calculus is a branch of mathematics that deals with rates of change and the accumulation of quantities.

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circle: $(x-a)^2 + (y-b)^2 = r^2$ where (a, b) is the center and the radius is r

midpoint between $P_1 = (x_1, y_1)$ and $P_2 = (x_2, y_2)$ is $\bar{P_1P_2}$ divided by 2

distance: $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

laws of exponents:

- $x^m x^n = x^{m+n}$
- $\frac{x^m}{x^n} = x^{m-n}$
- $(x^m)^n = x^{mn}$
- $x^{-n} = \frac{1}{x^n}$
- $(xy)^n = x^n y^n$
- $(\frac{x}{y})^n = \frac{x^n}{y^n}$
- $x^{1/n} = \sqrt[n]{x}$
- $x^{\frac{m}{n}} = \sqrt[n]{x^m} = (\sqrt[n]{x})^m$

special factorizations

- $x^2 - y^2 = (x+y)(x-y)$
- $x^3 + y^3 = (x+y)(x^2 - xy + y^2)$
- $x^3 - y^3 = (x-y)(x^2 + xy + y^2)$

binomial theorem:

- $(x+y)^2 = x^2 + 2xy + y^2$
- $(x-y)^2 = x^2 - 2xy + y^2$
- $(x+y)^3 = x^3 + 3x^2y + 3xy^2 + y^3$
- $(x-y)^3 = x^3 - 3x^2y + 3xy^2 - y^3$
- $(x+y)^n = x^n + nx^{n-1}y + \frac{n(n-1)}{2}x^{n-2}y^2 + \ldots + \binom{n}{k}x^{n-k}y^k + \cdots + nxy^{n-1} + y^n$ where $\binom{n}{k} = \frac{n(n-1)\ldots(n-k+1)}{1 \cdot 2 \cdot 3 \cdot \ldots \cdot k}$

quadratic formulae:

1. $ax^2 + bx + c = 0$
2. $x^2 + \frac{b}{a}x + \frac{c}{a} = 0$
3. $x^2 + \frac{b}{a}x = -\frac{c}{a}$
4. $x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2 = -\frac{c}{a} + \left(\frac{b}{2a}\right)^2$
5. $\left(x + \frac{b}{2a}\right)^2 = \frac{b^2-4ac}{4a^2}$
6. $x + \frac{b}{2a} = \pm\frac{\sqrt{b^2-4ac}}{2a}$
7. $x = -\frac{b}{2a} \pm \frac{\sqrt{b^2-4ac}}{2a}$
8. $x = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$

polynomials $a_n x^n + a_{n-1}x^{n-1} + ... + a_1 x + a_0$ where n is non-neg and represents the degree

rational $p(x)/q(x)$

root $\sqrt[n]{g(x)}$

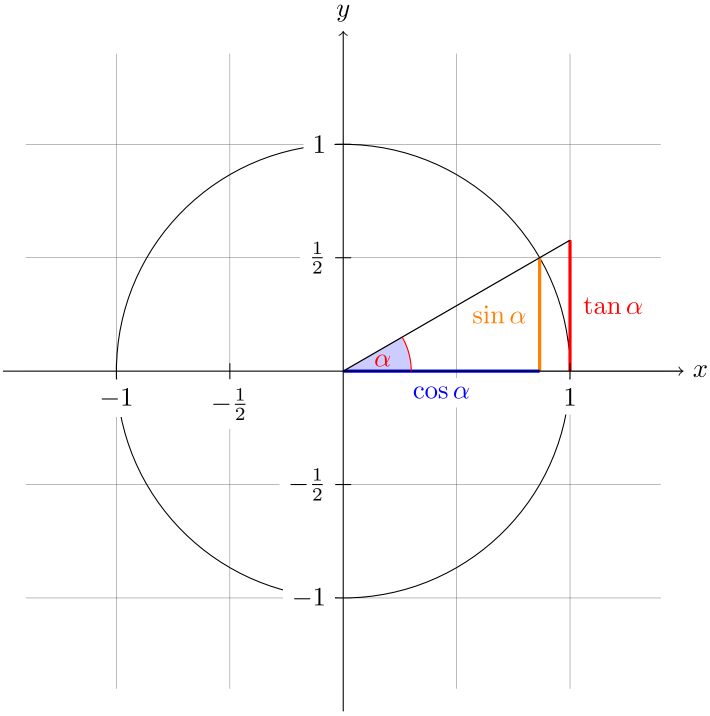
properties:
domain: set of all input values for which the function is defined
range: set of all possible output values of the function

continuity: most algebaic functions are continuous (no breaks of jumps), but rational functions have discontinuities at points

behavior: the functions behavior is influenced bu the degree of the polynomial and the nature of the function

scaling:

- vertical scaling $y = kf(x)$: If $k \geq 1$, the graph is expanded vertically by the factor k. If $0 < k < 1$, the graph is compressed vertically. When the scale factor k is negative ($k < 0$), the graph is also reflected across the x-axis.
- horizontal scaling $y = f(kx)$: If $K \geq 1$, the graph is compressed in the horizontal direction. If $0 < k < 1$, the graph is expanded. If $k \leq 0$, then the graph is also reflected across the y-axis.



Angle (Degrees)	Angle (Radians)	$\cos(\theta)$	$\sin(\theta)$
$0°$	0	1	0
$30°$	$\frac{\pi}{6}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$
$45°$	$\frac{\pi}{4}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$
$60°$	$\frac{\pi}{3}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$
$90°$	$\frac{\pi}{2}$	0	1
$120°$	$\frac{2\pi}{3}$	$-\frac{1}{2}$	$\frac{\sqrt{3}}{2}$
$135°$	$\frac{3\pi}{4}$	$-\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$
$150°$	$\frac{5\pi}{6}$	$-\frac{\sqrt{3}}{2}$	$\frac{1}{2}$
$180°$	π	-1	0
$210°$	$\frac{7\pi}{6}$	$-\frac{\sqrt{3}}{2}$	$-\frac{1}{2}$
$225°$	$\frac{5\pi}{4}$	$-\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{2}}{2}$
$240°$	$\frac{4\pi}{3}$	$-\frac{1}{2}$	$-\frac{\sqrt{3}}{2}$
$270°$	$\frac{3\pi}{2}$	0	-1
$300°$	$\frac{5\pi}{3}$	$\frac{1}{2}$	$-\frac{\sqrt{3}}{2}$
$315°$	$\frac{7\pi}{4}$	$\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{2}}{2}$
$330°$	$\frac{11\pi}{6}$	$\frac{\sqrt{3}}{2}$	$-\frac{1}{2}$
$360°$	2π	1	0

$1\,\mathrm{deg} = \frac{\pi}{180}\mathrm{rad}$

trigonometric functions are mathematical functions that relate the angle of a trianlge to the lengths of its sides... and can also be generalized to all real numbers using the unit circle.

To derive the rest of the fundamental trigonometric identities, you need a combination of a few key identities and principles. The most important starting point is the Pythagorean identity, but you'll also need the basic relationships between the trigonometric functions, such as the definitions of sine, cosine, tangent, secant, cosecant, and cotangent in terms of a right triangle or the unit circle.

$\sin^2\theta + \cos^2\theta = 1$

$\sec\theta = \frac{1}{\cos\theta}, \quad \csc\theta = \frac{1}{\sin\theta}, \quad \cot\theta = \frac{1}{\tan\theta}$

$\tan\theta = \frac{\sin\theta}{\cos\theta}, \quad \cot\theta = \frac{\cos\theta}{\sin\theta}$

$1 + \tan^2\theta = \sec^2\theta$

$1 + \cot^2\theta = \csc^2\theta$

$\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$

$\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$

$\tan(\alpha+\beta) = \frac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}$

$\sin(\alpha-\beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$

$\cos(\alpha-\beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$

$\tan(\alpha-\beta) = \frac{\tan\alpha-\tan\beta}{1+\tan\alpha\tan\beta}$

$\sin(2\theta) = 2\sin\theta\cos\theta$

$\cos(2\theta) = \cos^2\theta - \sin^2\theta = 2\cos^2\theta - 1 = 1 - 2\sin^2\theta$

