

my review

May 25, 2025

calculus is a branch of mathematics that deals with rates of change and the accumulation of quantities.

precalculus \rightarrow calculus II

$$|a| = |-a|, |ab| = |a||b|$$

The **distance** between two real numbers a and b is $|b - a|$, which is the length of the line segment joining a and b .

Two real numbers a and b are close to each other if $|b - a|$ is small, and this is the case if their decimal expansions agree to many places. More precisely, *if the decimal expansions of a and b agree to k places (to the right of the decimal point), then the distance $|b - a|$ is at most 10^{-k} . Thus, the distance between $a = 3.1415$ and $b = 3.1478$ is at most 10^{-2} because a and b agree to two places. In fact, the distance is exactly $|3.1415 - 3.1478| = 0.0063$.*

Beware that $|a + b|$ is not equal to $|a| + |b|$ unless a and b have the same sign or at least one of a and b is zero. If they have opposite signs, cancellation occurs in the sum $a + b$ and $|a + b| < |a| + |b|$. For example, $|2 + 5| = |2| + |5|$ but $|-2 + 5| = 3$, which is less than $|-2| + |5| = 7$. In any case, $|a + b|$ is never larger than $|a| + |b|$ and this gives us the simple but important **triangle inequality**: $|a + b| \leq |a| + |b|$

$$\begin{aligned}[a, b] &= \{x \in \mathbb{R} : a \leq x \leq b\} \\ (a, b) &= \{x \in \mathbb{R} : a < x < b\} \\ [a, b) &= \{x \in \mathbb{R} : a \leq x < b\} \\ (a, b] &= \{x \in \mathbb{R} : a < x \leq b\}\end{aligned}$$

$$(-r, r) = \{x : |x| < r\}$$

circle: $(x - a)^2 + (y - b)^2 = r^2$ where (a, b) is the center and the radius is r

midpoint between $P_1 = (x_1, y_1)$ and $P_2 = (x_2, y_2)$ is $(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2})$

distance: $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

composing new functions

If f and g are functions, we may construct new functions by forming the sum, difference, product, and quotient functions:

- $(f + g)(x) = f(x) + g(x)$

- $(f - g)(x) = f(x) - g(x)$
- $(fg)(x) = f(x)g(x)$
- $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$

We can also multiply functions by constants. A function of the form: $c_1f(x) + c_2g(x)$ is called a **linear combination**.

Composition is another important way of constructing new functions. The composition of f and g is the function $f \circ g$ defined by $(f \circ g)(x) = f(g(x))$, defined for values of x in the domain of g such that $g(x)$ lies in the domain of f .

ex. Compute the composite functions $f \circ g$ and $g \circ f$ and discuss their domains where $f(x) = \sqrt{x}$ and $g(x) = 1 - x$

solution: $(f \circ g)(x) = f(g(x)) = f(1 - x) = \sqrt{1 - x}$ The square root $\sqrt{1 - x}$ is defined if $1 - x \geq 0$ or $x \leq 1$, so the domain of $f \circ g$ is $x : x \leq 1$.

On the other hand, $(g \circ f)(x) = g(f(x)) = g(\sqrt{x}) = 1 - \sqrt{x}$ The domain of $g \circ f$ is $x : x \geq 0$.

invertable functions

"is this function invertible?" \Leftrightarrow "does an inverse function exist for this function" \Leftrightarrow "is the function one-to-one?" (horizontal line test)

- if it is, then the inverse function exists
- if it is not, then the inverse function does not exist, and the function is not invertible (as a function)

consider $f(x) = x^2$ this function is not one-to-one (horizontal line test) this it is not invertible unless you restrict the domain to be $x \geq 0$.

to find the inverse algebraically you can swap the x's and y's and then solve for y

complex numbers

- rectangular form $z = x + yi$ where s is the real part and y is the imaginary part, and $i = \sqrt{-1}, i^2 = -1$
- polar form $z = r(\cos(\theta) + i \sin(\theta))$ where $r = \sqrt{x^2 + y^2}, \theta = \tan^{-1}\left(\frac{y}{x}\right)$
- Euler's formula $z = re^{i\theta}$

the fundamental theorem of algebra

Let $p(z)$ be a non-constant polynomial with complex coefficients, i.e.,

$$p(z) = a_n z^n + a_{n-1} z^{n-1} + \cdots + a_1 z + a_0,$$

where $a_n \neq 0$ and $n \geq 1$. Then, there exists at least one complex number $c \in \mathbb{C}$ such that

$$p(c) = 0.$$

This theorem says that any polynomial equation — no matter how complicated — always has at least one solution if we allow the solutions to be complex numbers (numbers that can include the square root of negative one). Even if the polynomial doesn't have any real solutions, it will have complex ones. This means every polynomial can be "broken down" completely into simpler parts based on its roots.

