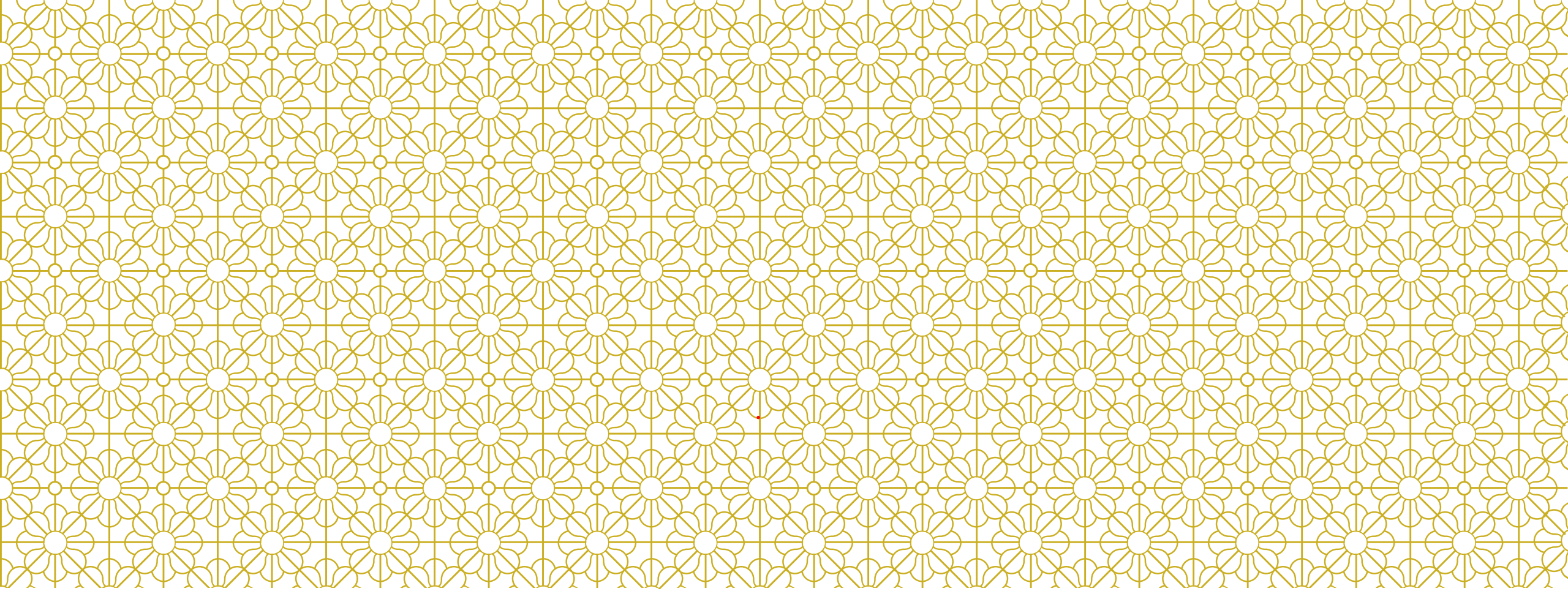


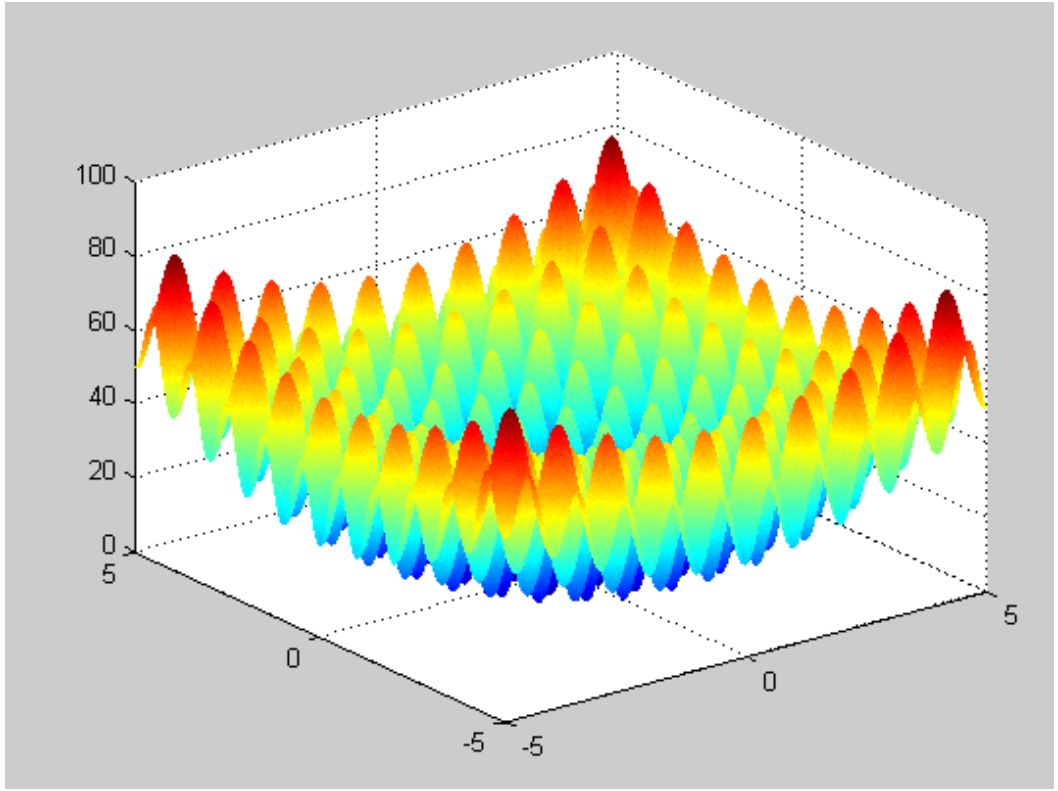
PROBLEM FORMULATION AND SEARCH TREE FUNDAMENTALS OF AI(COMP1037)

Dr Qian Zhang
