Hashbrown

Developed by Hyungho Choi

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Introduction

This program was proposed and developed in order to help the solution and representation of the multivariable first-order differential equations. The program embeds a Runge-Kutta Fourth order solution Method

The author of this program had inspiration from the Mathematical Modeling and Differential Equations class from the Korea Science Academy. And this program is conversely fit for students looking for a reliable solution software.

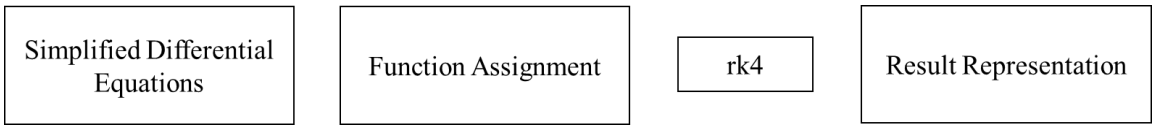
Special Thanks to prof. Y.D.Kim for the intellectual aid.

Documentation

Please read and follow the instructions on <README.txt> to properly install and use this program

Overview

The overall system workflow



0. Simplification

Before you look for the solution, the differential equation must be simplified to a simple multivariable first-order differential equation which has the form

$$\mathrm{x}_1' = f_1(t, x_1, x_2, \dots, x_n)$$

$$\mathrm{x}_2' = f_2(t, x_1, x_2, \dots, x_n)$$

...

$$\mathrm{x}_\mathrm{n}' = f_n(t, x_1, x_2, \dots, x_n)$$

The number of equations is irrelevant as long as the equations are given as the form above

For example, consider the forced harmonic equation

$$\mathrm{m}x'' + \mathrm{b}x' + \mathrm{k}x = F_0 \sin(wt)$$

We could simplify this by appointing $\mathrm{y} = \mathrm{x}'$ and concluding with a series of equations given as

$$\mathrm{x}' = y$$

$$\mathrm{y}' = -\frac{b}{m}y - \frac{k}{m}x + \frac{\mathrm{F}_0}{m}\sin(wt)$$

This process must be done by the user, the program can NOT (and should not) handle the simplification. You should also have the initial values ($\mathrm{x}_1(0), x_2(0) \dots \mathrm{x}_\mathrm{n}(0)$) Figured out accordingly

Developed in python v3.5.1 Any feedbacks or questions are welcome planner.king@gmail.com

1. Assigning Function Class Variables

Now the coding begins. Before we start, all you need to import is the **hashbrown** library. The library imports itelf in a hierarchy.

```
from hashbrown import *
```

From the simplified solutions from section 0, we assign **[Function]** class variables for each equation.

```
class Function:

    def __init__(self, f, var, name):
```

f is a python function that governs each equation. **var** is the number of variables that the fuction takes (including t) **name** is the name of this variable.

f could be separately defined and then plugged into the **[Function]** class constructor.

Following with the example from section 0, the **[Function]** class variables could be defined as follows

```
from hashbrown import *
b = 1
m = 1
k = 1
F0 = 1
w = 10
def xp(t,x,y):
    return y
def yp(t,x,y):
    return -(b/m)*y - (k/m)*x + (F0/m)*sin(w*t)
F1 = Function(xp,3,'Displacement')
F2 = Function(yp,3,'Speed')
```

Make good use of the global variables

Make sure you match the number of variables with all the fuctions. Even with functions like `xp`, we need all three parameters mentioned

We could also get creative with the fuctions. Suppose the force is no longer applied after `tf`. Then we could define

```
from hashbrown import *
b = 1
m = 1
k = 1
F0 = 1
w = 10
tf = 10
def F(t):
    if t>=tf:
        return 0
    return F0
def xp(t,x,y):
    return y
def yp(t,x,y):
    return -(b/m)*y - (k/m)*x - (F(t)/m)*sin(w*t)
```

2. Obtaining result dataset

The Runge-Kutta function (**`rk4`**) generates the solution dataset for the equation. It takes in four major variables

```
rk4(ts, te, step, s, name)
```

`ts, te` denotes intial and final time, *step* denotes the timestep of each computation and `name` is the desired name of the result

The most crucial parameter is the parmeter ***`s`***.

It takes a tuple of tuples ***`( (values) , (functions) ).`*** Each element in ***`values`*** are the initial values of each variable, and each element in ***`functions`*** takes a **`[Function]`** class element.

It is very important that the values are in the same order as functions.

Continued from the above example (the force stopping after `tf`) Let's generate a numerical solution. Where the initial state is at equilibrium (x,y are both zero)

```
ts = 0
te = 20
R = rk4(ts,te,0.0001,((0,0),(F1,F2)),'Forced Oscillation')
```

Now, R contains the result for the differential equation.

