Implementation of Quantum Galton Boards

1. Overview

A Quantum Galton Board (QGB) represents the complete set of classical Galton-board trajectories within a quantum superposition, producing binomial/normal-like statistics while using only polynomial resources ($\approx O(n^2)$) to represent 2^n classical trajectories. A quantum peg module is used, it is designed by using a small set of gates Hadamard or R_x, CNOT, controlled-SWAP, and ancilla qubits. Through a sequence of swaps and controlled resets, this module replicates the left/right branching behavior of a physical peg, and can be systematically repeated to build the complete Quantum Galton Board. Key features include a modular "quantum peg" design, per-peg bias control via $R_x(\theta)$, fine-grained distribution tuning, reduced circuit depth for improved performance on noisy hardware, and simple measurement outputs that map directly to classical results with minimal post-processing.

2.Background

The Galton board, also referred to as a quincunx or bean machine,is a statistical apparatus, named after English scientist Sir Francis Galton, designed to demonstrate principles of probability and distribution. In this apparatus, balls are released from the top onto an array of pegs arranged in successive rows of increasing width. Upon striking a peg, each ball is deflected either to the left or to the right, typically with an assumed probability of 50 percent for each direction.

3.Motivation

The motivation behind this work is to present a quantum version of the Galton board that uses superposition to sample all possible trajectories efficiently. The model used here is the quantum peg, designed to shorten circuits, operate on current quantum hardware, and allow individual biasing of each peg. It can be used in many types of statistical distributions, with uses in education, simulation, and data modeling.

4. Core Contributions

Model: Quantum Peg

- The fundamental quantum peg module employs three working qubits and a control qubit.
- Control qubit is placed into superposition by applying H or $R_x(\theta)$, and ball is placed on the middle working qubit by applying X gate.
- The controlled-SWAP directs the state between path qubits, and CNOT/RESET operations prepare the control qubit for reuse.
- Each measurement produces a bitstring with a single qubit in state $|1\rangle$ whose position directly indicates the corresponding bin index in the Galton board.

Biased Peg

- Hadamard gate is replaced with $R_x(\theta)$ to bias probabilities.
- The fine-grained design allows each peg to have its own bias angle θ with additional gates ensuring the control qubits remain correctly prepared for subsequent operations.

5. Implementation

- Most quantum devices lack a native Fredkin gate, requiring it to be decomposed into a sequence of CNOT and Toffoli gates, which increases circuit depth and susceptibility to noise.
- The design reuses the control qubit by resetting it to $|0\rangle$ during circuit execution, a feature that requires backend support for mid-circuit resets.

Measurements

- The raw measurement results consist of bitstrings in which exactly one output qubit is in the $|1\rangle$ state per shot, indicating the ball's final position.
- Convert each bitstring into its corresponding bin index, aggregate the counts over all shots, and normalize them to reconstruct the final probability distribution.

Noise Mitigation

- Apply readout calibration to correct measurement errors and use zero-noise extrapolation to estimate the ideal, noise-free results from noisy hardware executions.

- Minimize the number of SWAP operations by optimizing the mapping of logical qubits to physical qubits, ensuring that frequently interacting qubits are placed in close proximity on the hardware.
- Use classical quantum simulators to test larger board configurations, and execute smaller circuit modules on actual quantum hardware to evaluate real-device performance within hardware constraints.

6. Conclusion

This task laid the theoretical foundation for building and analyzing quantum circuits simulating statistical distributions. Quantum walks demonstrate the ability of quantum systems to simulate classical stochastic behavior with potential quantum advantage.