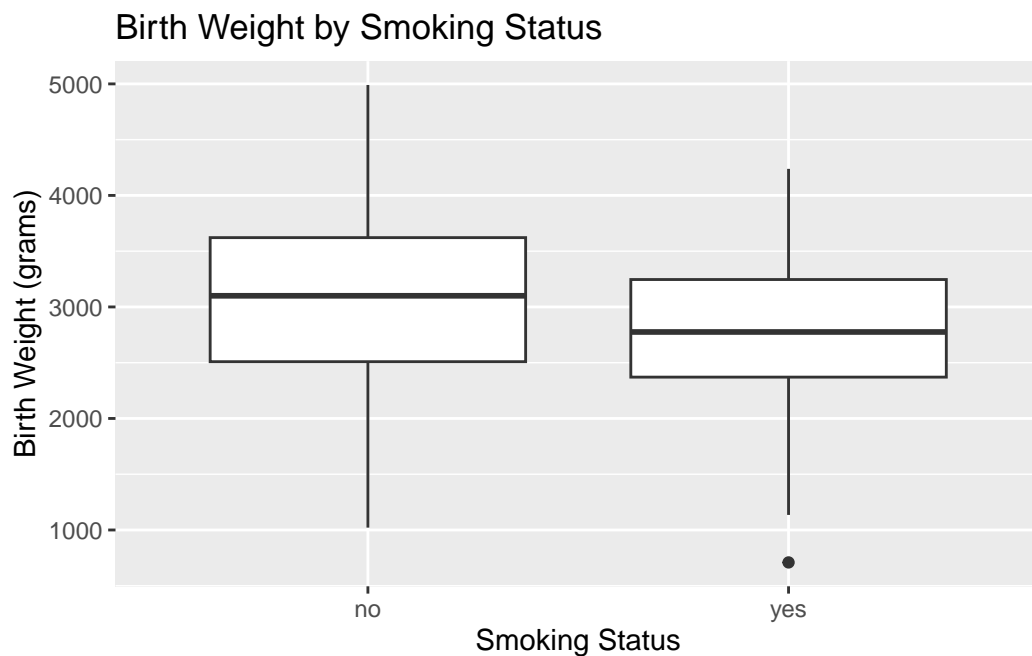


# Advanced medical Statistics – Answers lab 3

## Part 1: Independent Samples t-test and Mann-Whitney U Test

### Exploratory data analysis

```
# Create boxplot comparing birth weights for mothers who smoked and those who did not
ggplot(lowbwt, aes(x = smoke, y = bwt)) +
  geom_boxplot() +
  labs(x = "Smoking Status", y = "Birth Weight (grams)", title = "Birth Weight by Smoking Status")
```



### Question 1

Based on the boxplot, do you expect the smoking status to have an effect on birth weight?

### Answer question 1

Based on the boxplot, it appears that mothers who smoked have lower birth weights on average compared to those who did not smoke. This suggests that smoking status may indeed have an effect on birth weight.

## Independent samples t-test

```
# Perform independent samples t-test
independent_t_test <- t.test(bwt ~ smoke, data = lowbwt, var.equal = TRUE)

# Print results
print(independent_t_test)
```

### Two Sample t-test

```
data:  bwt by smoke
t = 2.6336, df = 187, p-value = 0.009156
alternative hypothesis: true difference in means between group no and group yes is not equal
95 percent confidence interval:
 70.69274 492.73382
sample estimates:
mean in group no mean in group yes
 3054.957          2773.243
```

