1+4+9+...+n^2 = n(n+1)(2n+1)/6

From ProofWiki

Theorem:

$$\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$$

Proof by Induction

Base Case: $n = 1$

So, when $n = 1$, we have $\sum_{i=1}^{1} i^2 = 1^2 = 1$

Now, we have $\frac{n(n+1)(2n+1)}{6} = \frac{1(1+1)(2 \cdot 1+1)}{6} = \frac{6}{6} = 1$

So, for the base case, we have shown the result.

Induction Hypothesis: $\sum_{i=1}^{k} i^2 = \frac{k(k+1)(2k+1)}{6}$ for some $k > 1$

Inductive Step: Now consider the case of $k + 1$. So we have $\sum_{i=1}^{k+1} i^2$

Now, using the properties of summation, we have $\sum_{i=1}^{k+1} i^2 = \sum_{i=1}^{k} i^2 + (k+1)^2$

We can now apply our induction hypothesis, obtaining

$$\sum_{i=1}^{k+1} i^2 = \frac{k(k+1)(2k+1)}{6} + (k+1)^2$$

$$= \frac{k(k+1)(2k+1) + 6(k+1)^2}{6}$$

$$= \frac{(k+1)[k(2k+1) + 6(k+1)]}{6}$$

$$= \frac{(k+1)(2k^2+7k+6)}{6}$$

$$= \frac{(k+1)(k+2)(2(k+1)+1)}{6}$$

Thus, we have shown the result by induction.

QED

Proof by using telescoping sum

Observe that $3i(i + 1) = i(i + 1)(i + 2) - i(i + 1)(i - 1)$, by taking the sum we'll get a telescoping one on the RHS and the conclusion follows.

Categories: Number Theory | Proofs by Induction | Algebra