University of Nottingham,
Ningbo China 2020

MATHEMATIC PROBLEM

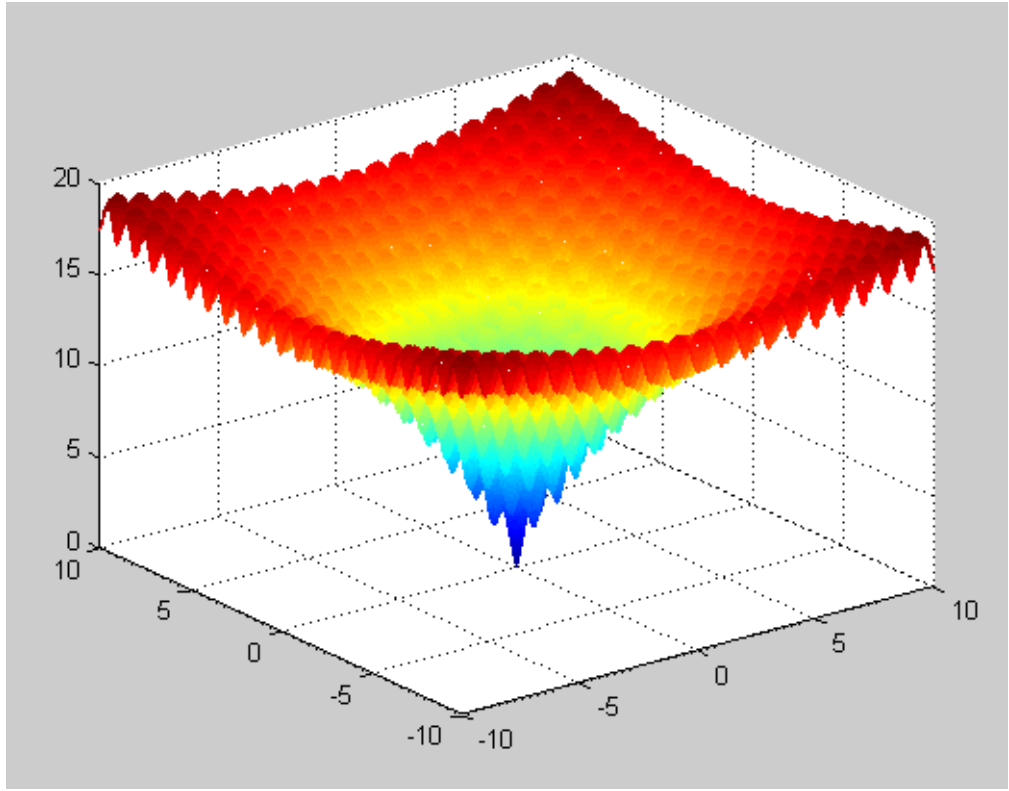
$$Ras(x) = 20 + x_1^2 + x_2^2 - 10(\cos 2\pi x_1 + \cos 2\pi x_2).$$

