

$$1+4+9+\dots+n^2 = n(n+1)(2n+1)/6$$

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Theorem:

$$\sum_{i=1}^n i^2 = \frac{n(n+1)(2n+1)}{6}$$

Proof by Induction

Base Case: $n = 1$

So, when $n = 1$, we have $\sum_{i=1}^1 i^2 = 1^2 = 1$

Now, we have $\frac{n(n+1)(2n+1)}{6} = \frac{1(1+1)(2 \cdot 1 + 1)}{6} = \frac{6}{6} = 1$

So, for the base case, we have shown the result.

Induction Hypothesis: $\sum_{i=1}^k i^2 = \frac{k(k+1)(2k+1)}{6}$ for some $k > 1$

Inductive Step: Now consider the case of $k + 1$. So we have $\sum_{i=1}^{k+1} i^2$

Now, using the properties of summation, we have $\sum_{i=1}^{k+1} i^2 = \sum_{i=1}^k i^2 + (k+1)^2$

We can now apply our induction hypothesis, obtaining

$$\begin{aligned} \sum_{i=1}^{k+1} i^2 &= \frac{k(k+1)(2k+1)}{6} + (k+1)^2 \\ &= \frac{k(k+1)(2k+1) + 6(k+1)^2}{6} \\ &= \frac{(k+1)[k(2k+1) + 6(k+1)]}{6} \end{aligned}$$

$$\begin{aligned}
 &= \frac{(k+1)(2k^2+7k+6)}{6} \\
 &= \frac{(k+1)(k+2)(2(k+1)+1)}{6}
 \end{aligned}$$

Thus, we have shown the result by induction.

QED

Proof by using telescoping sum

Observe that $3i(i+1) = i(i+1)(i+2) - i(i+1)(i-1)$, by taking the sum we'll get a telescoping one on the RHS and the conclusion follows.

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