3. Result Representation

The result obtained from section 2 is a **`[Result]`** class object. This class contains multiple methods to represent data.

3.0. Simple Methods

`print`

simply printing the object prints the whole dataset in line order (in some IDE, the data will get omitted)

`.get(t,varN='')`

`.get` method gets the closest dataset to the desired time entered. Putting in just **`t`** will get all the parameters at that time(tuple), and specifying **`varN`** will get the result in that specified time in that specified variable

Continuing from section 2,

```
print (R.get(10))        #(9.999999999990033, [-0.004728505645778045, 0.09209838842791386])
print (R.get(10,'Speed'))# 0.09209838842791386
```

3.1. Export Data

There are a number of ways to export data out of the program

`.writeCSV(s=1)`

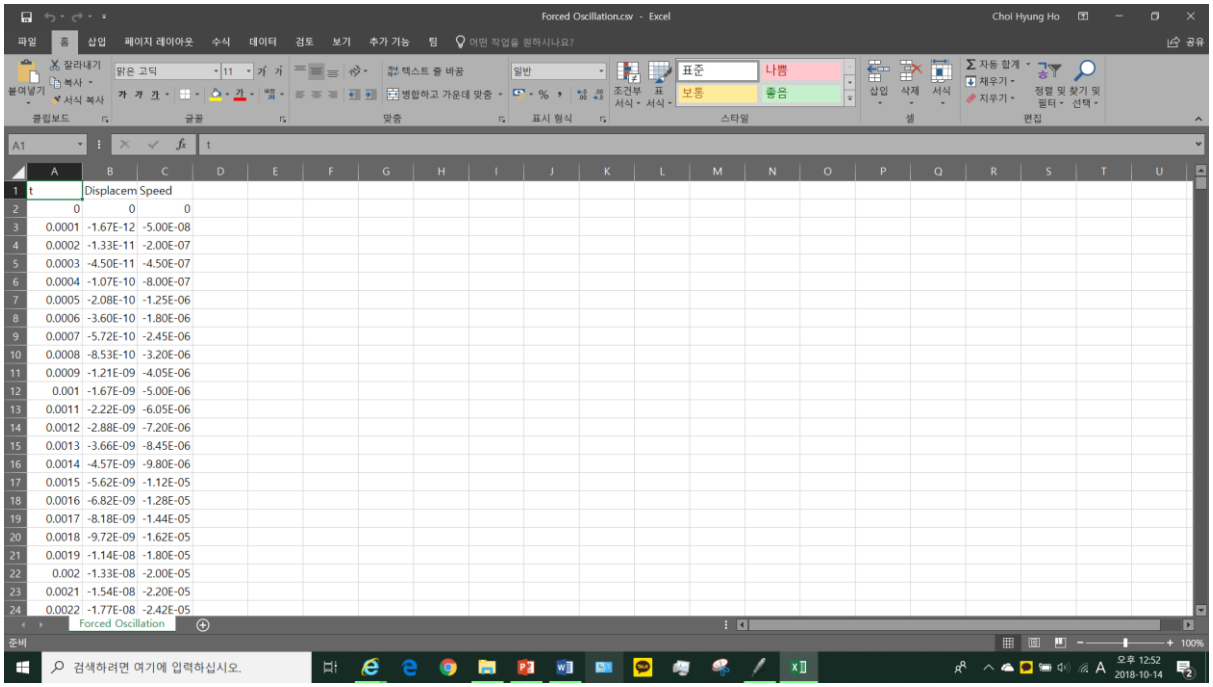
This method exports the dataset into the current folder. Parameter s denotes how many datas to actually

save. **`.writeCSV()`** will save everything, and **`.writeCSV(10)`** will save every 10th data. **`writeCSV(10)`** is recommended since the saving of CSV and EXCEL files are a long process

`.writeXLSX(s=1)`

Same deal with excel. Note that saving the excel file takes much longer than that of CSV. So use this when the datasets are small. Saving to CSV and converting to EXCEL is recommended (Until you have extra knowledge about **`openpyxl`**).