$$x_i \in [-5.12, 5.12]$$



$$F(\vec{x}) = -20 \cdot \exp\left(-0.2\sqrt{\frac{1}{n} \cdot \sum_{i=1}^{n} x_i^2}\right) - \exp\left(\frac{1}{n}\sum_{i=1}^{n} \cos(2\pi \cdot x_i)\right) + 20 + e$$

$$x_i \in [-30, 30]$$



REAL-WORLD PROBLEM

Tree SEARCH

- Do you drive? Have you thought about how the route is planed in your GPS?

- How would you implement a cross-and-nought computer program?

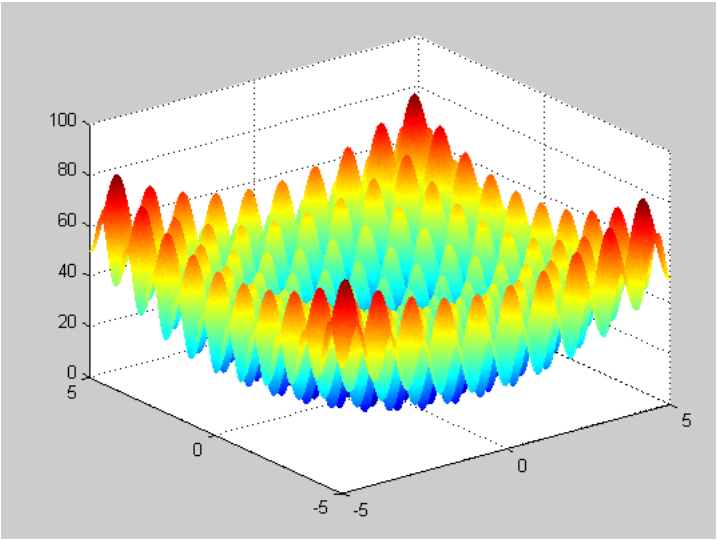
SOLVING PROBLEM BY SEARCHING

- Many problems in real-world exhibit no detectable regular structure to be exploited, they appear "**chaotic**", and do not yield to efficient algorithms
- Often we can't simply write down and solve the equations for a problem
- Exhaustive search of large state spaces appears to be the only viable approach

$$Ras(x) = 20 + x_1^2 + x_2^2 - 10(\cos 2\pi x_1 + \cos 2\pi x_2).$$

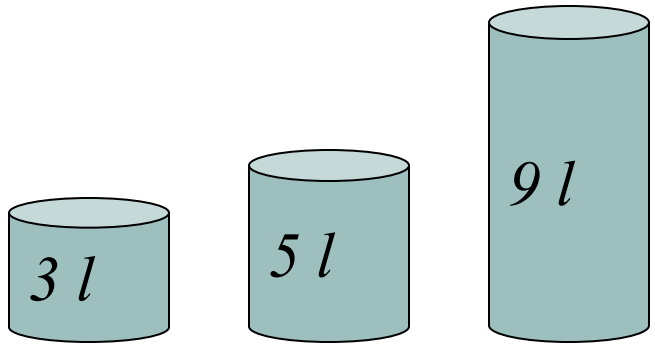
SOLVING PROBLEM BY SEARCHING

- The concept of **search** plays an important role in science and engineering
- In one way, any problem whatsoever can be seen as a search for "the **right answer**"
- Search **space**
  - Set of all possible solutions to a problem
- Search **algorithms**
  - Take a problem as input
  - Return a solution to the problem



