Integration

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FTC

- if f is a continuous function on an interval $[a, b]$, and $F(x) = \int_a^x f(t)\,dt$ is defined for $x \in [a, b]$, then $F'(x) = f(x)$
- if f is continuous function on $[a, b]$, and F is any andiderivative of f, meaning $F'(x) = f(x)$, then $\int_a^b f(x)\,dx = F(b) - F(a)$

integration power rule

- $\frac{d}{dx}[\frac{x^{n+1}}{n+1}] = x^n \Rightarrow \int x^n\,dx = \frac{x^{n+1}}{n+1} + C$

u-substitution

- $\int f(g(x))g'(x)\,dx \Rightarrow \int f(u)\,du$

integration by parts

- $(uv)'(x) = u'(x)v(x) + u(x)v'(x) \Rightarrow \int u\,dv = uv - \int v\,du$

trigonometric substitution

- $\sin^2(\theta) + \cos^2(\theta) = 1 \Rightarrow \cos(\theta) = \sqrt{1 - \sin^2(\theta)}$
- $\sqrt{a^2 - x^2} = \sqrt{a^2(1 - (\frac{x}{a})^2)} = a\sqrt{1 - (\frac{x}{a})^2}$
- $a\sin(\theta) = x$ and $a\cos(\theta) = \sqrt{a^2 - x^2}$

integration with partial fraction

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riemann sums

- given a definite integral like so: $\int_a^b f(x)\,dx$ you can approximate it by breaking $[a, b]$ into smaller subintervals with a width of $\Delta x = \frac{b-a}{n}$. you now have subintervals like so: $[x_0, x_1], [x_1, x_2], \ldots, [x_{n-1}, x_n]$ where $x_i = a + i\Delta x$.
- $L_n = \sum_{i=0}^{n-1} f(x_i) \cdot \Delta x$

- $R_n = \sum_{i=1}^{n} f(x_i) \cdot \Delta x$
- $M_n = \sum_{i=1}^{n} f(m_i) \cdot \Delta x$ where $m_i = \frac{x_{i-1}+x_i}{2}$
- $T_n = \sum_{i=1}^{n} \frac{f(x_{i-1}+f(x_i)}{2} \cdot \Delta x$

signed areas
area between graphs
average value
mean value theorem for intgrals
disk, shell, washer
exponential functions
inverse functions
L'Hopital's rule
hyperbolic trig functions
simpsons rule
improper integrals