Continuing from section 2, **`R.writeCSV()`** saves as follows.



t	Displacem	Speed
0	0	0
0.0001	-1.67E-12	-5.00E-08
0.0002	-1.33E-11	-2.00E-07
0.0003	-4.50E-11	-4.50E-07
0.0004	-1.07E-10	-8.00E-07
0.0005	-2.08E-10	-1.25E-06
0.0006	-3.60E-10	-1.80E-06
0.0007	-5.72E-10	-2.45E-06
0.0008	-8.53E-10	-3.20E-06
0.0009	-1.21E-09	-4.05E-06
0.001	-1.67E-09	-5.00E-06
0.0011	-2.22E-09	-6.05E-06
0.0012	-2.88E-09	-7.20E-06
0.0013	-3.66E-09	-8.45E-06
0.0014	-4.57E-09	-9.80E-06
0.0015	-5.62E-09	-1.12E-05
0.0016	-6.82E-09	-1.28E-05
0.0017	-8.18E-09	-1.44E-05
0.0018	-9.72E-09	-1.62E-05
0.0019	-1.14E-08	-1.80E-05
0.002	-1.33E-08	-2.00E-05
0.0021	-1.54E-08	-2.20E-05
0.0022	-1.77E-08	-2.42E-05

Notice that the name is already in place

3.3. Graphic representation

There are ways to immediately represent the data graphically

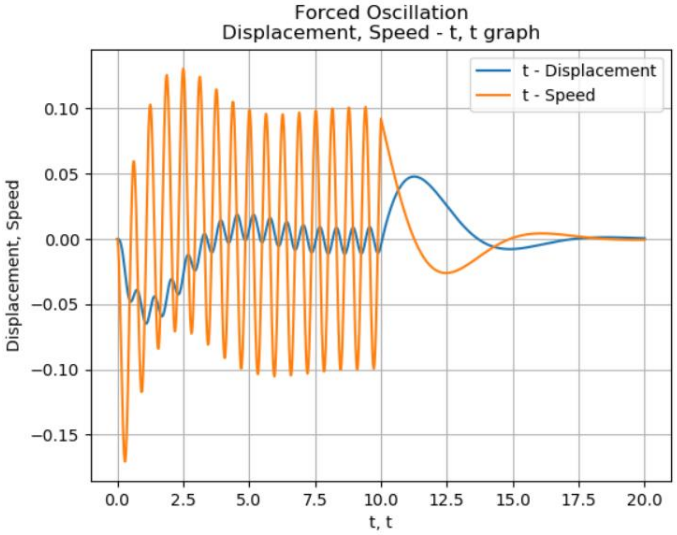
```
.writePLOT(points = 1000,s=1,subj = False)
```

The **`.writePLOT`** method grpahs the result in the desired format. **`points`** govern how many points the whole graph is drawn. 1000 is recommended since the rendering could take extensive time. Setting it to 0 overrides the parameters and follows **`s`**, which is how many to skip (same as that of **`.writeCSV`**).

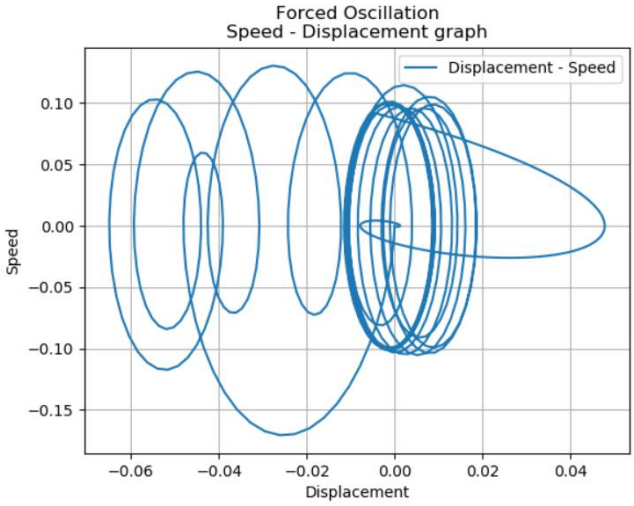
`subj` governs how the graph is drawn. It is a list of tuples containing variable names. The name that denotes time is **`'t'`**. For example, setting **`subj = [('t','x'),('t','y')]`** draws x-t, y-t graphs in the same graph. **`.writeCSV`** also supports 3d plotting. So **`subj`** can have also contain tuples of length 3. Setting **`subj = [('t','x','y')]`** graphs the result on a 3D space. If nothing is given for **`subj`**, it graphs everything with respect to time in 2D.

