integration and more

alexander

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trigonometric functions and the inverse
to obtain the inverse of a function, the function must be one-to-one, meaning that each input corresponds to exactly one unique output and no two different inputs share the same output value. however, the trigonometric functions such as sine, cosine, and tangent are not one-to-one over their entire domains because they are periodic and repeat their values infinitely many times. therefore, to define their inverses $\arcsin(x)$, $\arccos(x)$, and $\arctan(x)$ we must restrict the domain of each trigonometric function to an interval where it passes the horizontal line test. this ensures that each inverse function is well-defined and produces a single, unique ouput for every input within its range.

- $y = \sin(x)$, domain: $x \in (-\infty, \infty)$, range: $y \in [-1, 1]$, period: 2π
  $y = \arcsin(x)$, domain: $x \in [-1, 1]$, range: $y = [-\frac{\pi}{2}, \frac{\pi}{2}$
- $y = \cos(x)$, domain: $x \in (-\infty, \infty)$, range: $y \in [-1, 1]$, period: 2π
  $y = \arccos(x)$, domain: $x \in [-1, 1]$, range: $y \in [0, \pi]$
- $y = \tan(x) = \frac{\sin(x)}{\cos(x)}$, domain: $x \neq \frac{\pi}{2}, n \in \mathbb{Z}$, range: $y \in (\infty, \infty)$, period: π
  $y = \arctan(x)$, domain: $x \in (-\infty, \infty)$, range: $\in (-\frac{\pi}{2}, \frac{\pi}{2})$

FTC

- if f is a continuous function on an interval $[a, b]$, and $F(x) = \int_a^x f(t)\,dt$ is defined for $x \in [a, b]$, then $F'(x) = f(x)$
- if f is continuous function on $[a, b]$, and F is any andiderivative of f, meaning $F'(x) = f(x)$, then $\int_a^b f(x)\,dx = F(b) - F(a)$

integration power rule

- $\frac{d}{dx}[\frac{x^{n+1}}{n+1}] = x^n \Rightarrow \int x^n\,dx = \frac{x^{n+1}}{n+1} + C$

u-substitution

- $\int f(g(x))g'(x)\,dx \Rightarrow \int f(u)\,du$
- $\int_a^b f(g(x))g'(x)\,dx = \int_{u(a)}^{u(b)} f(u)\,du$

integration by parts

- $(uv)'(x) = u'(x)v(x) + u(x)v'(x) \Rightarrow \int u\,dv = uv - \int v\,du$

trigonometric substitution

1. $\sqrt{a^2 - b^2x^2} = \sqrt{a^2(1 - \frac{b^2x^2}{a^2})} = \sqrt{a^2(1 - (\frac{bx}{a})^2)} = \sqrt{a^2(1 - (\frac{b}{a}x)^2)} = a\sqrt{1 - (\frac{b}{a}x)^2}$
2. $\sin^2(\theta) = \cos^2(\theta) = 1 \Leftrightarrow \cos(\theta) = \sqrt{1 - (\sin(\theta))^2}$
3. $\frac{b}{a}x = \sin(\theta) \Leftrightarrow x = \frac{a}{b}\sin(\theta)$
4. $a\cos(\theta) = \sqrt{a^2 - b^2x^2}$

when you do a trig substitution, you often let something like $x = \sin(\theta)$. that automatically restrics θ to the range of the inverse sine function, which is $-\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}$. within this range, $\sin(\theta)$ can be positive or negative, but $\cos(\theta)$ is always nonnegative. now, when you use the pythagorean identity $\sin^2(\theta) + \cos^2(\theta) = 1$, and solve for $\cos(\theta)$, you get $\cos(\theta) = \pm\sqrt{1 - \sin^2(\theta)}$. mathematically, both the positive and negative square roots are valid - but you have to choose which sign is correct for the range of θ you are working in. because we restriced θ to $[-\frac{\pi}{2}, \frac{\pi}{2}]$ when we said $x = \sin(\theta)$, we know $\cos(\theta) \geq 0$ there. Therefore, we choose the positive square root: $\cos(\theta) = +\sqrt{1 - \sin^2(\theta)}$. if instead we had chosen a substitution involving $\cos(\theta)$, we would pick a domain where $\sin(\theta)$ has a definite sign and make the corresponding choice.
integration with partial fraction

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riemann sums

- given a definite integral like so: $\int_a^b f(x)\,dx$ you can approximate it by breaking $[a, b]$ into smaller subintervals with a width of $\Delta x = \frac{b-a}{n}$. you now have subintervals like so: $[x_0, x_1], [x_1, x_2], \ldots, [x_{n-1}, x_n]$ where $x_i = a + i\Delta x$.
- $L_n = \sum_{i=0}^{n-1} f(x_i) \cdot \Delta x$
- $R_n = \sum_{i=1}^{n} f(x_i) \cdot \Delta x$
- $M_n = \sum_{i=1}^{n} f(m_i) \cdot \Delta x$ where $m_i = \frac{x_{i-1}+x_i}{2}$
- $T_n = \sum_{i=1}^{n} \frac{f(x_{i-1}+f(x_i)}{2} \cdot \Delta x$

prooving the binomial theorem and special factorizations
diff. implies cont.
simple limits revisited
derivative of ln(x) vs integrating 1/x
differentiating inverse csc

IBP problems, power reduction, improper integration
why trig subs prefer certain substiutions
partial fractions
e exponentials and compounding, logs review (slide rule)
$e^{i\theta} = \cos(\theta) + i\sin(\theta)$, $\sinh(x)$, $\sin^2(\theta) - \cos^2(\theta) = 1$
signed areas
area between graphs
average value
mean value theorem for intgrals
disk, shell, washer
exponential functions
L'Hopital's rule
hyperbolic trig functions
simpsons rule