### Question 2

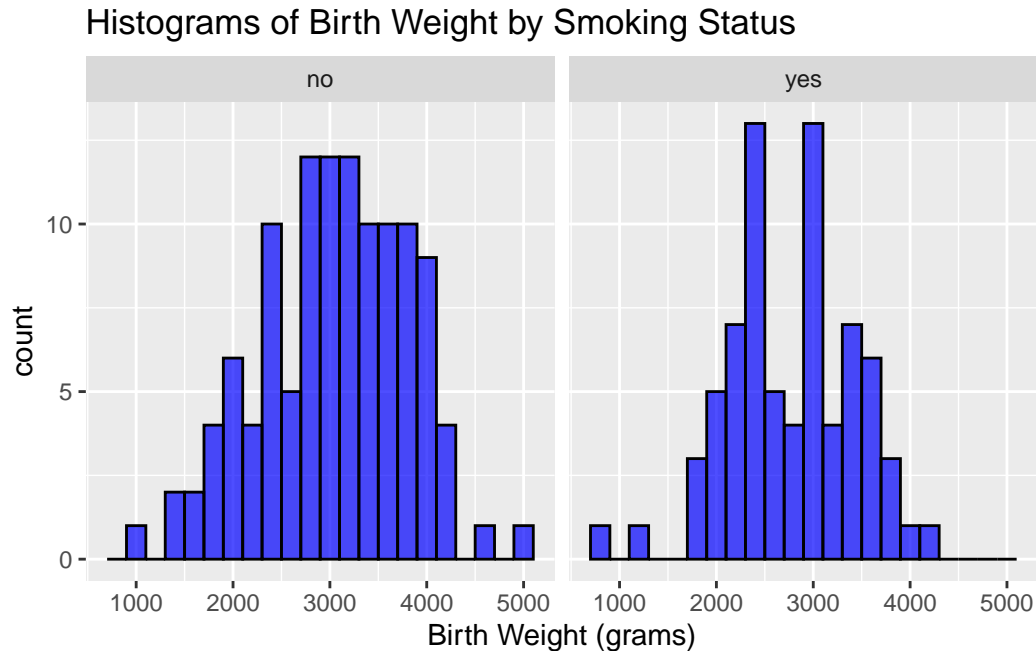
Based on the results of the independent samples t-test, is there a statistically significant difference in birth weight between mothers who smoked and those who did not?

### Answer question 2

The p-value of the independent samples t-test ( $p=0.009$ ) is less than 0.05, which indicates that there is a statistically significant difference in the mean birth weight between mothers who smoked and those who did not.

## Checking of assumptions

```
# Create histograms of birth weight by smoking status
ggplot(lowbwt, aes(x = bwt)) +
  geom_histogram(binwidth = 200, fill = "blue", colour="black", alpha = 0.7) +
  facet_wrap(~smoke) +
  labs(x = "Birth Weight (grams)", title = "Histograms of Birth Weight by Smoking Status")
```



### Question 3

Do the histograms indicate that the birth weight data are approximately normally distributed for both groups?

### Answer question 3

Based on the histograms, the distributions in both groups appear reasonably normal for the purposes of conducting an independent samples t-test.

```
# Perform Levene's test for homogeneity of variances
levene_test <- leveneTest(bwt ~ smoke, data = lowbwt)
levene_test
```

```
Levene's Test for Homogeneity of Variance (center = median)
      Df F value Pr(>F)
group  1  1.3901 0.2399
      187
```

#### Question 4

Based on the Levene test, does the assumption of equal variances hold?

#### Answer question 4

The p-value of the Levene test ( $p = 0.2399$ ) is greater than 0.05, indicating insufficient evidence to reject the null hypothesis of equal variances. Therefore, we can assume that the assumption of equal variances holds.

### 95% Confidence Interval for the mean difference

```
# Calculate summary statistics by smoking status
summary_stats <- lowbwt %>%
  group_by(smoke) %>%
  summarise(mean = mean(bwt), sd = sd(bwt), n = n())
summary_stats
```

```
# A tibble: 2 x 4
  smoke mean    sd     n
<fct> <dbl> <dbl> <int>
1 no    3055.  752.  115
2 yes   2773.  660.   74
```

#### Exercise

Based on these summary statistics, calculate the pooled standard deviation and the standard error of the mean difference. Then compute the 95% confidence interval for the mean difference in birth weight between mothers who smoked and those who did not. You may simplify the calculation by using the 97.5th percentile from the standard normal distribution (1.96) rather than the corresponding percentile from the t-distribution.