$\tan(2\theta) = \frac{2\tan\theta}{1-\tan^2\theta}$

$\sin^2(2\theta) = \frac{1-\cos^2(2\theta)}{2}$

$\cos^2(2\theta) = \frac{1+\cos(2\theta)}{2}$

$\sin(90^\circ-\theta) = \cos\theta, \quad \cos(90^\circ-\theta) = \sin\theta$

$\tan(90^\circ-\theta) = \cot\theta, \quad \cot(90^\circ-\theta) = \tan\theta$

$\sec(90^\circ-\theta) = \csc\theta, \quad \csc(90^\circ-\theta) = \sec\theta$

$\sin(-\theta) = -\sin(\theta), \quad \cos(-\theta) = \cos(\theta)$

$\tan(-\theta) = -\tan(\theta), \quad \sec(-\theta) = \sec(\theta)$

$\csc(-\theta) = -\csc(\theta), \quad \cot(-\theta) = -\cot(\theta)$

$\sin\alpha\sin\beta = \frac{1}{2}[\cos(\alpha-\beta) - \cos(\alpha+\beta)]$

$\cos\alpha\cos\beta = \frac{1}{2}[\cos(\alpha-\beta) + \cos(\alpha+\beta)]$

$\sin\alpha\cos\beta = \frac{1}{2}[\sin(\alpha+\beta) + \sin(\alpha-\beta)]$

law of sines: $\frac{\sin(A)}{a} = \frac{\sin(B)}{b} = \frac{\sin(C)}{c}$ (uppercase are the angles)

law of cosines: $a^2 = b^2 + c^2 - 2bc\cos(A)$

power functions: $f(x) = x^n$....if even the function behaves symmetrical around the y-axis...if odd then the function has point symmetry (x^4, x^3, $x^{-n} = \frac{1}{x^n}$)

inverse trig functions: $\arcsin(x) = \sin^{-1}(x) = \theta$, $\arccos(x) = \cos^{-1}(x) = \theta$...etc

logs: $\log_a x = y \leftrightarrow a^y = x$, $\ln(x) = y \leftrightarrow e^y = x$

hyperbolic functions: $\sinh(x) = \frac{e^x - e^{-x}}{2}$, $\cosh(x) = \frac{e^x + e^{-x}}{2}$, $\tanh(x) = \frac{\sinh(x)}{\cosh(x)}$

differentiation rules:

1. $\frac{d}{dx}(c) = 0$
2. $\frac{d}{dx}x = 1$
3. $\frac{d}{dx}(x^n) = nx^{-1}$ (power rule)
4. $\frac{d}{dx}[cf(x)] = cf'(x)$
5. $\frac{d}{dx}[f(x) + g(x)] = f'(x) + g'(x)$
6. $\frac{d}{dx}[f(x)g(x)] = f(x)g'(x) + g(x)f'(x)$ (product rule)
7. $\frac{d}{dx}[\frac{f(x)}{g(x)}] = \frac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^2}$ (quotient rule)
8. $\frac{d}{dx}f(g(x)) = f'(g(x))g'(x)$ (chain rule)
9. $\frac{d}{dx}f(x)^n = nf(x)^{n-1}f'(x)$ (general power rule)
10. $\frac{d}{dx}\sin(x) = \cos(x)$
11. $\frac{d}{dx}\cos(x) = -\sin(x)$
12. $\frac{d}{dx}\tan(x) = \sec^2(x)$
13. $\frac{d}{dx}\csc(x) = -\csc(x)\cot(x)$
14. $\frac{d}{dx}\sec(x) = \sec(x)\tan(x)$
15. $\frac{d}{dx}\cot(x) = -\csc^2(x)$
16. $\frac{d}{dx}\sin^{-1}(x) = \frac{1}{\sqrt{(1-x^2)}}$
17. $\frac{d}{dx}\cos^{-1}(x) = -\frac{1}{\sqrt{(1-x^2)}}$
18. $\frac{d}{dx}\tan^{-1}(x) = \frac{1}{1+x^2}$

19. $\frac{d}{dx}(e^x) = e^x$

20. $\frac{d}{dx}(a^x)(\ln a)a^x$

21. $\frac{d}{dx} \ln \mid x \mid = \frac{1}{x}$

22. $\frac{d}{dx} \log_a x = \frac{1}{(\ln a)x}$

Essential Theorems:

- **Intermediate Value Theorem (IVT):** Guarantees that a continuous function takes every value between $f(a)$ and $f(b)$ at some point in the interval $[a, b]$.
- **Mean Value Theorem (MVT):** States that for a continuous and differentiable function, there is at least one point where the instantaneous rate of change equals the average rate of change over the interval.
- **Extreme Value Theorem:** Guarantees that a continuous function on a closed interval attains a maximum and minimum value.
- **Fundamental Theorem of Calculus:**
    1. **First Part:** The derivative of the integral of a function is the original function.
    2. **Second Part:** The definite integral of a function can be computed using its antiderivative.

review problems

verifying trigonometric identities:

1. $\frac{1+\sin(x)}{\cos(x)} + \frac{1-\sin(x)}{\cos(x)} = 2\sec(x)$
2. $\frac{\sin(x)}{1+\cos(x)} + \frac{\sin(x)}{1-\cos(x)} = \frac{2\sin(x)}{1-\cos^2(x)}$
3. $\frac{\tan(x)+\cot(x)}{\sec(x)+\csc(x)} = \sin(x)\cos(x)$
4. $\frac{\sin(x)-\cos(x)}{\sin(x)+\cos(x)} = \frac{1-\tan(x)}{1+\tan(x)}$
5. $\frac{1-\cos(2x)}{2\sin(x)\cos(x)} = \tan(x)$

solving trigonometric equations:
bounded:

1. $\sin(x) = \frac{1}{2}$ for $0 \le x < 2\pi$
2. $2\cos(x) - 1 = 0$ for $0 \le x < 2\pi$
3. $\sin^2(x) - \sin(x) = 0$ for $0 \le x < 2\pi$
4. $2\cos^2(x) - 3\cos(x) + 1 = 0$ for $0 \le x < 2\pi$
5. $\sin(x)\cos(x) = \frac{1}{2}$ for $0 \le x < 2\pi$
6. $\tan(x) + \cot(x) = 2$ for $0 < x < \pi$
7. $1 - 2\sin^2(x) = \cos(2x)$ for $0 \le x < 2\pi$
8. $\sec(x) = 2$ for $0 \le x < 2\pi$
9. $\sin(2x) = \sqrt{3}\cos(x)$ for $0 \le x < 2\pi$
10. $\tan^2(x) = 3$ for $0 \le x < 2\pi$

unbounded:

1. $\sin(x) = \frac{1}{2}$
2. $\cos(x) = -\frac{\sqrt{2}}{2}$
3. $\tan(x) = \sqrt{3}$
4. $2\sin(x) - 1 = 0$
5. $\cos^2(x) = \frac{1}{4}$
6. $\sec(x) = -2$
7. $\tan^2(x) = 1$
8. $\cot(x) + 1 = 0$
9. $\sin(2x) = 0$
10. $\cos(3x) = 1$

direct substitution:

1. $\lim_{x\to 3} \left(2x^2 - 5x + 4\right)$
2. $\lim_{x\to 0} \left(\frac{3x^2+2x-1}{x+2}\right)$
3. $\lim_{x\to -1} \left(4x^3 - 2x + 6\right)$
4. $\lim_{x\to 2} \left(\frac{x^2-4}{x-2}\right)$
5. $\lim_{x\to 1} \left(3x^3 - 2x + 5\right)$
6. $\lim_{x\to 0} \left(\frac{5x^2+3x}{x^2+2x+1}\right)$
7. $\lim_{x\to 4} (\sqrt{x} - 2)$
8. $\lim_{x\to 0} \left(\frac{x^3+2x}{x^2-3x+2}\right)$
9. $\lim_{x\to 1} \left(\frac{x^2+x-2}{x-1}\right)$
10. $\lim_{x\to -3} \left(\frac{2x+1}{x^2+5x+6}\right)$

indeterminate forms and algebraic simplification

1. $\lim_{x\to 2} \frac{x^2-4}{x-2}$
2. $\lim_{x\to 0} \frac{\sin(x)}{x}$
3. $\lim_{x\to 0} \frac{e^x-1}{x}$
4. $\lim_{x\to 0} \frac{1-\cos(x)}{x^2}$
5. $\lim_{x\to 0} \frac{x^2+3x}{x}$
6. $\lim_{x\to 1} \frac{x^3-1}{x-1}$
7. $\lim_{x\to \infty} \frac{1}{x}$
8. $\lim_{x\to 0} \frac{\ln(x)}{x}$
9. $\lim_{x\to 0} \frac{x^2+2x-3}{x^2-1}$
10. $\lim_{x\to 0} \frac{x^3+x}{x^2-1}$

trigonometic limits:

1. $\lim_{x\to 0} \frac{\sin(x)}{x}$
2. $\lim_{x\to 0} \frac{1-\cos(x)}{x^2}$
3. $\lim_{x\to 0} \frac{\tan(x)}{x}$
4. $\lim_{x\to 0} \frac{\sin(3x)}{x}$
5. $\lim_{x\to 0} \frac{1-\cos(2x)}{x^2}$
6. $\lim_{x\to 0} \frac{\sin(x)-\sin(2x)}{x}$

7. $\lim_{x\to\infty} \frac{\sin(x)}{x}$
8. $\lim_{x\to 0} \frac{\cos(x)-1}{x^2}$
9. $\lim_{x\to 0} \frac{\sin(x)}{x^3}$
10. $\lim_{x\to 0} \frac{\tan(2x)}{x}$

piecewise functions:

1. $\lim_{x\to 0} \begin{cases} \sin(x) & \text{if } x \geq 0 \\ -\sin(x) & \text{if } x < 0 \end{cases}$
2. $\lim_{x\to 0} \begin{cases} \frac{x^2-4}{x-2} & \text{if } x \neq 2 \\ 4 & \text{if } x = 2 \end{cases}$
3. $\lim_{x\to 0} \begin{cases} \frac{\sin(x)}{x} & \text{if } x \neq 0 \\ 1 & \text{if } x = 0 \end{cases}$
4. $\lim_{x\to \pi} \begin{cases} \cos(x) & \text{if } x < \pi \\ \sin(x) & \text{if } x \geq \pi \end{cases}$
5. $\lim_{x\to 0} \begin{cases} \frac{\sin(2x)}{x} & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$
6. $\lim_{x\to 0} \begin{cases} \frac{1-\cos(x)}{x} & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$
7. $\lim_{x\to 0} \begin{cases} \tan(x) & \text{if } x \geq 0 \\ -\tan(x) & \text{if } x < 0 \end{cases}$
8. $\lim_{x\to 0} \begin{cases} \frac{\sin(x)-\sin(2x)}{x} & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$
9. $\lim_{x\to 0} \begin{cases} \frac{\sin(3x)}{x} & \text{if } x \neq 0 \\ 3 & \text{if } x = 0 \end{cases}$
10. $\lim_{x\to 0} \begin{cases} \frac{x^2}{\sin(x)} & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$

infinite limits and vertical asymptotes:

1. $\lim_{x\to 0^+} \frac{1}{x}$
2. $\lim_{x\to 0^-} \frac{1}{x}$
3. $\lim_{x\to \infty} \frac{1}{x^2}$
4. $\lim_{x\to 0} \frac{1}{x^2}$
5. $\lim_{x\to 2^+} \frac{1}{x-2}$
6. $\lim_{x\to -2^-} \frac{1}{x+2}$
7. $\lim_{x\to 0^+} \frac{\ln(x)}{x}$
8. $\lim_{x\to \infty} \frac{x^2}{x+1}$

9. $\lim_{x\to 3} \frac{1}{(x-3)^2}$
10. $\lim_{x\to 1} \frac{1}{x-1}$

squeeze theorem:

1. If $0 \leq x \leq 1$, show that $\lim_{x\to 0} x^2 \cos\left(\frac{1}{x}\right) = 0$
2. Show that $\lim_{x\to 0} \frac{\sin(x)}{x} = 1$ using the Squeeze Theorem
3. If $0 \leq x \leq 1$, show that $\lim_{x\to 0} x \sin\left(\frac{1}{x}\right) = 0$
4. Show that $\lim_{x\to 0} x^2 \sin\left(\frac{1}{x}\right) = 0$ using the Squeeze Theorem
5. Use the Squeeze Theorem to show that $\lim_{x\to 0} x^3 \cos(x) = 0$
6. Show that $\lim_{x\to 0} \frac{x^2 \cos(x)}{\sin(x)} = 0$ using the Squeeze Theorem
7. Use the Squeeze Theorem to show that $\lim_{x\to 0} \frac{x^2 \sin\left(\frac{1}{x}\right)}{x} = 0$
8. Show that $\lim_{x\to 0} \frac{\cos(x)}{x^2} = \infty$ using the Squeeze Theorem
9. Show that $\lim_{x\to 0} x^2 \cos(x) \sin\left(\frac{1}{x}\right) = 0$ using the Squeeze Theorem
10. Use the Squeeze Theorem to show that $\lim_{x\to 0} \frac{x \sin(x)}{x^2+1} = 0$

epsilon-delta:

