

Mal Condicionamento

$$\boxed{Ax = b} \quad \textcircled{1}$$

Perturbar b : \swarrow

$$A(x + \delta x) = b + \delta b$$

$$\cancel{Ax} + A\delta x = \cancel{b} + \delta b$$

Por $\textcircled{1}$: $A\delta x = \delta b \Rightarrow \boxed{\delta x = A^{-1} \delta b}$ \nwarrow $\textcircled{2}$

$\det(A) \neq 0$

De ② e ①

$$\|Sx\| = \|A^{-1}Sb\| \leq \|A^{-1}\| \cdot \|Sb\| \quad ③$$

$$\|b\| = \|Ax\| \leq \|A\| \cdot \|x\| \quad ④$$

Usando 3. e 4.

$$\|Sx\| \cdot \|b\| \leq \|A^{-1}\| \cdot \|Sb\| \cdot \|A\| \cdot \|x\|$$

$$\div \|x\| \quad \div \|b\|$$

$$\frac{\|Sx\|}{\|x\|} \leq \underbrace{\|A^{-1}\| \cdot \|A\|}_{\text{Cond}(A)} \cdot \frac{\|Sb\|}{\|b\|}$$

$$\text{cond}(A) = \|A^{-1}\| \cdot \|A\| \geq \|A \cdot A^{-1}\| =$$

$$= \|I\| = 1$$

$$\boxed{\text{Cond}(A) \geq 1}$$

$$\text{Cond}(A) \gg 1$$

Matriz é mal
condicionada.

Particular A : $Ax = b$

$$(A + SA)(x + \delta x) = b$$

↙
⋮
↘

$$\frac{\|\delta x\|}{\|x\|} \leq \overbrace{\text{cond}(A)}^{\text{cond}(A)} \cdot \frac{\|SA\|}{\|A\|}$$

↘

Inversa
computacionalmente
custoso

$$A \cdot A^{-1} = I$$

$$A b_1 = e_1 \quad \hookrightarrow \text{1ª coluna de } I$$

$$\hookrightarrow \text{1ª coluna de } A^{-1}$$

$$A \cdot A^{-1} = I$$

$$A^{-1} = [b_1 | b_2 | \dots | b_n]$$

$$A \cdot b_1 = e_1$$

$$A \cdot b_2 = e_2$$

$$A \cdot b_3 = e_3$$

\vdots

$$A \cdot b_n = e_n$$

Resolvent

\approx sistemas
lineares!