my review

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calculus is a branch of mathematics that deals with rates of change and the accumulation of quantities.

precalculus $\rightarrow$ calculus II

$|a| = |-a|$, $|ab| = |a||b|$

The **distance** between two real numbers a and b is $|b - a|$, which is the length of the line segment joining a and b.
Two real numbers a and b are close to each other if $|b - a|$ is small, and this is the case if their decimal expansions agree to many places. More precisely, *if the decimal expansions of a and b agree to k places (to the right of the decimal point), then the distance $|b - a|$ is at most 10^-k. Thus, the distance between a = 3.1415 and b = 3.1478 is at most 10^{-2} because a and b agree to two places. In fact, the distance is exactly* $|3.1415 - 3.1478| = 0.0063$.

Beware that $|a + b|$ is not equal to $|a| + |b|$ unless a and b have the same sign or at least one of a and b is zero. If they have opposite sins, cancellation occurs in the sum $a + b$ and $|a + b| < |a| + |b|$. For example, $|2 + 5| = |2| + |5|$ but $|-2 + 5| = 3$, which is less than $|-2| + |5| = 7$. In any case, $|a + b|$ is never larger than $|a| + |b|$ and this gives us the simple but important **triangle inequality**: $|a + b| \leq |a| + |b|$

$[a, b] = x \in \mathbb{R} : a \leq x \leq b$
$(a, b) = x \in \mathbb{R} : a < x < b$
$[a, b) = x \in \mathbb{R} : a \leq x < b$
$(a, b] = x \in \mathbb{R} : a < x \leq b$

$(-r, r) = x : |x| < r$

circle: $(x - a)^2 + (y - b)^2 = r^2$ where (a, b) is the center and the radius is r

midpoint between $P_1 = (x_1, y_1)$ and $P_2 = (x_2, y_2)$ is $(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2})$

distance: $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

composing new functions
If f and g are functions, we may construct new functions by forming the sum, difference, product, and quotient functions:

- $(f + g)(x) = f(x) + g(x)$

- $(f-g)(x) = f(x) - g(x)$
- $(fg)(x) = f(x)g(x)$
- $(\frac{f}{g}(x) = \frac{f(x)}{g(x)})$

We can also multiply functions by constants. A function of the form: $c_1 f(x) + c_2 g(x)$ is called a **linear combination**.
Composition is another important way of constructing new functions. The composition of f and g is the function $f \circ g$ defined by $(f \circ g)(x) = f(g(x))$, defined for values of x in the domain of g such that $g(x)$ lies in the domain of f.
ex. Compute the composite functions $f \circ g$ and $g \circ f$ and discuss their domains where $f(x) = \sqrt{x}$ and $g(x) = 1 - x$
solution: $(f \circ g)(x) = f(g(x)) = f(1-x) = \sqrt{1-x}$ The square root $\sqrt{1-x}$ is defined if $1 - x \geq 0$ or $x \leq 1$, so the domain of $f \circ g$ is $x : x \leq 1$.
On the other hand, $(g \circ f)(x) = g(f(x)) = g(\sqrt{x}) = 1 - \sqrt{x}$ The domain of $g \circ f$ is $x : x \geq 0$.

invertable functions
"is this function invertible?" $\Leftrightarrow$ "does an inverse function exist for this function" $\Leftrightarrow$ "is the function one-to-one?" (horizontal line test)

- if it is, then the inverse function exists
- if it is not, then the inverse function does not exist, and the function is not invertible (as a function)

consider $f(x) = x^2$ this function is not one-to-one (horizontal line test) this it is not invertable unless you restrict the domain to be $x \geq 0$.
to find the inverse algeraically you can swap the x's and y's and then solve for y

complex numbers

- rectangular form $z = x + yi$ where s is the real part and y is the imaginary part, and $i = \sqrt{-1}, i^2 = -1$
- polar form $z = r(\cos(\theta) + i\sin(\theta))$ where $r = \sqrt{x^2 + y^2}, \theta = \tan^{-1}(\frac{y}{x})$
- exponential form $z = re^{i\theta}$

the fundamental theorem of algebra
Let $p(z)$ be a non-constant polynomial with complex coefficients, i.e.,
$p(z) = a_n z^n + a_{n-1} z^{n-1} + \cdots + a_1 z + a_0$,
where $a_n \neq 0$ and $n \geq 1$. Then, there exists at least one complex number $c \in \mathbb{C}$ such that
$p(c) = 0$.
This theorem says that any polynomial equation — no matter how complicated — always has at least one solution if we allow the solutions to be complex numbers (numbers that can include the square root of negative one). Even if the polynomial doesn't have any real solutions, it will have complex ones. This means every polynomial can be "broken down" completely into simpler parts based on its roots.