## Answer

### Calculations for Birth Weight Difference

#### 1. Pooled Standard Deviation

The pooled standard deviation is calculated using the formula:

$$s_p = \sqrt{\frac{(n_{no} - 1) \cdot s_{no}^2 + (n_{yes} - 1) \cdot s_{yes}^2}{n_{no} + n_{yes} - 2}}$$

Substituting the values:

$$s_p = \sqrt{\frac{(115 - 1) \cdot 752^2 + (74 - 1) \cdot 660^2}{115 + 74 - 2}} = 717.49$$

#### 2. Standard Error of the Mean Difference

The standard error (SE) of the mean difference is calculated as:

$$SE = s_p \cdot \sqrt{\frac{1}{n_{no}} + \frac{1}{n_{yes}}}$$

Substituting the values:

$$SE = 717.49 \cdot \sqrt{\frac{1}{115} + \frac{1}{74}} = 106.93$$

#### 3. Mean Difference

The difference in means between the groups is:

$$\text{Mean Difference} = \bar{X}_{no} - \bar{X}_{yes} = 3055 - 2773 = 282$$

#### 4. 95% Confidence Interval

The confidence interval (CI) is calculated as:

$$CI = \text{Mean Difference} \pm 1.96 \cdot SE$$

Substituting the values:

$$CI = 282 \pm 1.96 \cdot 106.93 = (72.43, 491.57)$$

### Results

- **Pooled Standard Deviation:** 717.49
- **Standard Error of the Mean Difference:** 106.93
- **95% Confidence Interval for the Mean Difference:** (72.43, 491.57)

### Question 5

Does your manually calculated 95% confidence interval for the mean difference in birth weight between the two groups agree with the one provided in the output of the `t.test()` function?

### Answer question 5

The manually calculated 95% confidence interval for the mean difference in birth weight between mothers who smoked and those who did not is (72.43, 491.57), is approximately equal to the one provided in the output of the `t.test()` function, which is (70.69, 492.73). The latter is slightly wider due to the use of the 97.5th percentile of the t-distribution with 187 degrees of freedom (1.97) rather than the standard normal distribution (1.96).

## Mann-Whitney U Test

```
# Perform Mann-Whitney U test
wilcoxon_test <- wilcox.test(bwt ~ smoke, data = lowbwt)
wilcoxon_test
```

Wilcoxon rank sum test with continuity correction

```
data:  bwt by smoke
W = 5243.5, p-value = 0.007109
alternative hypothesis: true location shift is not equal to 0
```

### Question 6

What are the null and alternative hypotheses for the Mann-Whitney U test, and what does the p-value indicate about the difference in birth weight between mothers who smoked and those who did not?

