Date: / / object:

$t(n) = O(g(n))$, $t(n) \le C_1 \cdot g(n)$, $n \ge n_0$

Q1

1) is $2^{n+1} = O(2^n)$? (True) #

$= 2 \cdot \cancel{2^n} \le C \cdot \cancel{2^n}$ $C \ge 2$ [Constant] ✓

2) is $2^{2n} = O(2^n)$? [False] #

$2^n \cdot \cancel{2^n} \le C \cdot \cancel{2^n}$ $C \ge 2^n$ ⊗ → increases with time

→ C → (not constant)

③ is $(0.25)^n = O(n)$? [True] #

$(0.25)^n \le C \cdot n$ ✓

→ Reaches 0 with more steps

Q2

1) $f(n) = 3n^3 + n^2 + n \le 5n^3 \Rightarrow C_1 \cdot g(n)$

$f(n) = O(n^3) \Rightarrow$ ①

$3n^3 + n^2 + n \ge 1 \cdot n^3 \rightarrow C_2 \cdot g(n)$

$f(n) = \Omega(n^3) \Rightarrow$ ②

From ①, ② $f(n) = \Theta(g(n))$

$g(n) = n^3$ #

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2) $g(n) = 2^n$, $F(n) = 2^{n+1}$

$F(n) = c \cdot g(n)$

$F(n) = 2^{n+1} = 2 \cdot 2^n$

$c \leftarrow \quad \hookrightarrow g(n)$

$c = c_1 = c_2 = 2$, $g(n) = 2^n$

$c_1 \, g(n) \leq F(n) \leq c_2 \, g(n)$

$2 \cdot 2^n \leq 2^{n+1} \leq 2 \cdot 2^n$

$F(n) = \Theta(g(n))$, $g(n) = 2^n$ #

3) $g(n) = \ln(n)$, $F(n) = \log(n) + \log(\log(n))$

??

??

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Q3

1) $f(n) = n^3$, $g(n) = n^2$

* $f(n) = O(g(n)) \rightsquigarrow n^{\cancel{3}} \leq c_1 n^{\cancel{2}}$

$n \leq c_1 \rightsquigarrow$ impossible, n (increases) c (constant)

* $f(n) = \Omega(g(n)) \rightsquigarrow n^{\cancel{3}} \geq c_2 n^{\cancel{2}}$

$n \geq c_2 \rightsquigarrow$ True ✓✓

$f(n) = \Omega(g(n))$ #

2) $f(n) = \log(n)$, $g(n) = \log^2(n)$

* $f(n) = O(g(n)) \rightsquigarrow \log(n) \leq c \cdot \log^{\cancel{2}}(n)$

$\frac{1}{\log(n)} \leq c \rightsquigarrow$ n → increase
log(n) → increase
$\frac{1}{\log(n)}$ → decrease
→ smaller than c

$\boxed{\text{True}}$

* $f(n) = \Omega(g(n))$

$\frac{1}{\log(n)} \geq c$ $\boxed{\text{False}}$

$f(n) = O(g(n))$ #

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Q4 (if) $t(n) = \Theta(g(n))$

$$c_1\, g(n) \leq t(n) \leq c_2\, g(n)$$

so, $c_1\, g(n) \leq t(n)$, $t(n) = \Omega(g(n))$
, $t(n)$ is lower bounded by $\Omega(g(n))$

Then $\Omega(g(n))$ is best case scenario

similarly, $t(n) \geq c_2\, g(n)$, $t(n) = O(g(n))$
, $t(n)$ is upper bounded by $\Omega(g(n))$

Then $O(g(n))$ is worst case scenario

Q5

(1) definition of $\boxed{\text{Big } O}$ $t(n) = O(g(n))$

$t(n) < c_1\, g(n)$, $n \geq n_0$ (1)

(2) definition of $\boxed{\text{Big } \Omega}$ $t(n) = \Omega(g(n))$

$t(n) > c_2\, g(n)$, $n \geq n_0$ (2)

from (1), (2)

✓✓ $c_2 . g(n) < t(n) < c_1 . g(n)$

✓✓ $O(g(n)) \cap \omega(g(n)) =$ empty

no common elements