rational functions
$f(x) = \frac{P(x)}{Q(x)}$ where $P(x)$ and $Q(x)$ are polynomials
$Q(x) \neq 0$

the domian is all real numbers except for where the denominator is 0
a vertial asymptote occurs where the denominator is zero (and not canceled by a common factor)
holes (removable discontinuities) occur if a factor cancels from both the numerator and denominator
horizontal asmptotes take n to be the degree of the numerator and m to be the degree if the denominator

- $n < m$: horizontal asymptote at $y = 0$
- $n = m$: horitzontal asymptote at $\frac{\text{leading coeff. of} P(x)}{\text{leading coeff. of} Q(x)}$
- $n > m$: no horizontal asymptote (however there may be an oblique/slant asymptote instead)

slant(oblique) asymptotes occur when $n = m+1$ ... use polynomial division to find slant asymptotes the x intercepts occur where the numerator is zero (where does the function $= 0$)
the y intercept: plug in $x = 0$
conic sections

- ellipses
  - (h, k) center of the ellipse
  - a semi-major axis (long radius)
  - b semi-minor axis (short radius)
  - c distance from center to each focus $c = \sqrt{a^2 - b^2}$
  - horizontal major axis $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$
  - vertical major axis $\frac{(x-h)^2}{b^2} + \frac{(y-k)}{a^2} = 1$
- hyperbolas
  - (h, k) center
  - a distance from center to each vertex (on transverse axis)
  - b related to the asymptotes
  - $c = \sqrt{a^2 + b^2}$ distance from center to each focus (note here add not subtract like ellipse)
  - asymptotes
    - for horizontal hyperbola $y - k = \pm\frac{b}{a}(x - h)$
    - for vertical hyperbola $y - k = \pm\frac{a}{b}(x - h)$
  - transverse axis: line through both vertices and foci
  - conjugate axis: perpendicular to the transverse axis
  - opens horizontally $\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$
  - opens vertically $\frac{(x-h)^2}{b^2} - \frac{(y-k)}{a^2} = 1$

vectors

matrices

probability and combinatorics

series

limits and continuity revisited

laws of exponents:

- $x^m x^n = x^{m+n}$
- $\frac{x^m}{x^n} = x^{m-n}$
- $(x^m)^n = x^{mn}$
- $x^{-n} = \frac{1}{x^n}$
- $(xy)^n = x^n y^n$
- $(\frac{x}{y})^n = \frac{x^n}{y^n}$
- $x^{1/n} = \sqrt[n]{x}$
- $x^{\frac{m}{n}} = \sqrt[n]{x^m} = (\sqrt[n]{x})^m$

exponential and logarithmic functions:

- $\log_a x = y \leftrightarrow a^y = x$
- $\ln x = y \leftrightarrow e^y = x$
- $\log_a(xy) = \log_a x + \log_a y$
- $\log_a(a^x) = x$
  $a^{\log_a x} = x$
- $\ln(e^x) = xe^{\ln x} = x$
- $\log_a(\frac{x}{y}) = \log_a x - \log_a y$
- $\log_a 1 = 0$
  $\log_a a = 1$
- $\ln 1 = 0$
  $\ln e = 1$
- $\log_a(\frac{x}{y}) = \log_a x - \log_a y$
- $\log_a 1 = 0$
  $\log_a a = 1$
- $\ln 1 = 0$
  $\ln e = 1$
- $\log_a(x^r) = r \log_a x$

special factorizations:

- $x^2 - y^2 = (x + y)(x - y)$
- $x^3 + y^3 = (x + y)(x^2 - xy + y^2)$
- $x^3 - y^3 = (x - y)(x^2 + xy + y^2)$

binomial theorem:

- $(x + y)^2 = x^2 + 2xy + y^2$
- $(x - y)^2 = x^2 - 2xy + y^2$
- $(x + y)^3 = x^3 + 3x^2y + 3xy^2 + y^3$
- $(x - y)^3 = x^3 - 3x^2y + 3xy^2 - y^3$
- $(x + y)^n = x^n + nx^{n-1}y + \frac{n(n-1)}{2}x^{n-2}y^2 + \ldots + \binom{n}{k}x^{n-k}y^k + \cdots + nxy^{n-1} + y^n$ where $\binom{n}{k} = \frac{n(n-1)\ldots(n-k+1)}{1 \cdot 2 \cdot 3 \cdot \ldots \cdot k}$

quadratic formulae:

1. $ax^2 + bx + c = 0$
2. $x^2 + \frac{b}{a}x + \frac{c}{a} = 0$
3. $x^2 + \frac{b}{a}x = -\frac{c}{a}$
4. $x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2 = -\frac{c}{a} + \left(\frac{b}{2a}\right)^2$
5. $\left(x + \frac{b}{2a}\right)^2 = \frac{b^2-4ac}{4a^2}$
6. $x + \frac{b}{2a} = \pm\frac{\sqrt{b^2-4ac}}{2a}$
7. $x = -\frac{b}{2a} \pm \frac{\sqrt{b^2-4ac}}{2a}$
8. $x = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$

polynomials $a_nx^n + a_{n-1}x^{n-1} + ... + a_1x + a_0$ where n is non-neg and represents the degree

rational $p(x)/q(x)$

root $\sqrt[n]{g(x)}$

properties:
domain: set of all input values for which the function is defined
range: set of all possible output values of the function

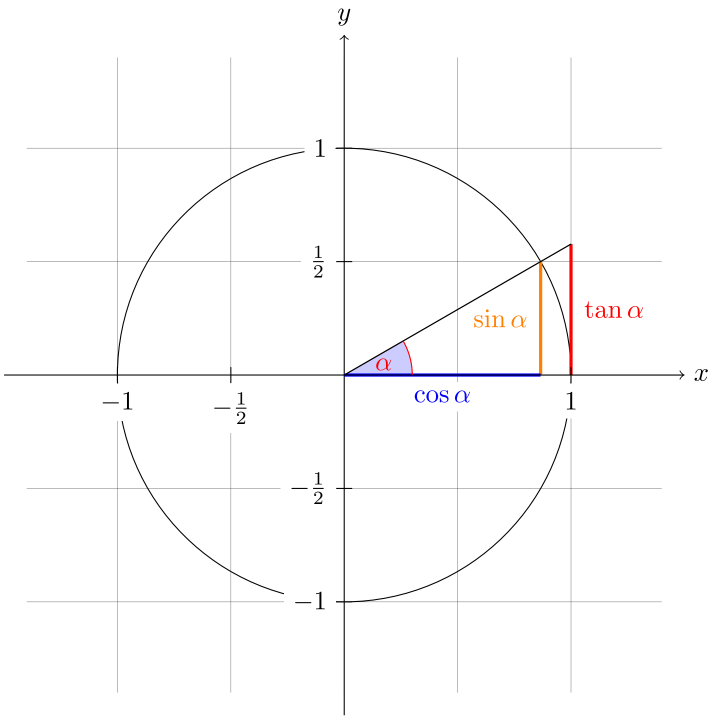
continuity: most algebaic functions are continuous (no breaks of jumps), but rational functions have discontinuities at points

behavior: the functions behavior is influenced by the degree of the polynomial and the nature of the function

scaling:

- vertical scaling $y = kf(x)$: If $k \geq 1$, the graph is expanded vertically by the factor k. If $0 < k < 1$, the graph is compressed vertically. When the scale factor k is negative ($k < 0$), the graph is also reflected across the x-axis.

- horizontal scaling $y = f(kx)$: If $K \geq 1$, the graph is compressed in the horizontal direction. If $0 < k < 1$, the graph is expanded. If $k \leq 0$, then the graph is also reflected across the y-axis.



Angle (Degrees)	Angle (Radians)	$\cos(\theta)$	$\sin(\theta)$
$0°$	0	1	0
$30°$	$\frac{\pi}{6}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$
$45°$	$\frac{\pi}{4}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$
$60°$	$\frac{\pi}{3}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$
$90°$	$\frac{\pi}{2}$	0	1
$120°$	$\frac{2\pi}{3}$	$-\frac{1}{2}$	$\frac{\sqrt{3}}{2}$
$135°$	$\frac{3\pi}{4}$	$-\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$
$150°$	$\frac{5\pi}{6}$	$-\frac{\sqrt{3}}{2}$	$\frac{1}{2}$
$180°$	π	-1	0
$210°$	$\frac{7\pi}{6}$	$-\frac{\sqrt{3}}{2}$	$-\frac{1}{2}$
$225°$	$\frac{5\pi}{4}$	$-\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{2}}{2}$
$240°$	$\frac{4\pi}{3}$	$-\frac{1}{2}$	$-\frac{\sqrt{3}}{2}$
$270°$	$\frac{3\pi}{2}$	0	-1
$300°$	$\frac{5\pi}{3}$	$\frac{1}{2}$	$-\frac{\sqrt{3}}{2}$
$315°$	$\frac{7\pi}{4}$	$\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{2}}{2}$
$330°$	$\frac{11\pi}{6}$	$\frac{\sqrt{3}}{2}$	$-\frac{1}{2}$
$360°$	2π	1	0

$1° = \frac{\pi}{180}\text{rad}$

$1rad = \frac{180^\circ}{pi}$

SOH-CAH-TOA is a mnemonic device that expresses the relationship between the basic trigonometric functions and the ratios of the sides in a right triangle.

trigonometric functions are mathematical functions that relate the angle of a trianlge to the lengths of its sides... and can also be generalized to all real numbers using the unit circle.

law of sines/cosines
$\frac{\sin(A)}{a} = \frac{\sin(B)}{b} \frac{\sin(C)}{c}$ $a^2 = b^2 + c^2 - 2bc\cos(\theta)$

To derive the rest of the fundamental trigonometric identities, you need a combination of a few key identities and principles. The most important starting point is the Pythagorean identity, but you'll also need the basic relationships between the trigonometric functions, such as the definitions of sine, cosine, tangent, secant, cosecant, and cotangent in terms of a right triangle or the unit circle.