rational functions

conic sections

- ellipses
 - (h, k) center of the ellipse
 - a semi-major axis (long radius)
 - b semi-minor axis (short radius)
 - c distance from center to each focus $c = \sqrt{a^2 - b^2}$
 - horizontal major axis $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$
 - vertical major axis $\frac{(x-h)^2}{b^2} + \frac{(y-k)^2}{a^2} = 1$
- hyperbolas
 - (h, k) center
 - a distance from center to each vertex (on transverse axis)
 - b related to the asymptotes
 - $c = \sqrt{a^2 + b^2}$ distance from center to each focus (note here add not subtract like ellipse)
 - asymptotes
 - * for horizontal hyperbola $y - k = \pm \frac{b}{a}(x - h)$
 - * for vertical hyperbola $y - k = \pm \frac{a}{b}(x - h)$
 - transverse axis: line through both vertices and foci
 - conjugate axis: perpendicular to the transverse axis
 - opens horizontally $\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$
 - opens vertically $\frac{(x-h)^2}{b^2} - \frac{(y-k)^2}{a^2} = 1$

vectors

matrices

probability and combinatorics

series

laws of exponents:

- $x^m x^n = x^{m+n}$
- $\frac{x^m}{x^n} = x^{m-n}$
- $(x^m)^n = x^{mn}$
- $x^{-n} = \frac{1}{x^n}$
- $(xy)^n = x^n y^n$
- $\left(\frac{x}{y}\right)^n = \frac{x^n}{y^n}$
- $x^{1/n} = \sqrt[n]{x}$
- $x^{\frac{m}{n}} = \sqrt[n]{x^m} = (\sqrt[n]{x})^m$

exponential and logarithmic functions:

- $\log_a x = y \leftrightarrow a^y = x$
- $\ln x = y \leftrightarrow e^y = x$
- $\log_a(xy) = \log_a x + \log_a y$
- $\log_a(a^x) = x$
 $a^{\log_a x} = x$
- $\ln(e^x) = xe^{\ln x} = x$
- $\log_a(\frac{x}{y}) = \log_a x - \log_a y$
- $\log_a 1 = 0$
 $\log_a a = 1$
- $\ln 1 = 0$
 $\ln e = 1$
- $\log_a(\frac{x}{y}) = \log_a x - \log_a y$
- $\log_a 1 = 0$
 $\log_a a = 1$
- $\ln 1 = 0$
 $\ln e = 1$
- $\log_a(x^r) = r \log_a x$

special factorizations:

- $x^2 - y^2 = (x + y)(x - y)$
- $x^3 + y^3 = (x + y)(x^2 - xy + y^2)$
- $x^3 - y^3 = (x - y)(x^2 + xy + y^2)$

binomial theorem:

- $(x + y)^2 = x^2 + 2xy + y^2$
- $(x - y)^2 = x^2 - 2xy + y^2$
- $(x + y)^3 = x^3 + 3x^2y + 3xy^2 + y^3$
- $(x - y)^3 = x^3 - 3x^2y + 3xy^2 - y^3$
- $(x + y)^n = x^n + nx^{n-1}y + \frac{n(n-1)}{2}x^{n-2}y^2 + \dots + \binom{n}{k}x^{n-k}y^k + \dots + nxy^{n-1} + y^n$
where $\binom{n}{k} = \frac{n(n-1)\dots(n-k+1)}{1 \cdot 2 \cdot 3 \dots k}$

quadratic formulae:

1. $ax^2 + bx + c = 0$
2. $x^2 + \frac{b}{a}x + \frac{c}{a} = 0$
3. $x^2 + \frac{b}{a}x = -\frac{c}{a}$
4. $x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2 = -\frac{c}{a} + \left(\frac{b}{2a}\right)^2$

$$5. \left(x + \frac{b}{2a}\right)^2 = \frac{b^2 - 4ac}{4a^2}$$

$$6. x + \frac{b}{2a} = \pm \frac{\sqrt{b^2 - 4ac}}{2a}$$

$$7. x = -\frac{b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a}$$

$$8. x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

polynomials $a_n x^n + a_{n-1} x^{n-1} + \dots + a_1 x + a_0$ where n is non-neg and represents the degree

rational $p(x)/q(x)$

root $\sqrt[n]{g(x)}$

properties:

domain: set of all input values for which the function is defined

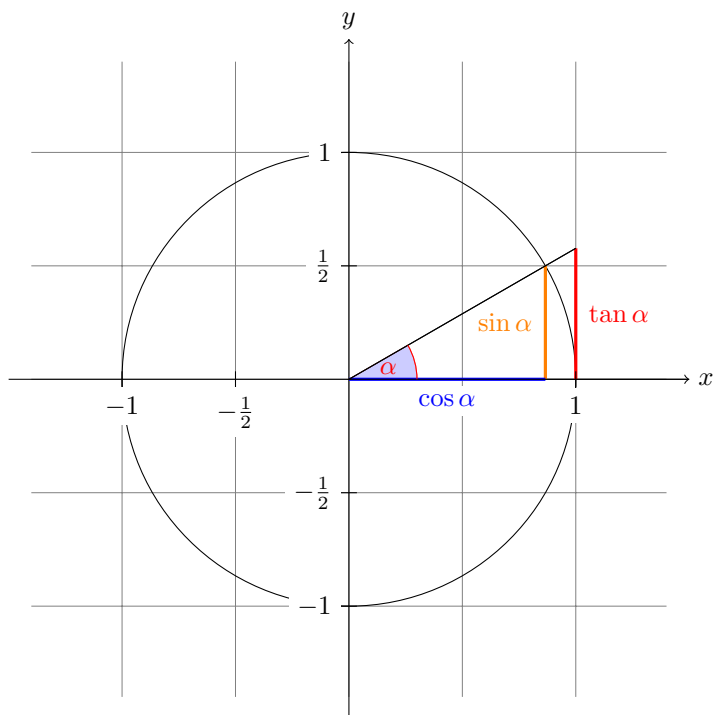
range: set of all possible output values of the function

continuity: most algebraic functions are continuous (no breaks or jumps), but rational functions have discontinuities at points

behavior: the functions behavior is influenced by the degree of the polynomial and the nature of the function

scaling:

- vertical scaling $y = kf(x)$: If $k \geq 1$, the graph is expanded vertically by the factor k . If $0 < k < 1$, the graph is compressed vertically. When the scale factor k is negative ($k < 0$), the graph is also reflected across the x-axis.
- horizontal scaling $y = f(kx)$: If $K \geq 1$, the graph is compressed in the horizontal direction. If $0 < k < 1$, the graph is expanded. If $k \leq 0$, then the graph is also reflected across the y-axis.



Angle (Degrees)	Angle (Radians)	$\cos(\theta)$	$\sin(\theta)$
0°	0	1	0
30°	$\frac{\pi}{6}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$
45°	$\frac{\pi}{4}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$
60°	$\frac{\pi}{3}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$
90°	$\frac{\pi}{2}$	0	1
120°	$\frac{2\pi}{3}$	$-\frac{1}{2}$	$\frac{\sqrt{3}}{2}$
135°	$\frac{3\pi}{4}$	$-\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$
150°	$\frac{5\pi}{6}$	$-\frac{\sqrt{3}}{2}$	$\frac{1}{2}$
180°	π	-1	0
210°	$\frac{7\pi}{6}$	$-\frac{\sqrt{3}}{2}$	$-\frac{1}{2}$
225°	$\frac{5\pi}{4}$	$-\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{2}}{2}$
240°	$\frac{4\pi}{3}$	$-\frac{1}{2}$	$-\frac{\sqrt{3}}{2}$
270°	$\frac{3\pi}{2}$	0	-1
300°	$\frac{5\pi}{3}$	$\frac{1}{2}$	$-\frac{\sqrt{3}}{2}$
315°	$\frac{7\pi}{4}$	$\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{2}}{2}$
330°	$\frac{11\pi}{6}$	$\frac{\sqrt{3}}{2}$	$-\frac{1}{2}$
360°	2π	1	0

$$1^\circ = \frac{\pi}{180} \text{rad}$$

$$1 \text{rad} = \frac{180^\circ}{\pi}$$

SOH-CAH-TOA is a mnemonic device that expresses the relationship between the

basic trigonometric functions and the ratios of the sides in a right triangle.

trigonometric functions are mathematical functions that relate the angle of a triangle to the lengths of its sides... and can also be generalized to all real numbers using the unit circle.

law of sines/cosines

$$\frac{\sin(A)}{a} = \frac{\sin(B)}{b} = \frac{\sin(C)}{c} \quad a^2 = b^2 + c^2 - 2bc \cos(\theta)$$

To derive the rest of the fundamental trigonometric identities, you need a combination of a few key identities and principles. The most important starting point is the Pythagorean identity, but you'll also need the basic relationships between the trigonometric functions, such as the definitions of sine, cosine, tangent, secant, cosecant, and cotangent in terms of a right triangle or the unit circle.