### Answer question 6

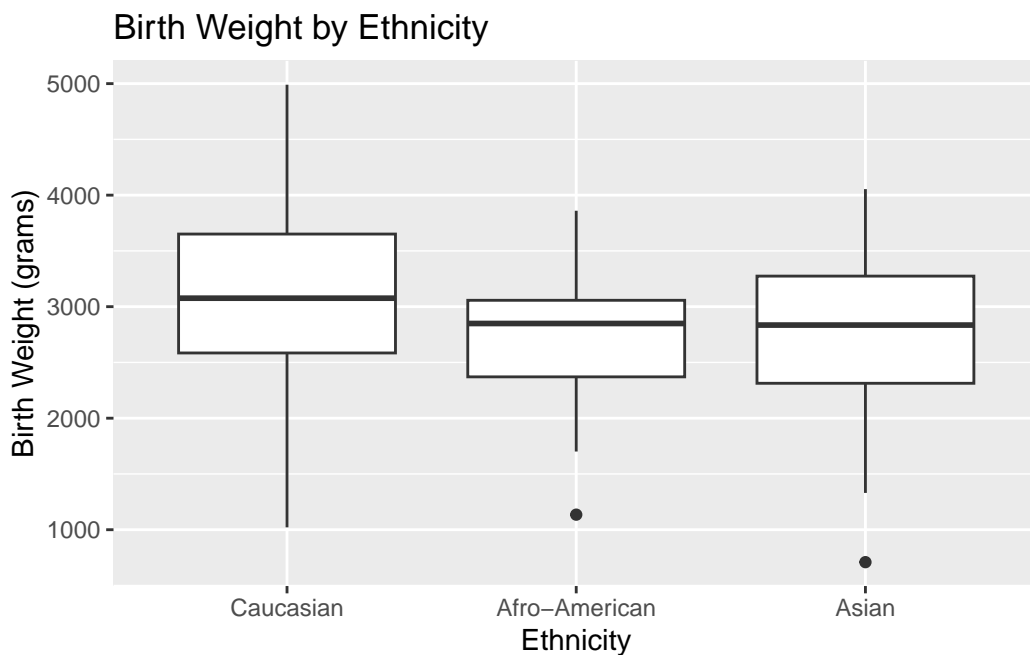
The null hypothesis for the Mann-Whitney U test is that there is no difference in the median birth weight between mothers who smoked and those who did not. The alternative hypothesis is that there is a difference in median birth weight between the two groups. The p-value of the test is 0.0071, which is less than 0.05. Therefore, we reject the null

hypothesis and conclude that there is a statistically significant difference in median birth weight between mothers who smoked and those who did not.

## Part 2: One-Way ANOVA and Kruskal-Wallis Test

### Exploratory data analysis

```
# Create boxplot comparing birth weights across ethnic groups
ggplot(lowbwt, aes(x = ethnicity, y = bwt)) +
  geom_boxplot() +
  labs(x = "Ethnicity", y = "Birth Weight (grams)", title = "Birth Weight by Ethnicity")
```



### Question 7

What does the boxplot suggest about the distribution of birth weights across different ethnic groups?

### Answer question 7

The boxplot suggests that there may be differences in birth weight across the three ethnic groups. More specifically, The mean birth weight appears to be higher for the Caucasian group compared to the Afro-American and Asian groups.

## One-way ANOVA

```
# Perform One-Way ANOVA
anova_result <- aov(bwt ~ ethnicity, data = lowbwt)

# Print summary of ANOVA
summary(anova_result)
```

```
              Df    Sum Sq Mean Sq F value    Pr(>F)
ethnicity      2  5070608 2535304    4.972 0.00788 **
Residuals    186 94846445  509927
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

### Question 8

What conclusions can be drawn from the results of the one-way ANOVA?

### Answer question 8

The p-value from the one-way ANOVA ( $p=0.0079$ ) is less than 0.05, indicating that there is at least one ethnic group with a significantly different birth weight.

## Post-hoc tests

```
# Perform pairwise comparisons with Bonferroni correction
posthoc <- pairwise.t.test(lowbwt$bwt, lowbwt$ethnicity, p.adjust.method = "bonferroni")
posthoc
```

Pairwise comparisons using t tests with pooled SD



```
data: lowbwt$bwt and lowbwt$ethnicity
```

	Caucasian	Afro-American
Afro-American	0.048	-
Asian	0.027	1.000

```
P value adjustment method: bonferroni
```

#### Question 9

What conclusions can be drawn from the post-hoc comparisons?

#### Answer question 9

It follows from the post-hoc comparisons with the Bonferroni-adjusted p-values that the mean birth weight of Caucasian infants is significantly different from that of Afro-American infants ( $p=0.048$ ) and Asian infants ( $p=0.027$ ). There is no significant difference in mean birth weight between Afro-American and Asian infants ( $p=1$ ).

### Kruskal-Wallis Test

If the assumptions of the one-way ANOVA are violated, we can use the Kruskal-Wallis test as a non-parametric alternative. The test can be performed in R using the `kruskal.test()` function:

```
# Perform Kruskal-Wallis test
kruskal_test <- kruskal.test(bwt ~ ethnicity, data = lowbwt)
kruskal_test
```

Kruskal-Wallis rank sum test

```
data: bwt by ethnicity
Kruskal-Wallis chi-squared = 8.5909, df = 2, p-value = 0.01363
```

#### Question 10

Are the results of the Kruskal-Wallis test consistent with the one-way ANOVA results?

#### Answer question 10

The p-value from the Kruskal-Wallis test ( $p=0.014$ ) is consistent with the one-way ANOVA results, indicating that there is at least one ethnic group with a significantly different median birth weight.

### Part 3: Unguided exercises

#### Effect of hypertension on birth weight

Examine the effect of history of hypertension on birth weight by performing the following steps:

- Create a boxplot to visualize the distribution of birth weights by history of hypertension
- Perform an independent samples t-test to compare the mean birth weights between mothers with and without a history of hypertension
- Check the assumptions of the t-test, including normality and homogeneity of variances
- If the assumptions of the t-test are violated, perform a Mann-Whitney U test as a non-parametric alternative

#### Example solution

```
# Create boxplot comparing birth weights by history of hypertension
ggplot(lowbwt, aes(x = ht, y = bwt)) +
  geom_boxplot() +
  labs(x = "History of Hypertension", y = "Birth Weight (grams)", title = "Birth Weight by H
```



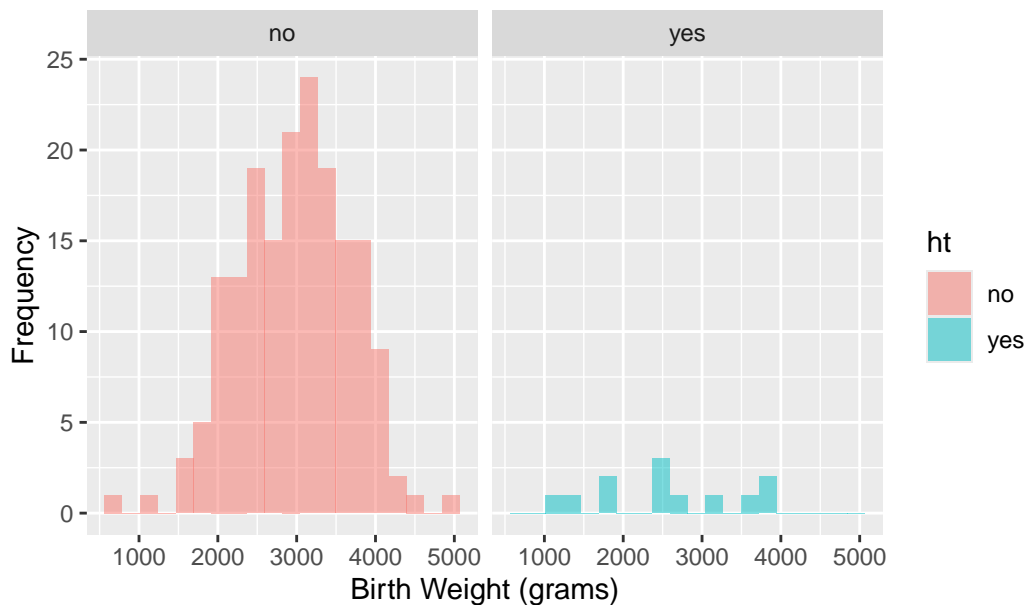
```
# Perform independent samples t-test
t_test <- t.test(bwt ~ ht, data = lowbwt, equal.var = TRUE)
t_test
```

Welch Two Sample t-test

```
data: bwt by ht
t = 1.6124, df = 11.908, p-value = 0.133
alternative hypothesis: true difference in means between group no and group yes is not equal
95 percent confidence interval:
 -153.4955 1024.6170
sample estimates:
mean in group no mean in group yes
      2972.311      2536.750
```