$\sin(-\theta) = -\sin(\theta)$
$\cos(-\theta) = \cos(\theta)$
$\tan(-\theta) = -\tan(\theta)$
$\sin(\frac{\pi}{2} - \theta) = \cos(\theta)$
$\cos(\frac{\pi}{2} - \theta) = \sin(\theta)$
$\tan(\frac{\pi}{2} - \theta) = \cot(\theta)$
$\sin^2\theta + \cos^2\theta = 1$
$\sec\theta = \frac{1}{\cos\theta}, \quad \csc\theta = \frac{1}{\sin\theta}, \quad \cot\theta = \frac{1}{\tan\theta}$
$\tan\theta = \frac{\sin\theta}{\cos\theta}, \quad \cot\theta = \frac{\cos\theta}{\sin\theta}$
$1 + \tan^2\theta = \sec^2\theta$
$1 + \cot^2\theta = \csc^2\theta$
$\sin(\alpha + \beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$
$\cos(\alpha + \beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$
$\tan(\alpha + \beta) = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta}$
$\sin(\alpha - \beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$
$\cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$
$\tan(\alpha - \beta) = \frac{\tan\alpha - \tan\beta}{1 + \tan\alpha\tan\beta}$
$\sin(2\theta) = 2\sin\theta\cos\theta$
$\cos(2\theta) = \cos^2\theta - \sin^2\theta = 2\cos^2\theta - 1 = 1 - 2\sin^2\theta$
$\tan(2\theta) = \frac{2\tan\theta}{1 - \tan^2\theta}$
$\sin^2(2\theta) = \frac{1 - \cos^2(2\theta)}{2}$
$\cos^2(2\theta) = \frac{1 + \cos(2\theta)}{2}$
$\sin(90^\circ - \theta) = \cos\theta, \quad \cos(90^\circ - \theta) = \sin\theta$
$\tan(90^\circ - \theta) = \cot\theta, \quad \cot(90^\circ - \theta) = \tan\theta$
$\sec(90^\circ - \theta) = \csc\theta, \quad \csc(90^\circ - \theta) = \sec\theta$
$\sin(-\theta) = -\sin(\theta), \quad \cos(-\theta) = \cos(\theta)$
$\tan(-\theta) = -\tan(\theta), \quad \sec(-\theta) = \sec(\theta)$
$\csc(-\theta) = -\csc(\theta), \quad \cot(-\theta) = -\cot(\theta)$
$\sin\alpha\sin\beta = \frac{1}{2}[\cos(\alpha - \beta) - \cos(\alpha + \beta)]$
$\cos\alpha\cos\beta = \frac{1}{2}[\cos(\alpha - \beta) + \cos(\alpha + \beta)]$
$\sin\alpha\cos\beta = \frac{1}{2}[\sin(\alpha + \beta) + \sin(\alpha - \beta)]$

power functions: $f(x) = x^n$....if even the function behaves symmetrical around the y-axis...if odd then the function has point symmetry (x^4, x^3, $x^{-n} = \frac{1}{x^n}$)

inverse trig functions: $\arcsin(x) = \sin^{-1}(x) = \theta$, $\arccos(x) = \cos^{-1}(x) = \theta$...etc

logs: $\log_a x = y \leftrightarrow a^y = x$, $\ln(x) = y \leftrightarrow e^y = x$

hyperbolic functions: $\sinh(x) = \frac{e^x - e^{-x}}{2}$, $\cosh(x) = \frac{e^x + e^{-x}}{2}$, $\tanh(x) = \frac{\sinh(x)}{\cosh(x)}$

differentiation rules:

