CS251 - Homework 3: Trees and Graphs

Out: March 04, 2016 @ 9:00 pm
Due: March 25, 2016 @ 9:00 pm

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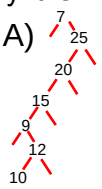
Read this before start: Not all questions are multiple choice, some of them may require an explanation or simulating an algorithm. In those questions give as much information you consider is required for the solution. Read carefully each question in order to identify what is being asked.

**** For all questions, any answer without an explanation (even though it is correct) will be graded with 0 points**. **

1 (10 pts). Consider binary search trees of size 30 containing keys 1 through 30. The query search(10) is executed. For each one of the following sequences explain whether there exists a tree so that the entries in the sequence are keys on the path from the root to the node with key 10. Either sketch a tree or explain why it is not possible.

a) 7→25→20→15→9→12→10
b) 7→25→15→20→16→10
c) 25→20→18→22→16→10

A)



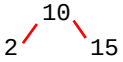
B) This is not valid because from 15 the search went to 20 (to the right) which is greater than 15 and the search needed to go lower (to the left).

C) This is not valid because from 18 the search went to 22 (to the right) which is greater than 18 and the search needed to go lower (to the left).

2 (10 pts). For a general 2-3 tree (i.e., each node has 2 or 3 children) of height h, what is the minimum and maximum number of nodes such a 2-3 tree may have?

a) $2^{h+1}-1$, $\frac{3^{h+1}-1}{2}$
b) $2^{h-1}-1$, $\frac{3^{h-1}-1}{2}$
c) 2^{h+1}, $\frac{3^{h+1}}{2}$
d) 2^{h-1}, $\frac{3^{h-1}}{2}$

The minimum number of nodes in a 2-3 tree is 2^(h+1)-1.
Given h=1 the minimum tree would resemble:



3 (10 pts). A graph G is an undirected graph with n vertices and each vertex has exactly degree d (i.e., d edges touching it). Assume n is odd. What can you tell about d?

a) d must be an odd number
b) d must be an even number
c) d can be odd or even
d) None of the above

If there are an odd number of vertices, and each vertex has the same number of edges touching it, it must be an even number of edges. (2 edges each below)

4 (10 pts). We have n elements. How many binary trees of height *n-1* can be built from them?

a) 1
b) 2
c) $n!$
d) $2^{(n-1)}$

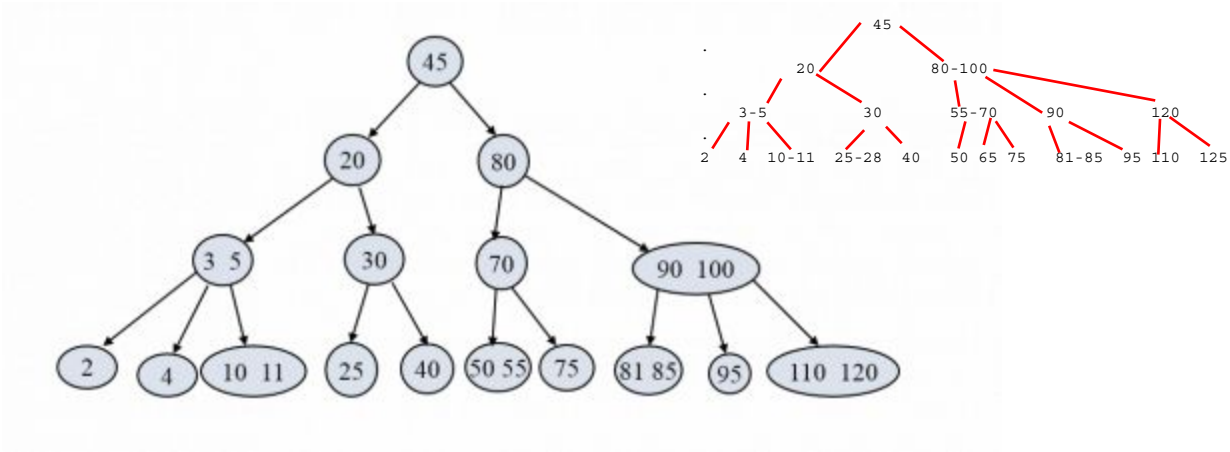
The tree would look like a linked list, so the height would be 2^(n-1) because each node would have exactly one child.

