Linear Algebra

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1. Linear Equations and Vectors
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linear algebra is a branch of mathematics that deals with the study of vector spaces, linear transformations, and systems of linear equations. it provides a framework for describing and analyzing linear relationships between variables

A linear equation in n variables $x_1, x_2, x_3, \ldots, x_n$: $a_1x_1 + a_2x_2 + a_3x_3 + \ldots + a_nx_n = b$ where the coefficients $a_1, a_2, \ldots, a_n$ and b are constants. The following is a system of linear equations:

$$\begin{aligned} x_1 + x_2 + x_3 &= 2 \\ 2x_1 + 3x_2 + x_3 &= 3 \\ x_1 - x_2 - 2x_3 &= -6 \end{aligned}$$

NOTE TO SELF: 1. three equations... so you are going to have three geometrical objects 2. three variables... those objects are going to be embedded in a three dimensional space ... 3. I cannot necessarily formulate the words here but observe the following...:

- equation: $x = a$, space: 1 dimension(s), object(s) 0 dimension(s)
- equation: $ax + by = c$, space: 2 dimension(s), object(s) 1 dimension(s)
- equation: $ax + by + cz = d$, space: 3 dimension(s), object(s) 2 dimension(s)
- equation: $a_1x_1 + \ldots + a_nx_n = d$, space: n dimension(s), object(s) n-1 dimension(s)

NOTE TO SELF: remember subsitution? imagine a system with two equations and two unknowns... you solve for one variable in terms of another ... allowing you to have an equation with only one unknown... once a variable is know... you can then use that known variable in an original equation to obtain the remaining unknown variable

As the number of variables increases, a geometrical interpretation os such a system of equations becomes increasingly complex. Each equation will represent a space embedded in a larger space. Solutions will be points that lie on all the embedded spaces.

$$\begin{bmatrix} 1 & 2 & 3 & 61 \\ 4 & 5 & 6 & 32 \\ 7 & 8 & 9 & 3 \end{bmatrix}$$

NOTE TO SELF: size is rows x columns so the size of the matrix above is 3 x 4 and the number 7 is in position row 3 column 1 or (3, 1)
An identity matrix is a square matrix with 1s in the diagonal locations (1,1), (2,2), (3,3), etc., and zeors elsewhere. We write I_n for the n x n identity matrix ... ex.:

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$