Melhor aproximação:
Método dos Mínimos
Quadrados (MMQ)
(lineares)

Caso Discreto

- $\{(x_i, y_i);\ i = 0, 1, \ldots, m\}$
- $\{\varphi_0, \varphi_1, \varphi_2, \ldots, \varphi_n\}$ funções L.I.

L.I.: linearmente independente

Problema: Encontrar

$$\varphi(x) = \underline{a_0}\,\varphi_0(x) + \underline{a_1}\,\varphi_1(x) + \cdots + \underline{a_n}\,\varphi_n(x).$$

que minimiza o erro

$$E(a_0, a_1, \ldots, a_n) = \|\varphi(x_i) - y_i\|_2^2$$

$$E(a_0, a_1, \dots, a_n) = \sum_{i=0}^{m} \left(\underbrace{\varphi(x_i) - y_i}_{} \right)^2$$

Devemos impor:

$$\boxed{\frac{\partial E(a_0, a_1, \dots, a_n)}{\partial a_j} = 0}$$

$j = 0, 1, \dots, n$

$$\Rightarrow$$

$$0 = 2 \sum_{i=0}^{m} \left(a_0 \varphi_0(x_i) + \cdots + a_n \varphi_n(x_i) - y_i \right) \cdot \varphi_j(x_i)$$

$$\sum_{i=0}^{m} \left(\varphi_0(x_i) \cdot \varphi_j(x_i)\, a_0 + \varphi_1(x_i) \cdot \varphi_j(x_i) \cdot a_1 + \cdots + \varphi_n(x_i) \cdot \varphi_j(x_i) \cdot a_n \right) = \sum_{i=0}^{m} y_i \cdot \varphi_j(x_i)$$

$$\boxed{j = 0, 1, \ldots, n}$$

$$
\left\{
\begin{array}{l}
\displaystyle\sum_{i=1}^{m}\Big[\underbrace{\varphi_0(x_i).\varphi_0(x_i)}.\underbrace{a_0}+\underbrace{\varphi_1(x_i)\varphi_0(x_i)}\underbrace{a_1}+\cdots+\underbrace{\varphi_n(x_i)\varphi_0(x_i)}\underbrace{a_n}\Big]=\underbrace{\sum_{i=1}^{m}y_i.\varphi_0(x_i)} \\
\qquad (n+1)\ \text{eqs.} \\
\displaystyle\sum_{i=1}^{m}\Big[\varphi_0(x_i).\varphi_1(x_i)a_0+\varphi_1(x_i).\varphi_1(x_i)a_1+\cdots+\varphi_n(x_i).\varphi_1(x_i)a_n\Big]=\underbrace{\sum_{i=1}^{m}y_i\varphi_1(x_i)} \\
\quad\vdots \\
\displaystyle\sum_{i=0}^{m}\Big[\varphi_0(x_i)\varphi_n(x_i)a_0+\varphi_1(x_i)\varphi_n(x_i)a_1+\cdots+\varphi_n(x_i)\varphi_n(x_i)a_n\Big]=\underbrace{\sum_{i=0}^{m}y_i.\varphi_n(x_i)}
\end{array}
\right.
$$

$$Ax = b \quad \boxed{(n+1)\times(n+1)} \quad \{1, x, x^2, \dots, x^n\}$$

$$A = \begin{bmatrix} \sum_{i=0}^{m} \underbrace{\varphi_0(x_i)}_{1}\cdot\underbrace{\varphi_0(x_i)}_{1} & \sum_{i=0}^{m} \underbrace{\varphi_1(x_i)}_{x_i}\underbrace{\varphi_0(x_i)}_{1} & \cdots & \sum_{i=0}^{m} \underbrace{\varphi_n(x_i)}_{x_i^n}\underbrace{\varphi_0(x_i)}_{1} \\ \sum_{i=0}^{m} \underbrace{\varphi_0(x_i)}_{1}\underbrace{\varphi_1(x_i)}_{x_i} & \sum_{i=0}^{m} \underbrace{\varphi_1(x_i)\varphi_1(x_i)}_{x_i^2} & \cdots & \sum_{i=0}^{m} \underbrace{\varphi_n(x_i)\varphi_1(x_i)}_{x_i^n\cdot x_i} \\ \vdots & & & \\ \sum_{i=0}^{m} \overbrace{\varphi_0(x_i)}^{1}\overbrace{\varphi_n(x_i)}^{x_i^n} & \sum_{i=0}^{m} \varphi_1(x_i)\varphi_n(x_i) & \cdots & \sum_{i=0}^{m} \overbrace{\varphi_n(x_i)\varphi_n(x_i)}^{x_i^{2n}} \end{bmatrix}$$

$(m+1)$

$$X = \begin{bmatrix} a_0 \\ a_1 \\ \vdots \\ a_n \end{bmatrix}_{(n+1) \times 1}$$

$$b = \begin{bmatrix} \sum_{i=0}^{m} y_i \underbrace{\varphi_0(x_i)}_{1} \\ \sum_{i=0}^{m} y_i \underbrace{\varphi_1(x_i)}_{x_i} \\ \vdots \\ \sum_{i=0}^{m} y_i \underbrace{\varphi_n(x_i)}_{x_i^n} \end{bmatrix}_{(n+1)\times 1} = B^T y$$

Outra notação $\boxed{Bx = y}$

$$\underbrace{B^T . B}_{A}\, x = \underbrace{B^T y}_{b}$$

$m+1$ $n+1$

$$B \Rightarrow \boxed{b_{ij} = \varphi_j (x_i)}$$

$$\underset{(n+1)(m+1)}{B^T}\ B_{(m+1)\times(n+1)} \cdot \underset{(n+1)\times 1}{x} = \underset{(n+1)(m+1)}{B^T} \cdot y$$

$(m+1)\times 1$

Caso Particular:

Polinomial (P_n)

$$\varphi(x) = a_0 + a_1 . x + a_2 x^2 + \dots + a_n x^n$$

$$\{\underset{\varphi_0}{1}, \underset{\varphi_1}{x}, \underset{\varphi_2}{x^2}, \dots, \underset{\varphi_n}{x^n}\}$$

$$A = \begin{bmatrix} \sum_{i=0}^{m} 1 & \sum_{i=0}^{m} x_i & \sum_{i=0}^{m} x_i^2 & \cdots & \sum_{i=0}^{m} x_i^n \\ \sum_{i=0}^{m} x_i & \sum_{i=0}^{m} x_i^2 & \sum_{i=0}^{m} x_i^3 & \cdots & \sum_{i=0}^{m} x_i^{n+1} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \sum_{i=0}^{m} x_i^n & \sum_{i=0}^{m} x_i^{n+1} & \sum_{i=0}^{m} x_i^{n+2} & \cdots & \sum_{i=0}^{m} x_i^{2n} \end{bmatrix}$$

$$b = \begin{bmatrix} \sum_{i=0}^{m} y_i \\ \sum_{i=0}^{m} y_i \cdot x_i \\ \vdots \\ \sum_{i=0}^{m} y_i \cdot x_i^{n} \end{bmatrix}$$