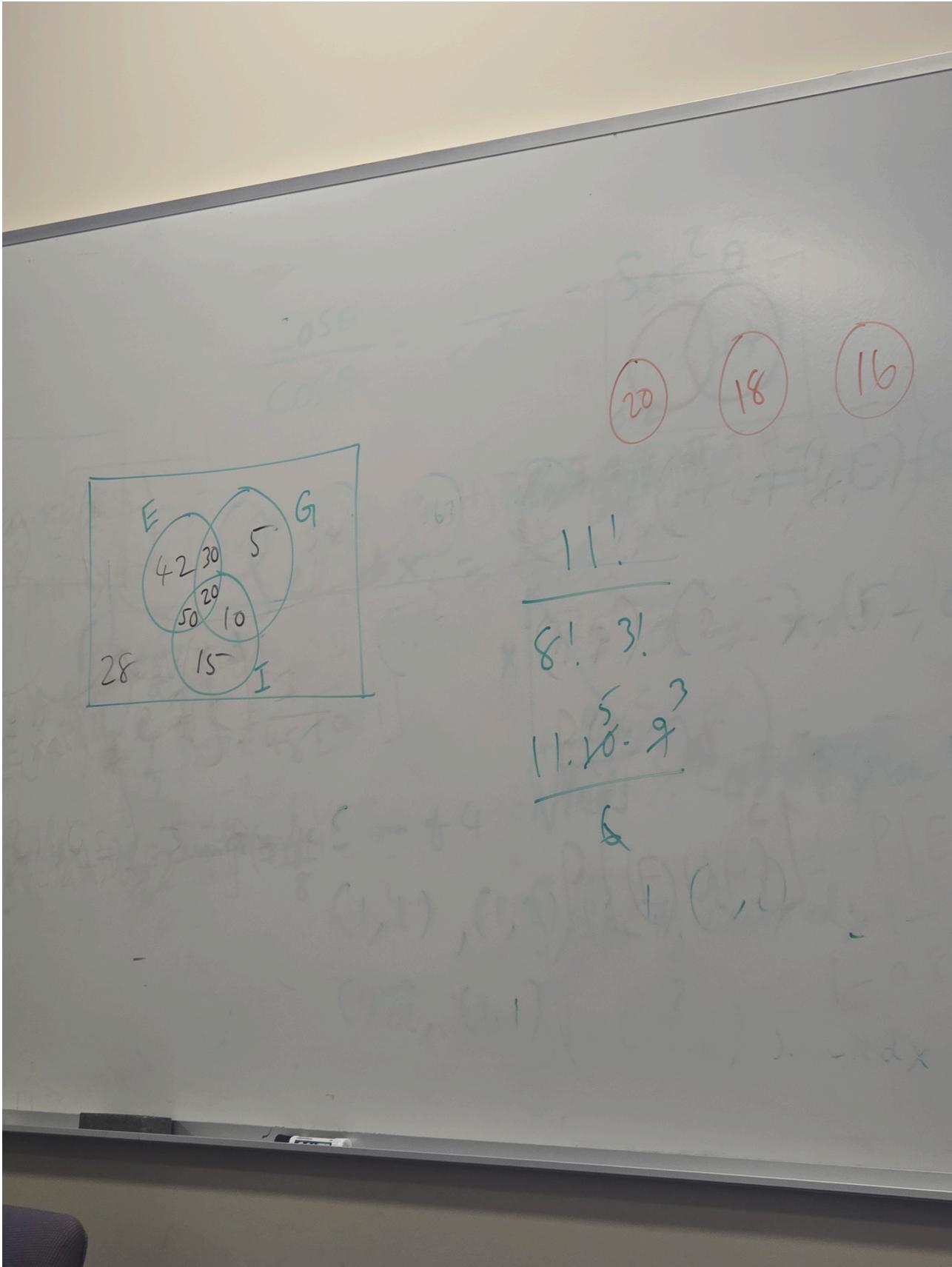
The total outcome is $(365)^5$

$$P(\text{at least two have same bday}) = 1 - P$$

In a survey of 200 people that just visited Europe

- 142 visited UK
- 95 visited Italy
- 65 visited Germany
- 70 visited both UK and Italy
- 50 visited UK and Germany
- 30 visited Italy and Germany

- 20 visited all



If 3 marbles are randomly drawn from a bowl containing 6 red and 5 blue what is the probability that one is red and two are blue.

$$n(s) = \binom{11}{3}$$

The number of ways choosing 1 red and two blues is

$$\binom{6}{1} \times \binom{5}{2}$$

Thus

$$\frac{\binom{6}{1} \times \binom{5}{2}}{\binom{11}{3}} = \frac{6 \times 10}{165} = \frac{60}{165} = \frac{4}{11}$$

Example 7: If 3 marbles are randomly drawn from a bowl containing six red and 5 blue marbles, what is the probability that one of the marbles is red and the other two are blue?

$$n(S) = \binom{11}{3}$$

The number of ways of choosing 1 red and two blue is $\binom{6}{1} \times \binom{5}{2}$

Thus $P(\text{choosing 1 red and two blue}) = \frac{\binom{6}{1} \times \binom{5}{2}}{\binom{11}{3}} = \frac{6 \times 10}{165} = \frac{60}{165} = \frac{4}{11}$

Example 8: Suppose 5 people are selected from a group of 20 individuals consisting of 10 married couples. Let N be the event that in the chosen group no two are married to each other. Find $P(N)$.

$$n(S) = 20 \times 19 \times 18 \times 17 \times 16$$
$$n(N) = 20 \times 18 \times 16 \times 14 \times 12$$
$$P(N) = \frac{n(N)}{n(S)} \approx 0.52$$