5 (10pts). Given a connected graph of n nodes and e edges, the minimum and maximum number of edges of the graph is:

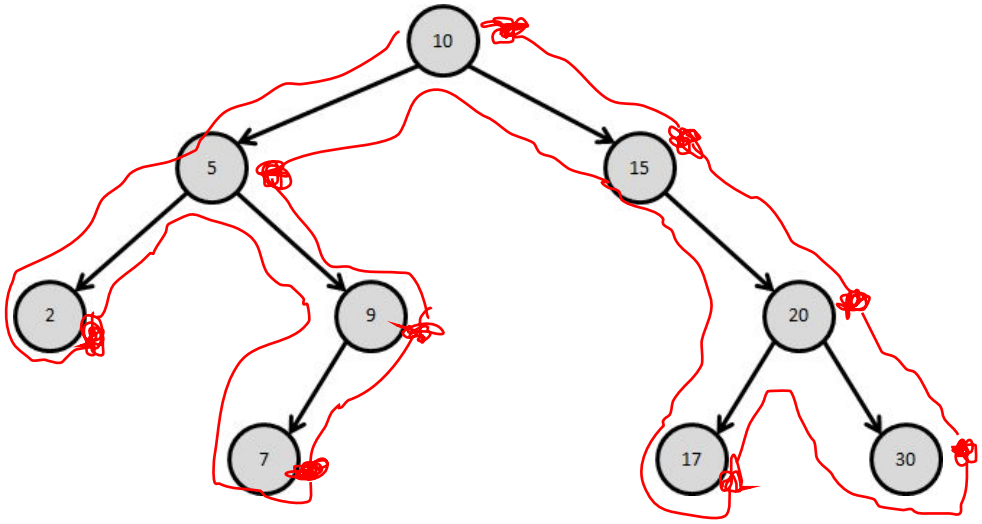
a) $n-1 \le e \le n(n-1)/2$
b) $n^2 \le e$
c) $n \le e \le n^2$
d) None of the above

The minimum is n (each node links to 2 others)
The maximum is 2n (each node links to all others)

6 (10 pts). In the 2-3 tree given below, execute insert(28), insert(125) and insert(65) using analogous re-balancing operations as in a 2-3-4 tree but using a 2-3 tree (i.e., attempt at all times to keep the tree balanced). Draw the resultant 2-3 tree after executing the operations.

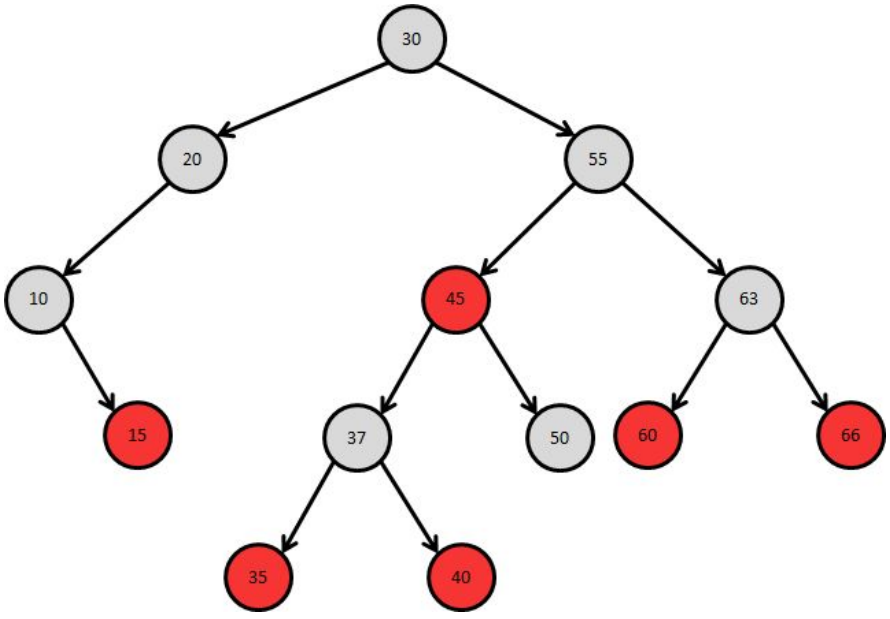


7 (10 pts). Given the tree below, which sequence is a post-order traversal?