$$\sin(-\theta) = -\sin(\theta)$$

$$\cos(-\theta) = \cos(\theta)$$

$$\tan(-\theta) = -\tan(\theta)$$

$$\sin\left(\frac{\pi}{2} - \theta\right) = \cos(\theta)$$

$$\cos\left(\frac{\pi}{2} - \theta\right) = \sin(\theta)$$

$$\tan\left(\frac{\pi}{2} - \theta\right) = \cot(\theta)$$

$$\sin^2 \theta + \cos^2 \theta = 1$$

$$\sec \theta = \frac{1}{\cos \theta}, \quad \csc \theta = \frac{1}{\sin \theta}, \quad \cot \theta = \frac{1}{\tan \theta}$$

$$\tan \theta = \frac{\sin \theta}{\cos \theta}, \quad \cot \theta = \frac{\cos \theta}{\sin \theta}$$

$$1 + \tan^2 \theta = \sec^2 \theta$$

$$1 + \cot^2 \theta = \csc^2 \theta$$

$$\sin(\alpha + \beta) = \sin \alpha \cos \beta + \cos \alpha \sin \beta$$

$$\cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta$$

$$\tan(\alpha + \beta) = \frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \tan \beta}$$

$$\sin(\alpha - \beta) = \sin \alpha \cos \beta - \cos \alpha \sin \beta$$

$$\cos(\alpha - \beta) = \cos \alpha \cos \beta + \sin \alpha \sin \beta$$

$$\tan(\alpha - \beta) = \frac{\tan \alpha - \tan \beta}{1 + \tan \alpha \tan \beta}$$

$$\sin(2\theta) = 2 \sin \theta \cos \theta$$

$$\cos(2\theta) = \cos^2 \theta - \sin^2 \theta = 2 \cos^2 \theta - 1 = 1 - 2 \sin^2 \theta$$

$$\tan(2\theta) = \frac{2 \tan \theta}{1 - \tan^2 \theta}$$

$$\sin^2(2\theta) = \frac{1 - \cos^2(2\theta)}{2}$$

$$\cos^2(2\theta) = \frac{1 + \cos(2\theta)}{2}$$

$$\sin(90^\circ - \theta) = \cos \theta, \quad \cos(90^\circ - \theta) = \sin \theta$$

$$\tan(90^\circ - \theta) = \cot \theta, \quad \cot(90^\circ - \theta) = \tan \theta$$

$$\sec(90^\circ - \theta) = \csc \theta, \quad \csc(90^\circ - \theta) = \sec \theta$$

$$\sin(-\theta) = -\sin(\theta), \quad \cos(-\theta) = \cos(\theta)$$

$$\tan(-\theta) = -\tan(\theta), \quad \sec(-\theta) = \sec(\theta)$$

$$\csc(-\theta) = -\csc(\theta), \quad \cot(-\theta) = -\cot(\theta)$$

$$\sin \alpha \sin \beta = \frac{1}{2} [\cos(\alpha - \beta) - \cos(\alpha + \beta)]$$

$$\cos \alpha \cos \beta = \frac{1}{2} [\cos(\alpha - \beta) + \cos(\alpha + \beta)]$$

$$\sin \alpha \cos \beta = \frac{1}{2} [\sin(\alpha + \beta) + \sin(\alpha - \beta)]$$

power functions: $f(x) = x^n$...if even the function behaves symmetrical around the y-axis...if odd then the function has point symmetry ($x^4, x^3, x^{-n} = \frac{1}{x^n}$)

inverse trig functions: $\arcsin(x) = \sin^{-1}(x) = \theta$, $\arccos(x) = \cos^{-1}(x) = \theta$...etc

logs: $\log_a x = y \leftrightarrow a^y = x$, $\ln(x) = y \leftrightarrow e^y = x$

hyperbolic functions: $\sinh(x) = \frac{e^x - e^{-x}}{2}$, $\cosh(x) = \frac{e^x + e^{-x}}{2}$, $\tanh(x) = \frac{\sinh(x)}{\cosh(x)}$

differentiation rules:

1. $\frac{d}{dx}(c) = 0$
2. $\frac{d}{dx}x = 1$
3. $\frac{d}{dx}(x^n) = nx^{n-1}$ (power rule)
4. $\frac{d}{dx}[cf(x)] = cf'(x)$
5. $\frac{d}{dx}[f(x) + g(x)] = f'(x) + g'(x)$
6. $\frac{d}{dx}[f(x)g(x)] = f(x)g'(x) + g(x)f'(x)$ (product rule)
7. $\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^2}$ (quotient rule)
8. $\frac{d}{dx}f(g(x)) = f'(g(x))g'(x)$ (chain rule)
9. $\frac{d}{dx}f(x)^n = nf(x)^{n-1}f'(x)$ (general power rule)
10. $\frac{d}{dx}\sin(x) = \cos(x)$
11. $\frac{d}{dx}\cos(x) = -\sin(x)$
12. $\frac{d}{dx}\tan(x) = \sec^2(x)$
13. $\frac{d}{dx}\csc(x) = -\csc(x)\cot(x)$
14. $\frac{d}{dx}\sec(x) = \sec(x)\tan(x)$
15. $\frac{d}{dx}\cot(x) = -\csc^2(x)$
16. $\frac{d}{dx}\sin^{-1}(x) = \frac{1}{\sqrt{1-x^2}}$
17. $\frac{d}{dx}\cos^{-1}(x) = -\frac{1}{\sqrt{1-x^2}}$
18. $\frac{d}{dx}\tan^{-1}(x) = \frac{1}{1+x^2}$
19. $\frac{d}{dx}(e^x) = e^x$
20. $\frac{d}{dx}(a^x) = (\ln a)a^x$
21. $\frac{d}{dx}\ln|x| = \frac{1}{x}$
22. $\frac{d}{dx}\log_a x = \frac{1}{(\ln a)x}$

Essential Theorems:

- **Intermediate Value Theorem (IVT):** Guarantees that a continuous function takes every value between $f(a)$ and $f(b)$ at some point in the interval $[a, b]$.

- **Mean Value Theorem (MVT):** States that for a continuous and differentiable function, there is at least one point where the instantaneous rate of change equals the average rate of change over the interval.
- **Extreme Value Theorem:** Guarantees that a continuous function on a closed interval attains a maximum and minimum value.
- **Fundamental Theorem of Calculus:**
 1. **First Part:** The derivative of the integral of a function is the original function.
 2. **Second Part:** The definite integral of a function can be computed using its antiderivative.

review problems

factoring:

1. $x^2 + 5x + 6$
2. $x^2 - 9$
3. $x^2 - x - 12$
4. $3x^2 + 2x - 1$
5. $x^2 + 2x$
6. $x^3 + 3x^2 + 2x + 6$
7. $2x^2 + 7x + 3$
8. $x^3 - 8$
9. $x^3 + 27$
10. $x^2 + 6x + 9$
11. $4x^2 - 12x + 9$
12. $x^2 + 4x + 5$
13. $x^2 - 10x + 29$

solving triangles:

1. In triangle ABC , $A = 40^\circ$, $B = 65^\circ$, and $c = 12$. Find side a .
2. In triangle ABC , $a = 10$, $b = 14$, and $C = 40^\circ$. Find side c using the Law of Cosines.
3. In triangle ABC , $a = 7$, $b = 9$, and $c = 10$. Find angle A .
4. In triangle ABC , $A = 50^\circ$, $a = 12$, and $b = 10$. Find angle B .
5. In triangle ABC , $a = 13$, $b = 15$, and $C = 70^\circ$. Find the area of the triangle.
6. In triangle ABC , $A = 30^\circ$, $B = 45^\circ$, and $c = 18$. Find side a and side b .
7. In triangle ABC , $a = 8$, $b = 11$, and $c = 14$. Find all angles using the Law of Cosines.
8. In triangle ABC , $A = 110^\circ$, $b = 20$, and $c = 25$. Find side a .
9. In triangle ABC , $a = 16$, $b = 20$, and angle $C = 90^\circ$. Find side c and angles A and B .
10. In triangle ABC , $a = 9$, $b = 10$, $c = 11$. Find the area using Heron's Formula.

solving triangles (ambiguous case SSA):

1. Given triangle ABC with $A = 30^\circ$, $a = 10$, and $b = 15$. Determine the number of possible triangles and find all possible solutions.
2. In triangle ABC , $A = 50^\circ$, $a = 7$, and $b = 10$. Find how many triangles exist and solve for all possible angles and sides.