1. $\frac{d}{dx}(c) = 0$
2. $\frac{d}{dx}x = 1$
3. $\frac{d}{dx}(x^n) = nx^{-1}$ (power rule)
4. $\frac{d}{dx}[cf(x)] = cf'(x)$
5. $\frac{d}{dx}[f(x) + g(x)] = f'(x) + g'(x)$
6. $\frac{d}{dx}[f(x)g(x)] = f(x)g'(x) + g(x)f'(x)$ (product rule)
7. $\frac{d}{dx}[\frac{f(x)}{g(x)}] = \frac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^2}$ (quotient rule)
8. $\frac{d}{dx}f(g(x)) = f'(g(x))g'(x)$ (chain rule)
9. $\frac{d}{dx}f(x)^n = nf(x)^{n-1}f'(x)$ (general power rule)
10. $\frac{d}{dx}\sin(x) = \cos(x)$
11. $\frac{d}{dx}\cos(x) = -\sin(x)$
12. $\frac{d}{dx}\tan(x) = \sec^2(x)$
13. $\frac{d}{dx}\csc(x) = -\csc(x)\cot(x)$
14. $\frac{d}{dx}\sec(x) = \sec(x)\tan(x)$
15. $\frac{d}{dx}\cot(x) = -\csc^2(x)$
16. $\frac{d}{dx}\sin^{-1}(x) = \frac{1}{\sqrt{(1-x^2)}}$
17. $\frac{d}{dx}\cos^{-1}(x) = -\frac{1}{\sqrt{(1-x^2)}}$
18. $\frac{d}{dx}\tan^{-1}(x) = \frac{1}{1+x^2}$
19. $\frac{d}{dx}(e^x) = e^x$
20. $\frac{d}{dx}(a^x) = (\ln a)a^x$
21. $\frac{d}{dx}\ln \mid x \mid = \frac{1}{x}$
22. $\frac{d}{dx}\log_a x = \frac{1}{(\ln a)x}$

Essential Theorems:

- **Intermediate Value Theorem (IVT):** Guarantees that a continuous function takes every value between $f(a)$ and $f(b)$ at some point in the interval $[a, b]$.

- **Mean Value Theorem (MVT):** States that for a continuous and differentiable function, there is at least one point where the instantaneous rate of change equals the average rate of change over the interval.
- **Extreme Value Theorem:** Guarantees that a continuous function on a closed interval attains a maximum and minimum value.
- **Fundamental Theorem of Calculus:**
    1. **First Part:** The derivative of the integral of a function is the original function.
    2. **Second Part:** The definite integral of a function can be computed using its antiderivative.

review problems

factoring:

1. $x^2 + 5x + 6$
2. $x^2 - 9$
3. $x^2 - x - 12$
4. $3x^2 + 2x - 1$
5. $x^2 + 2x$
6. $x^3 + 3x^2 + 2x + 6$
7. $2x^2 + 7x + 3$
8. $x^3 - 8$
9. $x^3 + 27$
10. $x^2 + 6x + 9$
11. $4x^2 - 12x + 9$
12. $x^2 + 4x + 5$
13. $x^2 - 10x + 29$

solving triangles:

1. In triangle ABC, $A = 40°$, $B = 65°$, and $c = 12$. Find side a.
2. In triangle ABC, $a = 10$, $b = 14$, and $C = 40°$. Find side c using the Law of Cosines.
3. In triangle ABC, $a = 7$, $b = 9$, and $c = 10$. Find angle A.
4. In triangle ABC, $A = 50°$, $a = 12$, and $b = 10$. Find angle B.
5. In triangle ABC, $a = 13$, $b = 15$, and $C = 70°$. Find the area of the triangle.
6. In triangle ABC, $A = 30°$, $B = 45°$, and $c = 18$. Find side a and side b.
7. In triangle ABC, $a = 8$, $b = 11$, and $c = 14$. Find all angles using the Law of Cosines.
8. In triangle ABC, $A = 110°$, $b = 20$, and $c = 25$. Find side a.
9. In triangle ABC, $a = 16$, $b = 20$, and angle $C = 90°$. Find side c and angles A and B.
10. In triangle ABC, $a = 9$, $b = 10$, $c = 11$. Find the area using Heron's Formula.

solving triangles (ambiguous case SSA):

1. Given triangle ABC with $A = 30°$, $a = 10$, and $b = 15$. Determine the number of possible triangles and find all possible solutions.
2. In triangle ABC, $A = 50°$, $a = 7$, and $b = 10$. Find how many triangles exist and solve for all possible angles and sides.

3. For triangle ABC, $A = 40°$, $a = 8$, and $b = 5$. Decide the number of triangles possible and solve if applicable.
4. Given $A = 60°$, $a = 9$, and $b = 9$. How many triangles can be formed? Find all solutions.
5. In triangle ABC, $A = 25°$, $a = 4$, and $b = 10$. Determine the possible number of triangles and solve accordingly.
6. For triangle ABC, $A = 35°$, $a = 12$, and $b = 7$. Analyze the number of triangles and solve.
7. In triangle ABC, $A = 45°$, $a = 5$, and $b = 6$. Find the number of possible triangles and all possible solutions.
8. Given triangle ABC with $A = 20°$, $a = 3$, and $b = 8$. Determine how many triangles can be formed and solve.
9. For triangle ABC, $A = 70°$, $a = 15$, and $b = 12$. Find the number of possible triangles and solve accordingly.
10. In triangle ABC, $A = 10°$, $a = 2$, and $b = 5$. Determine the number of triangles possible and solve if any.

