Maths

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Algebra

Lines

Slope of the line through $P_1 = (x_1, y_1)$ and $P_2 = (x_2, y_2)$:

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Slope-intercept equation of line with slope m and y-intercept b:

$$y = mx + b$$

Point-slope equation of line through $P_1 = (x_1, y_1)$ with slope m:

$$y - y_1 = m(x - x_1)$$

Circles

Equation of the circle with center (a, b) and radius r:

$$(x - a)^2 + (y - b)^2 = r^2$$

Distance and Midpoint Formulas

Distance between $P_1 = (x_1, y_1)$ and $P_2 = (x_2, y_2)$:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Midpoint of $\bar{P_1P_2}$:

$$(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2})$$

Laws of Exponents

$$x^m x^n = x^{m+n}$$
$$\frac{x^m}{x^n} = x^{m-n}$$
$$(x^m)^n = x^{mn}$$
$$x^{-n} = \frac{1}{x^n}$$
$$(xy)^n = x^n y^n$$
$$(\frac{x}{y})^n = \frac{x^n}{y^n}$$
$$x^{\frac{1}{n}} = \sqrt[n]{x}$$
$$\sqrt[n]{xy} = \sqrt[n]{x}\sqrt[n]{y}$$
$$\sqrt[n]{\frac{x}{y}} = \frac{\sqrt[n]{x}}{\sqrt[n]{y}}$$
$$x^{\frac{m}{n}} = \sqrt[n]{x^m} = (\sqrt[n]{x})^m$$

Special Factorizations

$$x^2 - y^2 = (x + y)(x - y)$$
$$x^3 + y^3 = (x + y)(x^2 - xy + y^2)$$
$$x^3 - y^3 = (x - y)(x^2 + xy + y^2)$$

Binomial Theorem

$$(x + y)^2 = x^2 + 2xy + y^2$$
$$(x - y)^2 = x^2 - 2xy + y^2$$
$$(x + y)^3 = x^3 + 3x^2y + 3xy^2 + y^3$$
$$(x - y)^3 = x^3 - 3x^2y + 3xy^2 - y^3$$
$$(x + y)^n = x^n + nx^{n-1}y + \tfrac{n(n-1)}{2}x^{n-2}y^2 + \ldots + \binom{n}{k}x^{n-k}y^k + \ldots + nxy^{n-1} + y^n$$
$$\text{where } \binom{n}{k} = \tfrac{n(n-1)\ldots(n-k+1)}{1\cdot 2\cdot 3\ldots k}$$

Quadratic Formula

$$\text{If } ax^2 + bx + c = 0, \text{ then } x = \tfrac{-b \pm \sqrt{b^2-4ac}}{2a}.$$

Inequalities and Absolute Value

$$\text{If } a < b \text{ and } b < c, \text{ then } a < c.$$
$$\text{If } a < b, \text{ then } a + c < b + c.$$
$$\text{if } a < b \text{ and } c > 0, \text{ then } ca < cb.$$
$$\text{if } a < b \text{ and } c < 0, \text{ then } ca > cb.$$
$$|x| = x \text{ if } x >= 0$$
$$|x| = -x \text{ if } x <= 0$$

Geometry

Formulas for area A, circumference C, and volume V
Triangle

$$A = \tfrac{1}{2}bh$$
$$A = \tfrac{1}{2}ab\sin(\theta)$$

Circle

$$A = \pi r^2$$
$$C = 2\pi r$$

Sector of Circle

$$A = \tfrac{1}{2}r^2\theta$$
$$s = r\theta$$

Sphere

$$V = \tfrac{4}{3}\pi r^3$$
$$A = 4\pi r^2$$

Cylinder

$$V = \pi r^2 h$$

Cone

$$V = \frac{1}{3}\pi r^2 h$$
$$A = \pi r\sqrt{r^2 + h^2}$$

Cone with arbitrary base

$$V = \frac{1}{3}Ah$$

Trigonometry

Pythagorean Theorem: For a right trianlge with hypotenuse of length c and legs of lengths a and b, $c^2 = a^2 + b^2$.

Angle Measurement

π radians = 180°
$1^\circ = \frac{\pi}{180} rad$
1 rad = $\frac{180}{\pi}$
$s = r\theta$ (θ in radians)

Right Triangle Definitions

$\sin\theta = \frac{opp}{hyp}$
$\cos\theta = \frac{adj}{hyp}$
$\tan\theta = \frac{sin\theta}{cos\theta} = \frac{opp}{adj}$
$\sec\theta = \frac{1}{cos\theta}$
$\csc\theta = \frac{1}{sin\theta}$

Trigonometric Functions

$\sin\theta = \frac{y}{r}$
$\cos\theta = \frac{x}{r}$
$\tan\theta = \frac{y}{x}$
$\sec\theta = \frac{r}{x}$
$\csc\theta = \frac{r}{y}$
$\lim_{\theta\to 0} \frac{sin\theta}{\theta} = 1$
$\lim_{\theta\to 0} \frac{1-cos\theta}{\theta} = 0$

Fundamental Identities

$$sin^2\theta + cos^2\theta = 1$$
$$1 + tan^2\theta = sec^2\theta$$
$$1 + cot^2\theta = csc^2\theta$$
$$\sin(\tfrac{\pi}{2} - \theta) = \cos(\theta)$$
$$\cos(\tfrac{\pi}{2} - \theta) = \sin(\theta)$$
$$\tan(\tfrac{\pi}{2} - \theta) = \cot(\theta)$$
$$\sin(-\theta) = -sin\theta$$
$$\cos(-\theta) = cos\theta$$
$$\tan(-\theta) = -tan\theta$$

$$\sin(\theta + 2\pi) = sin\theta$$
$$\cos(\theta + 2\pi) = cos\theta$$
$$\tan(\theta + \pi) = tan\theta$$

The Law of Sines

$$\frac{sinA}{a} = \frac{sinB}{b} = \frac{sinC}{c}$$

The Law of Cosines

$$a^2 = b^2 + c^2 - 2bccosA$$

Addition and Subtraction Formulas

$$\sin(x + y) = sinxcosy + cosxsiny$$
$$\sin(x - y) = sinxcosy - cosxsiny$$
$$\cos(x + y) = cosxcosy - sinxsiny$$
$$\cos(x - y) = cosxcosy + sinxsiny$$
$$tan(x + y) = \frac{tanx+tany}{1-tanxtany}$$
$$tan(x - y) = \frac{tanx-tany}{1+tanxtany}$$

Double-Angle Formulas

$$sin2x = 2sinxcosx$$
$$cos2x = cos^2x - sin^2x = 2cos^2x - 1 = 1 - 2sin^2x$$
$$tan2x = \frac{2tanx}{1-tan^2x}$$
$$sin^2x = \frac{1-cos2x}{2}$$
$$cos^2x = \frac{1+cos2x}{2}$$

Precalculus Review