a) 10,5,15,2,9,20,7,17,30
b) 10,5,2,9,7,15,20,17,30
c) 2,5,7,9,10,15,17,20,30
d) 2,7,9,5,17,30,20,15,10

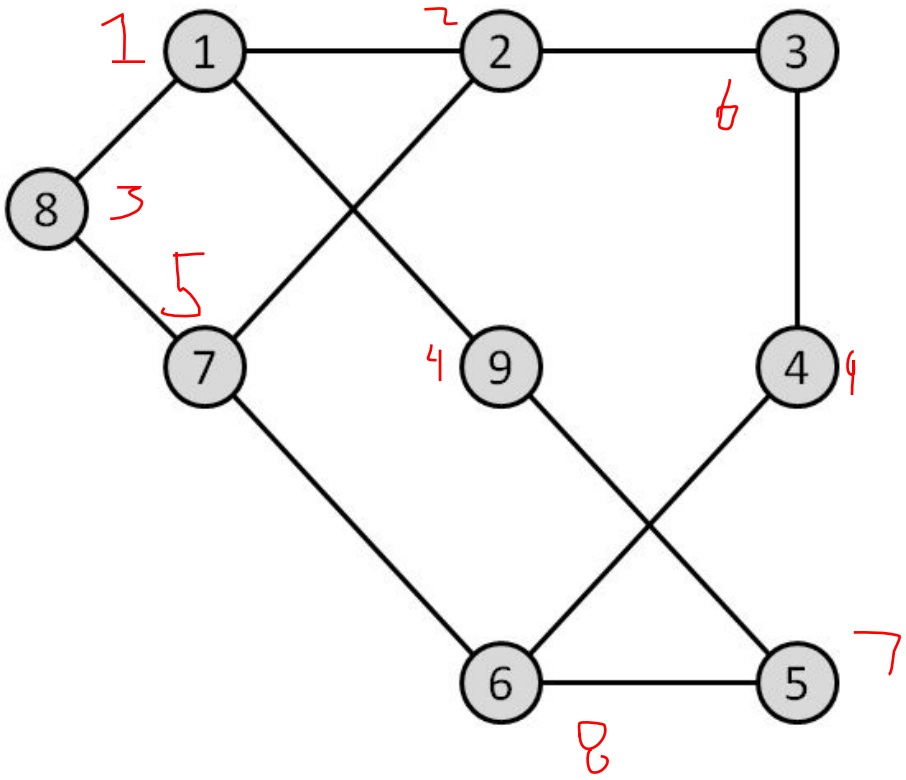
8 (10 pts). Given the tree below, is it a valid Red-Black tree?



a) Yes
b) No

35 and 40 are too far away from the root.

9 (10 pts). In a BFS of the graph the nodes are dequeued in a particular sequence for processing. Which of the following node sequences is from a breadth-first search of the graph shown, beginning at node 1:



a) 1, 2, 8, 9, 3, 5, 7, 4, 6
b) 1, 2, 8, 9, 5, 7, 3, 4, 6
c) 1, 2, 8, 9, 7, 5, 6, 3, 4
d) 1, 2, 8, 9, 7, 3, 5, 6, 4
e) All sequences can come from a BFS

The sequence explores all children before moving on to any grandchildren.

10 (10 pts). In a DFS the nodes are encountered for the first time in a particular sequence. Using the same graph from previous question, which of the following node sequences is not from a depth-first search of the graph shown:

a) 1, 2, 7, 8, 6, 5, 9, 4, 3
b) 1, 9, 5, 6, 4, 3, 2, 7, 8
c) 1, 2, 3, 8, 7, 4, 5, 6, 9
d) 1, 8, 7, 2, 3, 4, 6, 5, 9
e) All sequences can come from a DFS

The node 3 does not connect to 8, and it has a child that is unexplored when sitting at 3, therefore, moving on to 8 is not a DFS.