```
# Create histograms to check normality
ggplot(lowbwt, aes(x = bwt, fill = ht)) +
  geom_histogram(position = "identity", alpha = 0.5, bins = 20) +
  facet_wrap(~ht) +
  labs(x = "Birth Weight (grams)", y = "Frequency", title = "Distribution of Birth Weight by
```

Distribution of Birth Weight by History of Hypertension



```
# Perform Levene's test for homogeneity of variances
levene_test <- leveneTest(bwt ~ ht, data = lowbwt)
levene_test
```

```
Levene's Test for Homogeneity of Variance (center = median)
      Df F value Pr(>F)
group  1  1.2851 0.2584
      187
```

**Evaluation of the results:** Although the p-value from the independent samples t-test suggests that there is no statistically significant difference in birth weight between mothers with and without a history of hypertension, the group sizes are highly imbalanced (177 vs. 12). This imbalance reduces the reliability of the t-test results and may limit its power to detect true differences. Additionally, the small sample size in the hypertension group makes it harder to assess assumptions such as normality. To check the robustness of the findings, we also conduct the Mann-Whitney U test as a non-parametric alternative.

```
# Perform Mann-Whitney U test
wilcox_test <- wilcox.test(bwt ~ ht, data = lowbwt)
wilcox_test
```

Wilcoxon rank sum test with continuity correction

data: bwt by ht

W = 1350, p-value = 0.1169

alternative hypothesis: true location shift is not equal to 0

**Overall conclusion:** Both the independent samples t-test and the Mann-Whitney U test suggest that there is no statistically significant difference in birth weight between mothers with and without a history of hypertension. This consistency across methods supports the robustness of the findings, despite the imbalance in group sizes.

### Comparing red cell folate levels across ventilation strategies in cardiac bypass patients

Twenty-two patients undergoing cardiac bypass surgery were randomized to one of three ventilation groups:

- **Group I:** Received a 50% nitrous oxide and 50% oxygen mixture continuously for 24 hours
- **Group II:** Received a 50% nitrous oxide and 50% oxygen mixture only during the operation
- **Group III:** Received no nitrous oxide and a 35-50% oxygen mixture continuously for 24 hours

The data file `ex5_6.sav` (available on Brightspace) contains the red cell folate levels for the three groups after 24 hours of ventilation. The aim of this study is to compare the three groups and test whether they have the same red cell folate levels.

### Tasks

1. **Exploratory data analysis** Create a boxplot to visualize the distribution of red cell folate levels by ventilation group. Based on this plot:
  - What are your first conclusions regarding the means and variances of the different groups?
2. **Perform a one-way ANOVA:**
  - Interpret the results
  - Are the assumptions satisfied?
3. **Try a log transformation on the data:**
  - Perform another one-way ANOVA
  - Are the assumptions satisfied after the transformation?

4. **Determine which means differ:**

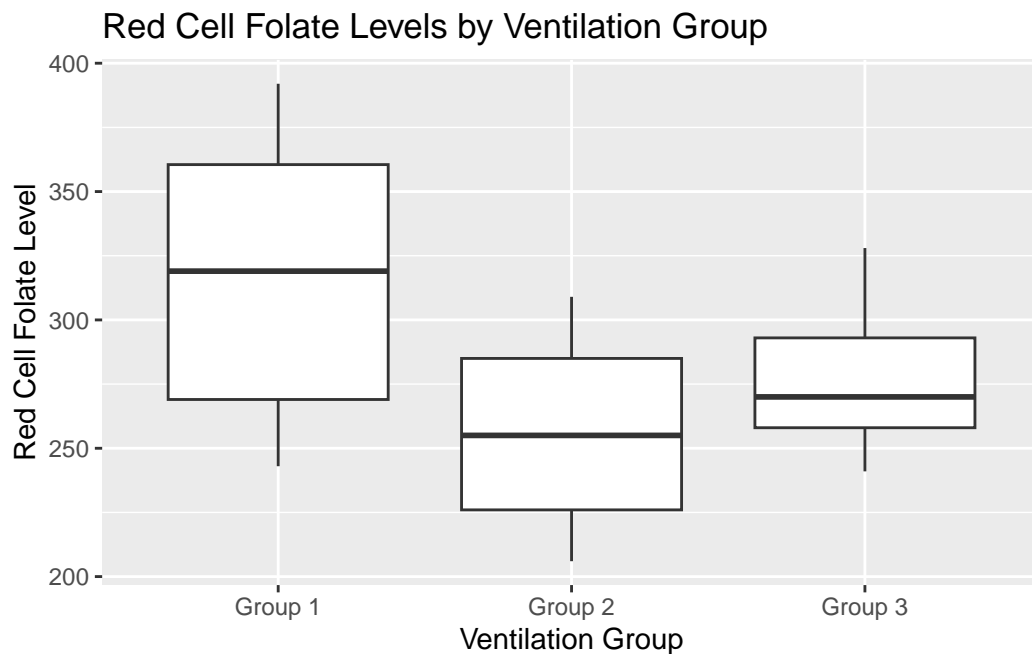
- Which means do you think differ?
- Explain your reasoning.

5. **Try a non-parametric approach:**

- What are your conclusions from this method?

**Example solution**

**Exploratory data analysis**



The groups seem to be different with respect to their means, but also with respect to their variation (range has different length)

**Perform a one way ANOVA, and interpret the results. Are the conditions satisfied?**