Giving more examples from the Oscillation,

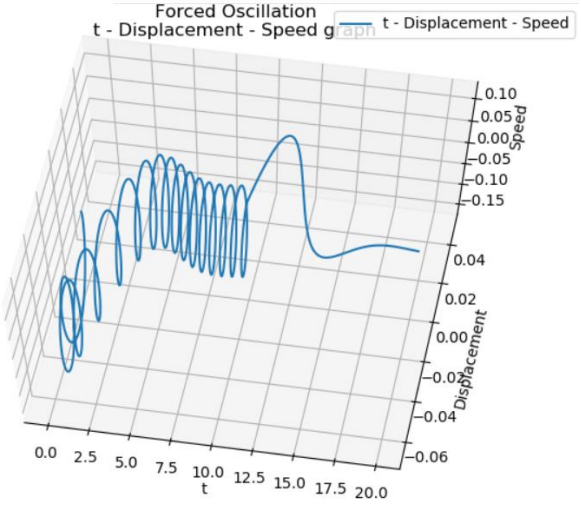
```
R.writePLOT()
```




```
R.writePLOT(subj = [('Displacement','Speed')]
```




```
R.writePLOT(subj = [('t','Displacement','Speed')]
```



`.animateF(subj = [],scale = 1, tail = 1):`

The **`.animateF`** method animates the result in a desired fashion. The important aspect of **`.animateF`** is that it animates entities only with respect to time (which is probably all that is needed for solutions of differential equations).

The **`subj`** parameter works the same with that of **`.writePLOT`**. It governs what is to be animated. **`subj = [('x','y')]`** will animate the variables with name **`'x','y'`** with respect to time. **`.animateF`** also supports 3D animations.

The **`scale`** parameter determines how fast the animation is played. Unfortunately, the program does not support connection to Realtime. So the **`scale`** parameter only controls relative speed. Larger **`scale`** means faster animation. It is recommended to be about one tenth of the total simulation time.

CAUTION: Do not close the Animation Window before the animation closes. The Window is designed to close two seconds after the animation ends.

The **`tail`** parameter rules the length of the tail behind the animation. If the animation scribbles too much, the **`tail`** parameter could be set smaller to clean up the animation. Setting **`tail`** to 0 Traces out the whole process without erasing any of it.

4. Summary

The functions and methods presented above are the critical parts of this program. They could each be encapsulated, giving extra versatility.

To summarize, the summary of the example is given.

```
from hashbrown import *

def interruptedForcedOsc(b,m,k,F0,w,tf,ts,te):
    def F(t):
        if t>=tf:
            return 0
        return F0

    def xp(t,x,y):
        return y
    def yp(t,x,y):
        return -(b/m)*y - (k/m)*x - (F(t)/m)*sin(w*t)

    F1 = Function(xp,3,'Displacement')
    F2 = Function(yp,3,'Speed')

    return rk4(ts,te,0.0001,((0,0),(F1,F2)),'Forced Oscillation')

Res = interruptedForcedOsc(1,1,1,10,1,10,0,20)

Res.writeCSV()
Res.writePLOT()
```

The `interruptedForcedOsc` function generates a solution that gives Displacement and Speed values when the Drag constant is `b`, mass is `m`, spring constant is `k`, driving force is a sinusoidal force of amplitude `F0` with the angular velocity `w`, that stops at `tf`. And it is simulated from `ts` to `te`. And this script uses the result to save a CSV file and draw a graph. The function itelf is in the tutorial. But representing the results is left for the user.

5. Additional Documentation

There are multiple methods and functions that were not covered in the documentation above. Although these features are not the major features, it is powerful in some mathematical and visual aspects.

- More on **`rk4(ts, te, step, s, name)`**

The full representation of the **`rk4`** function's parameters are

`rk4(ts, te, step, s, name, ti)`

and there are two additional features to this function.

1.Reverse calculation

If `ts` is greater than `ts`, the function automatically assumes the initial value is actually given for the `ts` and reverse calculates back to `te`. So, the **`rk4`** function is actually capable of taking 'final conditions' to calculate the result

2.Midpoint calculation

The same is true with giving 'middle conditions.' This is what the parameter `ti` is for. The parameter denotes the time the initial condition applies. For example, setting `ts = 0, te = 10, ti = 5` finds a solution where the initial conditions are true a t=5. Since the process takes two calculations, two computations will take place. (one forward and one reverse)

- **`mapF(ts,te,step,f,name)`**

mapF returns a **`[Result]`** class object of a simple time-based function. The parameter `f` takes a tuple of **`[Function]`** class elements to return the result as the value of that function form `ts` to `te` and by timestep `step`. Since it returns a **`[Result]`** class object, it could be represented as shown above in section 4.

- **`.merge(R)`**

The **`.merge(R)`** method returns a **`[Result]`** class object that merges two instances of results (`self, R`). If the time ranges are incompatible, the function simply pastes the first or last value as the result representing that time. Also if the timestep between two results are not compatible, It adopts the larger timestep as its own timestep. Although the merging for results are possible between for different timesteps, it is recommended that the one timestep is a multiple of another.

To avoid name collision, the merged values have their original names as prefixes. But any name with '_' will not be changed. Here is an example of a merged plot.