- For triangle ABC , $A = 40^\circ$, $a = 8$, and $b = 5$. Decide the number of triangles possible and solve if applicable.
- Given $A = 60^\circ$, $a = 9$, and $b = 9$. How many triangles can be formed? Find all solutions.
- In triangle ABC , $A = 25^\circ$, $a = 4$, and $b = 10$. Determine the possible number of triangles and solve accordingly.
- For triangle ABC , $A = 35^\circ$, $a = 12$, and $b = 7$. Analyze the number of triangles and solve.
- In triangle ABC , $A = 45^\circ$, $a = 5$, and $b = 6$. Find the number of possible triangles and all possible solutions.
- Given triangle ABC with $A = 20^\circ$, $a = 3$, and $b = 8$. Determine how many triangles can be formed and solve.
- For triangle ABC , $A = 70^\circ$, $a = 15$, and $b = 12$. Find the number of possible triangles and solve accordingly.
- In triangle ABC , $A = 10^\circ$, $a = 2$, and $b = 5$. Determine the number of triangles possible and solve if any.

polynomial long division (1. divide 2. multiply 3. subtract 4. repeat):

- divide $(2x^3 + 3x^2 - 5x + 6)$ by $(x - 2)$
- divide $(x^2 + 5x + 6)$ by $(x + 2)$
- divide $(2x^3 + 8x^2 + 10)$ by $(x - 2)$
- divide $6x^4 - 9x^2 + 18$ by $x - 3$

synthetic division (1. multiply 2. divide 3. repeat):

- divide $(x^3 - 2x^2 - 5x)$ by $(x - 3)$
- divide $(x^3 + 5x^2 + 7x + 2)$ by $(x + 2)$
- divide $(3x^2 + 7x - 20)$ by $(x + 5)$
- divide $(7x^3 + 6x - 8)$ by $(x - 4)$

verifying trigonometric identities:

- $\frac{1+\sin(x)}{\cos(x)} + \frac{1-\sin(x)}{\cos(x)} = 2\sec(x)$
- $\frac{\sin(x)}{1+\cos(x)} + \frac{\sin(x)}{1-\cos(x)} = \frac{2\sin(x)}{1-\cos^2(x)}$
- $\frac{\tan(x)+\cot(x)}{\sec(x)+\csc(x)} = \sin(x)\cos(x)$
- $\frac{\sin(x)-\cos(x)}{\sin(x)+\cos(x)} = \frac{1-\tan(x)}{1+\tan(x)}$
- $\frac{1-\cos(2x)}{2\sin(x)\cos(x)} = \tan(x)$

inequalities:

- $3x - 5 \leq 2x + 7$

2. $x^2 - 4x - 5 > 0$
3. $\frac{2x+3}{x-1} \geq 0$
4. $|2x - 3| \leq 5$
5. $x^3 - 3x^2 - 4x + 12 \leq 0$
6. $\sin(x) \geq 0.5 \quad 0 \leq x \leq 2\pi$
7. $\tan(x) < 1 \quad 0 \leq x \leq \pi$
8. $\sin(x) - \cos(x) \geq 0 \quad 0 \leq x \leq 2\pi$
9. $2 \cos(x) - \sin(x) \leq 1 \quad 0 \leq x \leq 2\pi$
10. $\cos^2(x) \geq \frac{1}{2} \quad 0 \leq x \leq 2\pi$
11. $x^2 + 5x + 6 \geq 0$
12. $2x - 7 < 3x + 1$
13. $\frac{3x-4}{x+2} \leq 1$
14. $|x + 4| > 2$
15. $x^3 - 2x^2 + x - 1 \geq 0$
16. $\sin(x) \leq -\frac{1}{2} \quad 0 \leq x \leq 2\pi$
17. $\cos(x) > 0 \quad 0 \leq x \leq 2\pi$
18. $2 \sin(x) + 1 \geq 0 \quad 0 \leq x \leq 2\pi$
19. $2 \tan(x) \leq 3 \quad 0 \leq x \leq \frac{\pi}{2}$
20. $\sin^2(x) + \cos^2(x) = 1$

solving trigonometric equations:
bounded:

1. $\sin(x) = \frac{1}{2} \quad \text{for } 0 \leq x < 2\pi$
2. $2 \cos(x) - 1 = 0 \quad \text{for } 0 \leq x < 2\pi$
3. $\sin^2(x) - \sin(x) = 0 \quad \text{for } 0 \leq x < 2\pi$
4. $2 \cos^2(x) - 3 \cos(x) + 1 = 0 \quad \text{for } 0 \leq x < 2\pi$
5. $\sin(x) \cos(x) = \frac{1}{2} \quad \text{for } 0 \leq x < 2\pi$
6. $\tan(x) + \cot(x) = 2 \quad \text{for } 0 < x < \pi$
7. $1 - 2 \sin^2(x) = \cos(2x) \quad \text{for } 0 \leq x < 2\pi$
8. $\sec(x) = 2 \quad \text{for } 0 \leq x < 2\pi$
9. $\sin(2x) = \sqrt{3} \cos(x) \quad \text{for } 0 \leq x < 2\pi$
10. $\tan^2(x) = 3 \quad \text{for } 0 \leq x < 2\pi$

unbounded:

1. $\sin(x) = \frac{1}{2}$
2. $\cos(x) = -\frac{\sqrt{2}}{2}$
3. $\tan(x) = \sqrt{3}$
4. $2\sin(x) - 1 = 0$
5. $\cos^2(x) = \frac{1}{4}$
6. $\sec(x) = -2$
7. $\tan^2(x) = 1$
8. $\cot(x) + 1 = 0$
9. $\sin(2x) = 0$
10. $\cos(3x) = 1$

direct substitution:

1. $\lim_{x \rightarrow 3} (2x^2 - 5x + 4)$
2. $\lim_{x \rightarrow 0} \left(\frac{3x^2 + 2x - 1}{x + 2} \right)$
3. $\lim_{x \rightarrow -1} (4x^3 - 2x + 6)$
4. $\lim_{x \rightarrow 2} \left(\frac{x^2 - 4}{x - 2} \right)$
5. $\lim_{x \rightarrow 1} (3x^3 - 2x + 5)$
6. $\lim_{x \rightarrow 0} \left(\frac{5x^2 + 3x}{x^2 + 2x + 1} \right)$
7. $\lim_{x \rightarrow 4} (\sqrt{x} - 2)$
8. $\lim_{x \rightarrow 0} \left(\frac{x^3 + 2x}{x^2 - 3x + 2} \right)$
9. $\lim_{x \rightarrow 1} \left(\frac{x^2 + x - 2}{x - 1} \right)$
10. $\lim_{x \rightarrow -3} \left(\frac{2x + 1}{x^2 + 5x + 6} \right)$

indeterminate forms and algebraic simplification

1. $\lim_{x \rightarrow 2} \frac{x^2 - 4}{x - 2}$
2. $\lim_{x \rightarrow 0} \frac{\sin(x)}{x}$
3. $\lim_{x \rightarrow 0} \frac{e^x - 1}{x}$
4. $\lim_{x \rightarrow 0} \frac{1 - \cos(x)}{x^2}$
5. $\lim_{x \rightarrow 0} \frac{x^2 + 3x}{x}$
6. $\lim_{x \rightarrow 1} \frac{x^3 - 1}{x - 1}$
7. $\lim_{x \rightarrow \infty} \frac{1}{x}$

8. $\lim_{x \rightarrow 0} \frac{\ln(x)}{x}$
9. $\lim_{x \rightarrow 0} \frac{x^2+2x-3}{x^2-1}$
10. $\lim_{x \rightarrow 0} \frac{x^3+x}{x^2-1}$

trigonometric limits:

1. $\lim_{x \rightarrow 0} \frac{\sin(x)}{x}$
2. $\lim_{x \rightarrow 0} \frac{1-\cos(x)}{x^2}$
3. $\lim_{x \rightarrow 0} \frac{\tan(x)}{x}$
4. $\lim_{x \rightarrow 0} \frac{\sin(3x)}{x}$
5. $\lim_{x \rightarrow 0} \frac{1-\cos(2x)}{x^2}$
6. $\lim_{x \rightarrow 0} \frac{\sin(x)-\sin(2x)}{x}$
7. $\lim_{x \rightarrow \infty} \frac{\sin(x)}{x}$
8. $\lim_{x \rightarrow 0} \frac{\cos(x)-1}{x^2}$
9. $\lim_{x \rightarrow 0} \frac{\sin(x)}{x^3}$
10. $\lim_{x \rightarrow 0} \frac{\tan(2x)}{x}$

piecewise functions:

1. $\lim_{x \rightarrow 0} \begin{cases} \sin(x) & \text{if } x \geq 0 \\ -\sin(x) & \text{if } x < 0 \end{cases}$
2. $\lim_{x \rightarrow 0} \begin{cases} \frac{x^2-4}{x-2} & \text{if } x \neq 2 \\ 4 & \text{if } x = 2 \end{cases}$
3. $\lim_{x \rightarrow 0} \begin{cases} \frac{\sin(x)}{x} & \text{if } x \neq 0 \\ 1 & \text{if } x = 0 \end{cases}$
4. $\lim_{x \rightarrow \pi} \begin{cases} \cos(x) & \text{if } x < \pi \\ \sin(x) & \text{if } x \geq \pi \end{cases}$
5. $\lim_{x \rightarrow 0} \begin{cases} \frac{\sin(2x)}{x} & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$
6. $\lim_{x \rightarrow 0} \begin{cases} \frac{1-\cos(x)}{x} & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$
7. $\lim_{x \rightarrow 0} \begin{cases} \tan(x) & \text{if } x \geq 0 \\ -\tan(x) & \text{if } x < 0 \end{cases}$
8. $\lim_{x \rightarrow 0} \begin{cases} \frac{\sin(x)-\sin(2x)}{x} & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$
9. $\lim_{x \rightarrow 0} \begin{cases} \frac{\sin(3x)}{x} & \text{if } x \neq 0 \\ 3 & \text{if } x = 0 \end{cases}$