```
      Df Sum Sq Mean Sq F value Pr(>F)
group    2  15516    7758   3.711 0.0436 *
Residuals 19  39716    2090
```

---

Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1

```

Levene's Test for Homogeneity of Variance (center = median)
      Df F value Pr(>F)
group  2  3.6413 0.04585 *
      19
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

```

The ANOVA reveals significant results  $P = 0.044$ ; this means that the null hypothesis of equal means in the three groups is rejected. However, according to the test of homogeneity of variances, the assumption of equal variances is violated ( $P\text{-value} = 0.044$ ).

**Try a log transformation on the data, and perform again a one-way ANOVA. Are now the assumptions satisfied?**

```

      Df Sum Sq Mean Sq F value Pr(>F)
group    2 0.1784 0.08922   3.539 0.0494 *
Residuals 19 0.4790 0.02521
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

```

```

Levene's Test for Homogeneity of Variance (center = median)
      Df F value Pr(>F)
group  2   1.889 0.1785
      19

```

The test of equal variances for the log-transformed data reveals that the assumption of equal variances can be made. Test results for the ANOVA:  $P\text{-value} = 0.049$ . This is on the boundary of significance. There is indication that at least one pair of groups have different means.

**Which means do differ according to you? Why?**

```

# Untransformed data
pairwise.t.test(df$rcfl, df$group, p.adj = "bonf")

```

Pairwise comparisons using t tests with pooled SD

data: df\$rcfl and df\$group

	Group 1	Group 2
Group 2	0.042	-
Group 3	0.464	1.000

P value adjustment method: bonferroni

```
# Log-transformed data
pairwise.t.test(df$log_rcfl, df$group, p.adj = "bonf")
```

Pairwise comparisons using t tests with pooled SD

data: df\$log\_rcfl and df\$group

	Group 1	Group 2
Group 2	0.047	-
Group 3	0.597	1.000

P value adjustment method: bonferroni

From the paired comparisons with Bonferroni-corrections it appears that group1 and group 2 differ significantly with respect to their means (but on the boundary, P-value = 0.047).

(e) Try a non-parametric approach on these data. What are now your conclusions?

```
kruskal.test(rcfl ~ group, data = df)
```

Kruskal-Wallis rank sum test

data: rcfl by group

Kruskal-Wallis chi-squared = 4.1852, df = 2, p-value = 0.1234

The Kruskal-Wallis test gives P-value = 0.123, indicating that there are no significant differences between the three groups.

### Caveat

In this example, the sample size is very limited, making it difficult — if not impossible — to verify whether the assumptions underlying the one-way ANOVA are met. The Kruskal-Wallis test is a good alternative in such cases.