1. Prove that $\lim_{x\to 2}(3x+1) = 7$ using the epsilon-delta definition of a limit.
2. Prove that $\lim_{x\to 0} \frac{x^2}{x+3} = 0$ using the epsilon-delta definition.
3. Prove that $\lim_{x\to 1}(x^2-1) = 0$ using the epsilon-delta definition of a limit.
4. Prove that $\lim_{x\to 3} \frac{x^2-9}{x-3} = 6$ using the epsilon-delta definition of a limit.
5. Prove that $\lim_{x\to 0} \sin(x) = 0$ using the epsilon-delta definition.
6. Prove that $\lim_{x\to 0} \frac{1}{x^2+1} = 1$ using the epsilon-delta definition.
7. Prove that $\lim_{x\to 2} \frac{x^2-4}{x-2} = 4$ using the epsilon-delta definition.
8. Prove that $\lim_{x\to 0} \frac{1}{x+1} = 1$ using the epsilon-delta definition.
9. Prove that $\lim_{x\to 2} \frac{3x^2+1}{x-1} = \infty$ using the epsilon-delta definition.
10. Prove that $\lim_{x\to 1}(x^3-1) = 0$ using the epsilon-delta definition of a limit.

basic polynomial differentiation:

1. Differentiate $f(x) = 3x^4 + 5x^3 - 2x + 7$
2. Differentiate $f(x) = 4x^5 - x^2 + 6x - 3$
3. Differentiate $f(x) = x^6 + 2x^4 - 3x^2 + x - 8$
4. Differentiate $f(x) = 5x^3 - 4x^2 + 7x + 1$
5. Differentiate $f(x) = 2x^5 - x^3 + x - 9$

6. Differentiate $f(x) = 3x^2 - 2x + 4$
7. Differentiate $f(x) = x^7 - 5x^3 + 2x^2 - x + 6$
8. Differentiate $f(x) = 6x^4 - 3x^3 + 2x^2 + x - 5$
9. Differentiate $f(x) = 2x^8 - x^6 + 4x^2 - 3$
10. Differentiate $f(x) = 9x^5 - 7x^4 + 3x^2 + 2x + 1$

implicit differentiation:

1. Differentiate $x^2 + y^2 = 25$
2. Differentiate $x^3 + y^3 = 6xy$
3. Differentiate $x^2y + y^2 = 10$
4. Differentiate $x^2 + 2xy + y^2 = 7$
5. Differentiate $x^3 + y^3 = 3xy$
6. Differentiate $x^2y^2 + 3x = 5$
7. Differentiate $x^2 + y^2 = x + y$
8. Differentiate $x^3 + y^3 = 6x + 2y$
9. Differentiate $xy = x + y$
10. Differentiate $x^2 - 3xy + y^2 = 10$

higher-order derivatives:

1. Find the second derivative of $f(x) = 3x^4 - 5x^2 + 2x - 7$
2. Find the third derivative of $f(x) = x^5 - 3x^3 + 4x^2 - 6$
3. Find the second derivative of $f(x) = \sin(x) + \cos(x)$
4. Find the first and second derivatives of $f(x) = e^{2x}\cos(x)$
5. Find the third derivative of $f(x) = 4x^3 + 3x^2 - 2x + 5$
6. Find the first and second derivatives of $f(x) = \ln(x^2 + 1)$
7. Find the second derivative of $f(x) = \tan(x)$
8. Find the fourth derivative of $f(x) = 7x^4 - 6x^2 + x - 3$
9. Find the second derivative of $f(x) = \frac{x^3}{x^2+1}$
10. Find the third derivative of $f(x) = x^4 \ln(x)$

applications of derivatives (tangent line equations, critical points, motion problems, related rates):
basic antiderivatives:
u-sub (reverse chain rule):
trig integrals:
partial fractions:
applications (area under curves, average value of a function, volume by disks/washers/shells, accumulation problems):