11 (10 pts). Let G be a graph with n vertices. If G has n or more edges, then G must have a cycle.

This guarantees that at least 2 nodes are connected to each other, forming a cycle

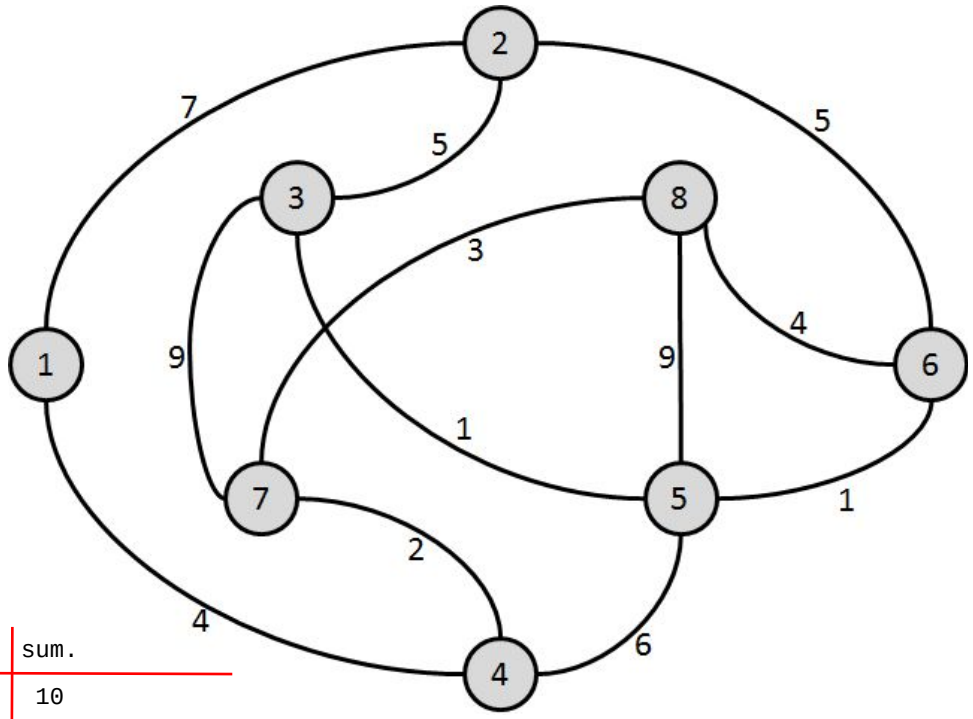
a) Always true
b) Sometimes true, other times false
c) Never true

12 (10 pts). Assume a *directed graph* G in which weight of all edges are equal. We need to compute the shortest path from node S to every other node in G. Which of the followings represent time complexity of this computation (n and e representing number of vertices of and number of edges of the graph, respectively):

a) $O(n^2)$
b) $O(n * e)$
c) $O(n\ log(e))$
d) $O(n + e)$
e) None of the above

You would have to travel every possible path from S to every node. Making you take n+e trips.

13 (10 pts). Given the graph below, list the vertices along the shortest path (and the total edge sum) to each node starting from node 5.



start	vert.	sum.
1:	4	10
2:	3	6
3:	N/A	1
4:	N/A	6
6:	N/A	1
7:	4	8
8:	6	5

Submit Instructions:
The homework must be turned in by the due date and time using the turnin command. Follow the next steps:

1. Please make sure your submission is legible! (No cellphone pictures please! All ITAP labs have scanners for you to use)
2. Login to data.cs.purdue.edu (you can use the labs or a ssh remote connection).
3. Make a directory named with your username and copy your solution (in pdf format) there. (**Important:** Such pdf file should be the only one contained within the folder).
4. Go to the upper level directory and execute the following command:
   turnin -c cs251 -p hw3 your_username
   (Important: previous submissions are overwritten with the new ones. Your last submission will be the official and therefore graded).
5. Verify what you have turned in by typing **turnin -v -c cs251 -p hw3**
   (**Important:** Do not forget the -v flag, otherwise your submission would be replaced with an empty one). If you submit the wrong file you will not receive credit.