$$10. \lim_{x \rightarrow 0} \begin{cases} \frac{x^2}{\sin(x)} & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$$

infinite limits and vertical asymptotes:

1. $\lim_{x \rightarrow 0^+} \frac{1}{x}$
2. $\lim_{x \rightarrow 0^-} \frac{1}{x}$
3. $\lim_{x \rightarrow \infty} \frac{1}{x^2}$
4. $\lim_{x \rightarrow 0} \frac{1}{x^2}$
5. $\lim_{x \rightarrow 2^+} \frac{1}{x-2}$
6. $\lim_{x \rightarrow -2^-} \frac{1}{x+2}$
7. $\lim_{x \rightarrow 0^+} \frac{\ln(x)}{x}$
8. $\lim_{x \rightarrow \infty} \frac{x^2}{x+1}$
9. $\lim_{x \rightarrow 3} \frac{1}{(x-3)^2}$
10. $\lim_{x \rightarrow 1} \frac{1}{x-1}$

squeeze theorem:

- 1.

epsilon-delta:

- 1.

basic polynomial differentiation:

1. find $\frac{d}{dx}[3x^4 + 5x^3 - 2x + 7]$
2. find $\frac{d}{dx}[4x^5 - x^2 + 6x - 3]$
3. find $\frac{d}{dx}[x^6 + 2x^4 - 3x^2 + x - 8]$
4. find $\frac{d}{dx}[5x^3 - 4x^2 + 7x + 1]$
5. find $\frac{d}{dx}[2x^5 - x^3 + x - 9]$
6. find $\frac{d}{dx}[3x^2 - 2x + 4]$
7. find $\frac{d}{dx}[x^7 - 5x^3 + 2x^2 - x + 6]$
8. find $\frac{d}{dx}[6x^4 - 3x^3 + 2x^2 + x - 5]$
9. find $\frac{d}{dx}[2x^8 - x^6 + 4x^2 - 3]$
10. find $\frac{d}{dx}[9x^5 - 7x^4 + 3x^2 + 2x + 1]$

implicit differentiation:

1. find $\frac{dy}{dx}[x^2 + y^2 = 25]$
2. find $\frac{dy}{dx}[x^3 + y^3 = 6xy]$

3. find $\frac{dy}{dx}[x^2y + y^2 = 10]$
4. find $\frac{dy}{dx}[x^2 + 2xy + y^2 = 7]$
5. find $\frac{dy}{dx}[x^3 + y^3 = 3xy]$
6. find $\frac{dy}{dx}[x^2y^2 + 3x = 5]$
7. find $\frac{dy}{dx}[x^2 + y^2 = x + y]$
8. find $\frac{dy}{dx}[x^3 + y^3 = 6x + 2y]$
9. find $\frac{dy}{dx}[xy = x + y]$
10. find $\frac{dy}{dx}[x^2 - 3xy + y^2 = 10]$

higher-order derivatives:

1. Find the second derivative of $f(x) = 3x^4 - 5x^2 + 2x - 7$
2. Find the third derivative of $f(x) = x^5 - 3x^3 + 4x^2 - 6$
3. Find the second derivative of $f(x) = \sin(x) + \cos(x)$
4. Find the first and second derivatives of $f(x) = e^{2x} \cos(x)$
5. Find the third derivative of $f(x) = 4x^3 + 3x^2 - 2x + 5$
6. Find the first and second derivatives of $f(x) = \ln(x^2 + 1)$
7. Find the second derivative of $f(x) = \tan(x)$
8. Find the fourth derivative of $f(x) = 7x^4 - 6x^2 + x - 3$
9. Find the second derivative of $f(x) = \frac{x^3}{x^2+1}$
10. Find the third derivative of $f(x) = x^4 \ln(x)$

applications of derivatives (tangent line equations, critical points, motion problems, related rates):

basic antiderivatives:

u-sub (reverse chain rule):

trig sub:

integration by parts:

partial fractions:

applications (area under curves, average value of a function, volume by disks/washers/shells, accumulation problems):