polynomial long division (1. divide 2. multiply 3. subtract 4. repeat):

1. divide $(2x^3 + 3x^2 - 5x + 6)$ by $(x - 2)$
2. divide $(x^2 + 5x + 6)$ by $(x + 2)$
3. divide $(2x^3 + 8x^2 + 10)$ by $(x - 2)$
4. divide $6x^4 - 9x^2 + 18$ by $x - 3$

synthetic division (1. multiply 2. divide 3. repeat):

- divide $(x^3 - 2x^2 - 5x)$ by $(x - 3)$
- divide $(x^3 + 5x^2 + 7x + 2)$ by $(x + 2)$
- divide $(3x^2 + 7x - 20)$ by $(x + 5)$
- divide $(7x^3 + 6x - 8)$ by $(x - 4)$

verifying trigonometric identities:

1. $\frac{1+\sin(x)}{\cos(x)} + \frac{1-\sin(x)}{\cos(x)} = 2\sec(x)$
2. $\frac{\sin(x)}{1+\cos(x)} + \frac{\sin(x)}{1-\cos(x)} = \frac{2\sin(x)}{1-\cos^2(x)}$
3. $\frac{\tan(x)+\cot(x)}{\sec(x)+\csc(x)} = \sin(x)\cos(x)$
4. $\frac{\sin(x)-\cos(x)}{\sin(x)+\cos(x)} = \frac{1-\tan(x)}{1+\tan(x)}$
5. $\frac{1-\cos(2x)}{2\sin(x)\cos(x)} = \tan(x)$

inequalities:

1. $3x - 5 \leq 2x + 7$

2. $x^2 - 4x - 5 > 0$

3. $\frac{2x+3}{x-1} \geq 0$

4. $|2x - 3| \leq 5$

5. $x^3 - 3x^2 - 4x + 12 \leq 0$

6. $\sin(x) \geq 0.5 \quad 0 \leq x \leq 2\pi$

7. $\tan(x) < 1 \quad 0 \leq x \leq \pi$

8. $\sin(x) - \cos(x) \geq 0 \quad 0 \leq x \leq 2\pi$

9. $2\cos(x) - \sin(x) \leq 1 \quad 0 \leq x \leq 2\pi$

10. $\cos^2(x) \geq \frac{1}{2} \quad 0 \leq x \leq 2\pi$

11. $x^2 + 5x + 6 \geq 0$

12. $2x - 7 < 3x + 1$

13. $\frac{3x-4}{x+2} \leq 1$

14. $|x + 4| > 2$

15. $x^3 - 2x^2 + x - 1 \geq 0$

16. $\sin(x) \leq -\frac{1}{2} \quad 0 \leq x \leq 2\pi$

17. $\cos(x) > 0 \quad 0 \leq x \leq 2\pi$

18. $2\sin(x) + 1 \geq 0 \quad 0 \leq x \leq 2\pi$

19. $2\tan(x) \leq 3 \quad 0 \leq x \leq \frac{\pi}{2}$

20. $\sin^2(x) + \cos^2(x) = 1$

solving trigonometric equations:
bounded:

1. $\sin(x) = \frac{1}{2} \quad \text{for } 0 \leq x < 2\pi$

2. $2\cos(x) - 1 = 0 \quad \text{for } 0 \leq x < 2\pi$

3. $\sin^2(x) - \sin(x) = 0 \quad \text{for } 0 \leq x < 2\pi$

4. $2\cos^2(x) - 3\cos(x) + 1 = 0 \quad \text{for } 0 \leq x < 2\pi$

5. $\sin(x)\cos(x) = \frac{1}{2} \quad \text{for } 0 \leq x < 2\pi$

6. $\tan(x) + \cot(x) = 2 \quad \text{for } 0 < x < \pi$

7. $1 - 2\sin^2(x) = \cos(2x) \quad \text{for } 0 \leq x < 2\pi$

8. $\sec(x) = 2 \quad \text{for } 0 \leq x < 2\pi$

9. $\sin(2x) = \sqrt{3}\cos(x) \quad \text{for } 0 \leq x < 2\pi$

10. $\tan^2(x) = 3 \quad \text{for } 0 \leq x < 2\pi$

unbounded:

1. $\sin(x) = \frac{1}{2}$
2. $\cos(x) = -\frac{\sqrt{2}}{2}$
3. $\tan(x) = \sqrt{3}$
4. $2\sin(x) - 1 = 0$
5. $\cos^2(x) = \frac{1}{4}$
6. $\sec(x) = -2$
7. $\tan^2(x) = 1$
8. $\cot(x) + 1 = 0$
9. $\sin(2x) = 0$
10. $\cos(3x) = 1$

complex numbers (basic arithmetic operations and graphical manipulation)

1. Simplify the following expression and write the result in the form $a + bi$:
   $(3 + 2i)(1 - 4i) - (2 - i)^2$
2. Let $z = 5 - 3i$. Find:
   (a) The complex conjugate of z
   (b) The modulus $|z|$
   (c) The multiplicative inverse $\frac{1}{z}$
3. Compute $(1 + i)^8$ and express your answer in the form $a + bi$.
   Hint: Convert to polar form first.
4. Find all cube roots of the complex number $8(\cos(\pi) + i\sin(\pi))$.
   Express your answers in both polar and rectangular form.
5. Solve the quadratic equation $z^2 + 4z + 13 = 0$ and graph the solutions in the complex plane.
6. Plot the following complex numbers on the complex plane:
   $A = 3 + 4i, \quad B = -2 + i, \quad C = 1 - 3i$
   Also compute and plot:
   $A + B, \quad B - C$
7. Let $z_1 = 2 + 2i$ and $z_2 = -2 + 2i$. Show that the triangle formed by z_1, z_2, and the origin is a right triangle. Justify your answer algebraically and graphically.
8. Describe and sketch the locus of points z in the complex plane such that $|z - (2 + i)| = 3$.
   Hint: This is a circle. State its center and radius.
9. Let $z = 4 + 4i$. Rotate z by $90°$ counterclockwise about the origin. Express the rotated point as a complex number and plot both the original and rotated points.
10. Consider multiplication by i. For a general complex number $z = x + yi$, what is the result of iz?
    Describe the geometric transformation and illustrate it on the Argand diagram.

direct substitution:

1. $\lim_{x \to 3} \left(2x^2 - 5x + 4\right)$
2. $\lim_{x \to 0} \left(\frac{3x^2+2x-1}{x+2}\right)$
3. $\lim_{x \to -1} \left(4x^3 - 2x + 6\right)$
4. $\lim_{x \to 2} \left(\frac{x^2-4}{x-2}\right)$
5. $\lim_{x \to 1} \left(3x^3 - 2x + 5\right)$
6. $\lim_{x \to 0} \left(\frac{5x^2+3x}{x^2+2x+1}\right)$
7. $\lim_{x \to 4} (\sqrt{x} - 2)$
8. $\lim_{x \to 0} \left(\frac{x^3+2x}{x^2-3x+2}\right)$
9. $\lim_{x \to 1} \left(\frac{x^2+x-2}{x-1}\right)$
10. $\lim_{x \to -3} \left(\frac{2x+1}{x^2+5x+6}\right)$

indeterminate forms and algebraic simplification

1. $\lim_{x \to 2} \frac{x^2-4}{x-2}$
2. $\lim_{x \to 0} \frac{\sin(x)}{x}$
3. $\lim_{x \to 0} \frac{e^x-1}{x}$
4. $\lim_{x \to 0} \frac{1-\cos(x)}{x^2}$
5. $\lim_{x \to 0} \frac{x^2+3x}{x}$
6. $\lim_{x \to 1} \frac{x^3-1}{x-1}$
7. $\lim_{x \to \infty} \frac{1}{x}$
8. $\lim_{x \to 0} \frac{\ln(x)}{x}$
9. $\lim_{x \to 0} \frac{x^2+2x-3}{x^2-1}$
10. $\lim_{x \to 0} \frac{x^3+x}{x^2-1}$

trigonometic limits:

1. $\lim_{x \to 0} \frac{\sin(x)}{x}$
2. $\lim_{x \to 0} \frac{1-\cos(x)}{x^2}$
3. $\lim_{x \to 0} \frac{\tan(x)}{x}$
4. $\lim_{x \to 0} \frac{\sin(3x)}{x}$
5. $\lim_{x \to 0} \frac{1-\cos(2x)}{x^2}$

6. $\lim_{x\to 0} \frac{\sin(x)-\sin(2x)}{x}$

7. $\lim_{x\to \infty} \frac{\sin(x)}{x}$

8. $\lim_{x\to 0} \frac{\cos(x)-1}{x^2}$

9. $\lim_{x\to 0} \frac{\sin(x)}{x^3}$

10. $\lim_{x\to 0} \frac{\tan(2x)}{x}$

piecewise functions:

1. $\lim_{x\to 0} \begin{cases} \sin(x) & \text{if } x \geq 0 \\ -\sin(x) & \text{if } x < 0 \end{cases}$

2. $\lim_{x\to 0} \begin{cases} \frac{x^2-4}{x-2} & \text{if } x \neq 2 \\ 4 & \text{if } x = 2 \end{cases}$

3. $\lim_{x\to 0} \begin{cases} \frac{\sin(x)}{x} & \text{if } x \neq 0 \\ 1 & \text{if } x = 0 \end{cases}$

4. $\lim_{x\to \pi} \begin{cases} \cos(x) & \text{if } x < \pi \\ \sin(x) & \text{if } x \geq \pi \end{cases}$

5. $\lim_{x\to 0} \begin{cases} \frac{\sin(2x)}{x} & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$

6. $\lim_{x\to 0} \begin{cases} \frac{1-\cos(x)}{x} & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$

7. $\lim_{x\to 0} \begin{cases} \tan(x) & \text{if } x \geq 0 \\ -\tan(x) & \text{if } x < 0 \end{cases}$

8. $\lim_{x\to 0} \begin{cases} \frac{\sin(x)-\sin(2x)}{x} & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$

9. $\lim_{x\to 0} \begin{cases} \frac{\sin(3x)}{x} & \text{if } x \neq 0 \\ 3 & \text{if } x = 0 \end{cases}$

10. $\lim_{x\to 0} \begin{cases} \frac{x^2}{\sin(x)} & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$

infinite limits and vertical asymptotes:

1. $\lim_{x\to 0^+} \frac{1}{x}$

2. $\lim_{x\to 0^-} \frac{1}{x}$

3. $\lim_{x\to \infty} \frac{1}{x^2}$

4. $\lim_{x\to 0} \frac{1}{x^2}$

5. $\lim_{x\to 2^+} \frac{1}{x-2}$

6. $\lim_{x\to -2^-} \frac{1}{x+2}$

7. $\lim_{x\to 0^+} \frac{\ln(x)}{x}$

8. $\lim_{x\to\infty} \frac{x^2}{x+1}$
9. $\lim_{x\to 3} \frac{1}{(x-3)^2}$
10. $\lim_{x\to 1} \frac{1}{x-1}$

squeeze theorem:

1.

epsilon-delta:

1.

basic polynomial differentiation:

1. find $\frac{d}{dx}[3x^4 + 5x^3 - 2x + 7]$
2. find $\frac{d}{dx}[4x^5 - x^2 + 6x - 3]$
3. find $\frac{d}{dx}[x^6 + 2x^4 - 3x^2 + x - 8]$
4. find $\frac{d}{dx}[5x^3 - 4x^2 + 7x + 1]$
5. find $\frac{d}{dx}[2x^5 - x^3 + x - 9]$
6. find $\frac{d}{dx}[3x^2 - 2x + 4]$
7. find $\frac{d}{dx}[x^7 - 5x^3 + 2x^2 - x + 6]$
8. find $\frac{d}{dx}[6x^4 - 3x^3 + 2x^2 + x - 5]$
9. find $\frac{d}{dx}[2x^8 - x^6 + 4x^2 - 3]$
10. find $\frac{d}{dx}[9x^5 - 7x^4 + 3x^2 + 2x + 1]$

implicit differentiation:

1. find $\frac{dy}{dx}[x^2 + y^2 = 25]$
2. find $\frac{dy}{dx}[x^3 + y^3 = 6xy]$
3. find $\frac{dy}{dx}[x^2y + y^2 = 10]$
4. find $\frac{dy}{dx}[x^2 + 2xy + y^2 = 7]$
5. find $\frac{dy}{dx}[x^3 + y^3 = 3xy]$
6. find $\frac{dy}{dx}[x^2y^2 + 3x = 5]$
7. find $\frac{dy}{dx}[x^2 + y^2 = x + y]$
8. find $\frac{dy}{dx}[x^3 + y^3 = 6x + 2y]$
9. find $\frac{dy}{dx}[xy = x + y]$
10. find $\frac{dy}{dx}[x^2 - 3xy + y^2 = 10]$

higher-order derivatives:

1. Find the second derivative of $f(x) = 3x^4 - 5x^2 + 2x - 7$

2. Find the third derivative of $f(x) = x^5 - 3x^3 + 4x^2 - 6$
3. Find the second derivative of $f(x) = \sin(x) + \cos(x)$
4. Find the first and second derivatives of $f(x) = e^{2x}\cos(x)$
5. Find the third derivative of $f(x) = 4x^3 + 3x^2 - 2x + 5$
6. Find the first and second derivatives of $f(x) = \ln(x^2 + 1)$
7. Find the second derivative of $f(x) = \tan(x)$
8. Find the fourth derivative of $f(x) = 7x^4 - 6x^2 + x - 3$
9. Find the second derivative of $f(x) = \frac{x^3}{x^2+1}$
10. Find the third derivative of $f(x) = x^4 \ln(x)$

applications of derivatives (tangent line equations, critical points, motion problems, related rates):

basic antiderivatives:

u-sub (reverse chain rule):

trig sub:

integration by parts:

partial fractions:

applications (area under curves, average value of a function, volume by disks/washers/shells, accumulation problems):