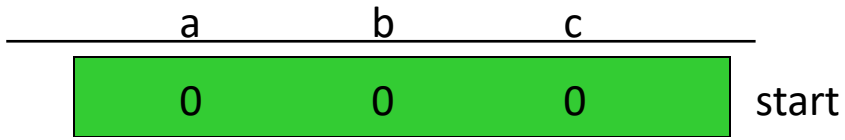
OUTLINES

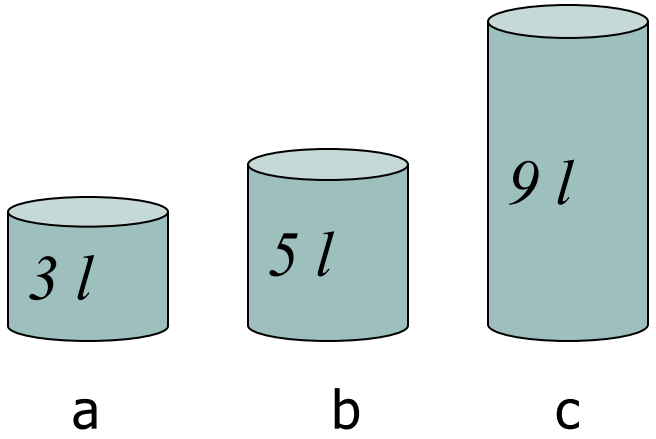
- Topic one: Definitions and examples of problems (2nd Session)
  - **Problem formulation**
  - Problem representation
- Topic two: Problem solving by searching
  - Uninformed(blind) search algorithms (3rd Session)
    - Simplest exhaustive search
    - Breadth first search, depth first search, Uniform cost search
  - Informed search algorithms (4th Session)
    - Use of heuristics that apply domain knowledge
    - A^* algorithm, Minimax algorithm



Problem: Using these three buckets, measure 7 liters of water.

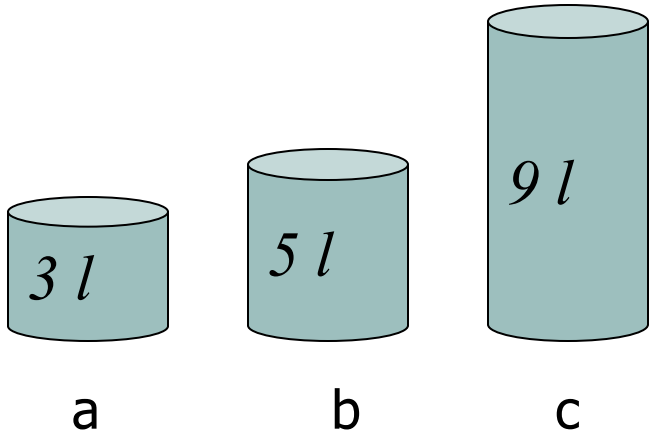
(one possible) Solution:





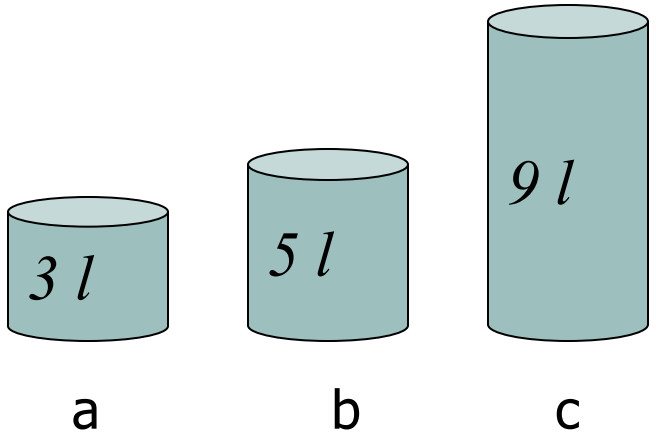
(one possible) Solution:

a	b	c	
0	0	0	start
3	0	0	



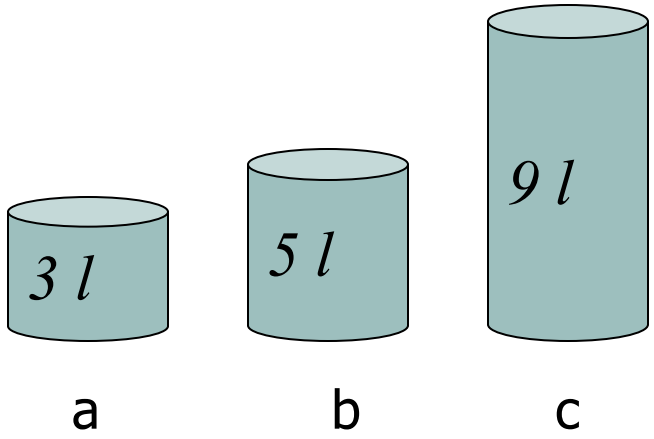
(one possible) Solution:

a	b	c	
0	0	0	start
3	0	0	
0	0	3	



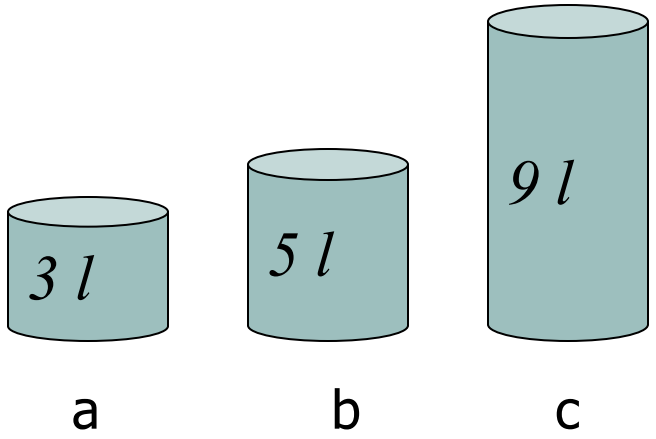
(one possible) Solution:

a	b	c	
0	0	0	start
3	0	0	
0	0	3	
3	0	3	



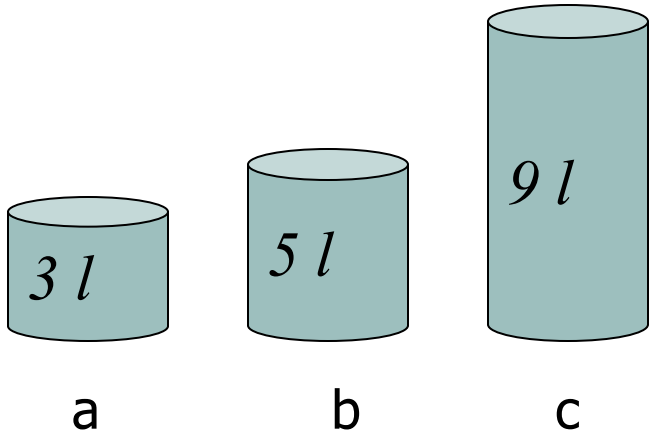
(one possible) Solution:

a	b	c	
0	0	0	start
3	0	0	
0	0	3	
3	0	3	
0	0	6	



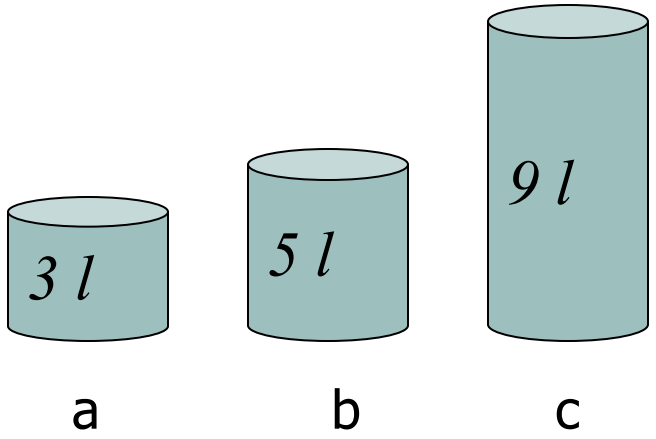
(one possible) Solution:

a	b	c	
0	0	0	start
3	0	0	
0	0	3	
3	0	3	
0	0	6	
3	0	6	



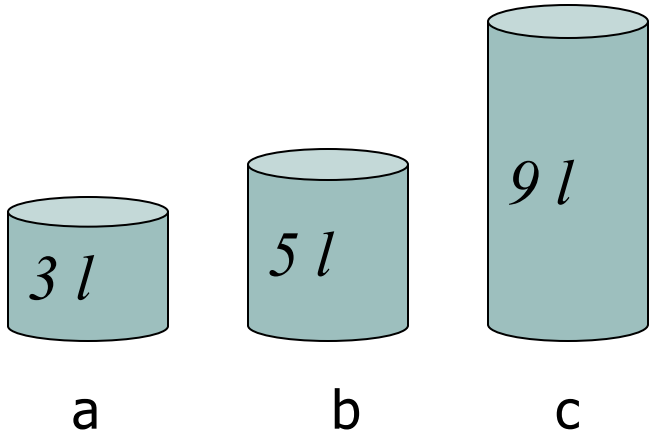
(one possible) Solution:

a	b	c	
0	0	0	start
3	0	0	
0	0	3	
3	0	3	
0	0	6	
3	0	6	
0	3	6	



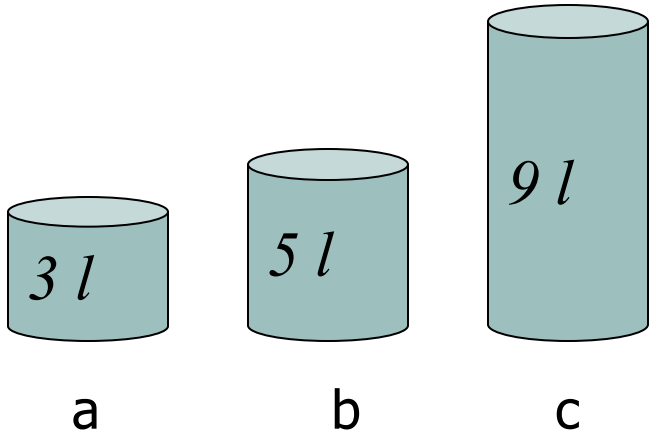
(one possible) Solution:

a	b	c	
0	0	0	start
3	0	0	
0	0	3	
3	0	3	
0	0	6	
3	0	6	
0	3	6	
3	3	6	



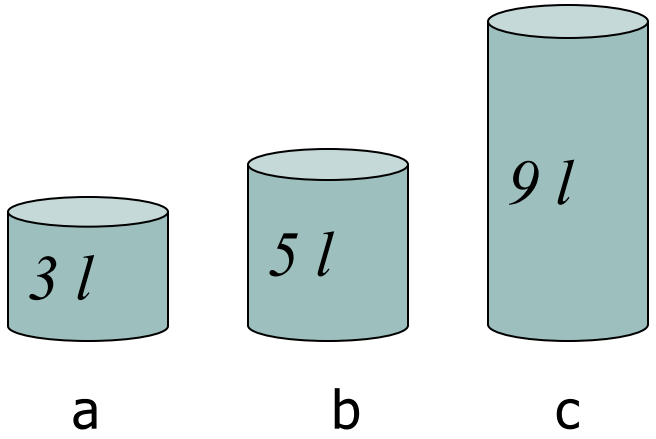
(one possible) Solution:

a	b	c	
0	0	0	start
3	0	0	
0	0	3	
3	0	3	
0	0	6	
3	0	6	
0	3	6	
3	3	6	
1	5	6	

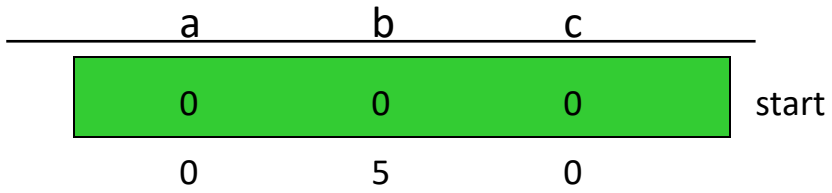


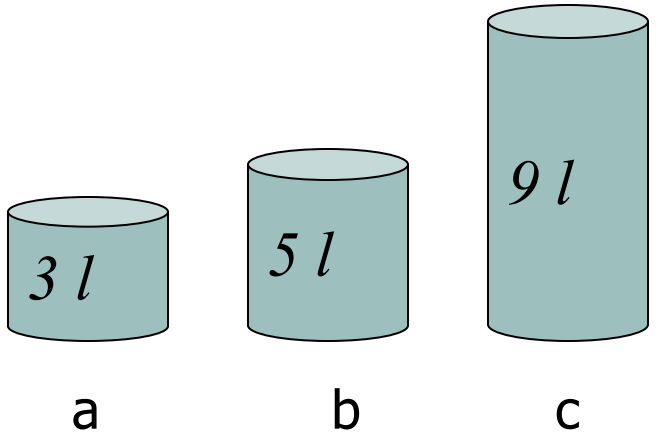
(one possible) Solution:

a	b	c	
0	0	0	start
3	0	0	
0	0	3	
3	0	3	
0	0	6	
3	0	6	
0	3	6	
3	3	6	
1	5	6	
0	5	**7**	**goal**



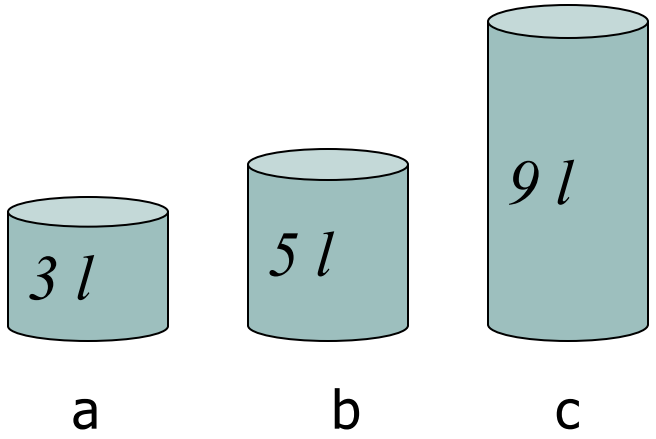
Another Solution:





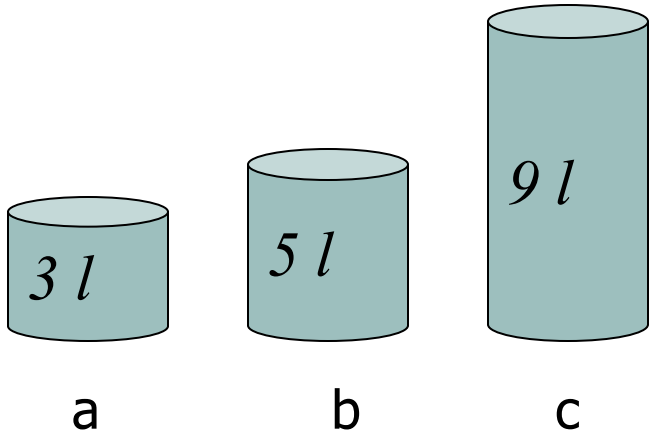
Another Solution:

a	b	c	
0	0	0	start
0	5	0	
3	2	0	



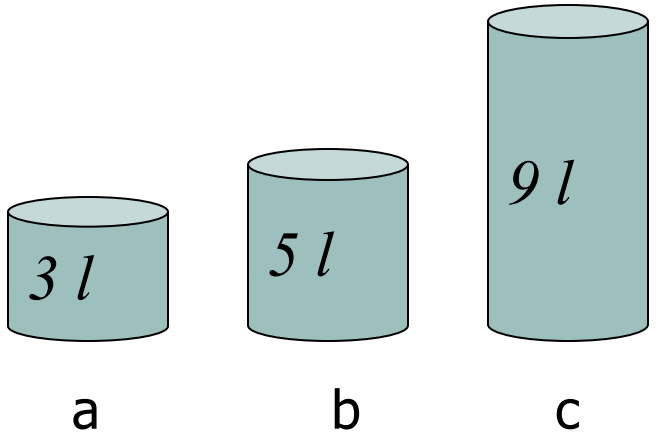
Another Solution:

a	b	c	
0	0	0	start
0	5	0	
3	2	0	
3	0	2	



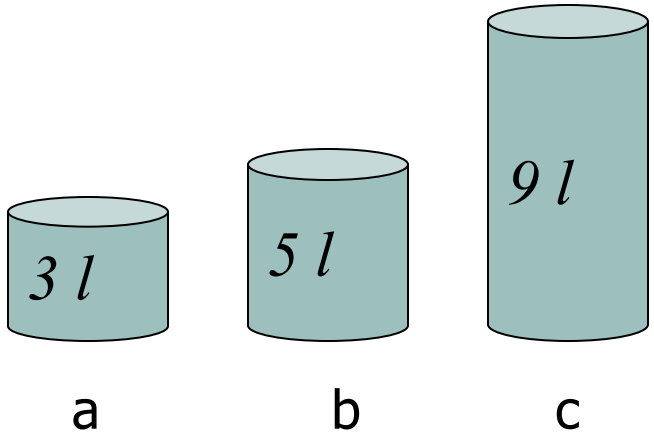
Another Solution:

a	b	c	
0	0	0	start
0	5	0	
3	2	0	
3	0	2	
3	5	2	



Another Solution:

a	b	c	
0	0	0	start
0	5	0	
3	2	0	
3	0	2	
3	5	2	
3	**0**	**7**	**goal**



- **Solution 1:**

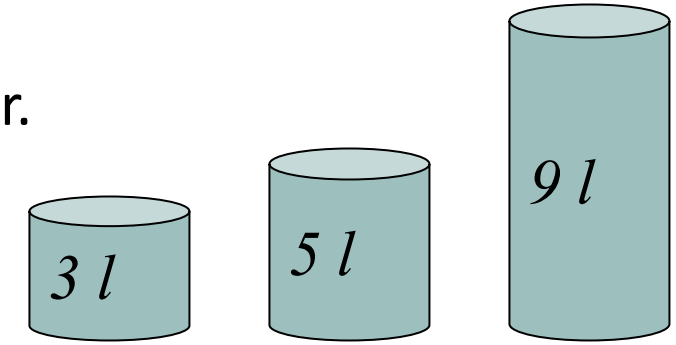
a	b	c	
0	0	0	start
3	0	0	
0	0	3	
3	0	3	
0	0	6	
3	0	6	
0	3	6	
3	3	6	
1	5	6	
0	5	**7**	**goal**

- **Solution 2:**

a	b	c	
0	0	0	start
0	5	0	
3	2	0	
3	0	2	
3	5	2	
3	**0**	**7**	**goal**

PROBLEM FORMULATION

Problem: Using these three buckets, measure 7 liters of water.



What is the environment?

What is the actions?

What does it mean by 'success'?

What is solution?

What to search?

Problem formulation is the process of deciding what actions and states to consider, given a goal.

PROBLEM COMPONENTS

Initial State

- The starting state of the problem, defined in a suitable manner

Actions (Operators)

- An *action* or a set of actions that moves the problem from one state to another
- The set of all possible states reachable from a given state s by applying all legal action is known as the *neighbourhood* and the action(s) can be recognized as the successor function

PROBLEM COMPONENTS

Goal Test

- A test applied to a state which returns true if we have reached a state that solves the problem

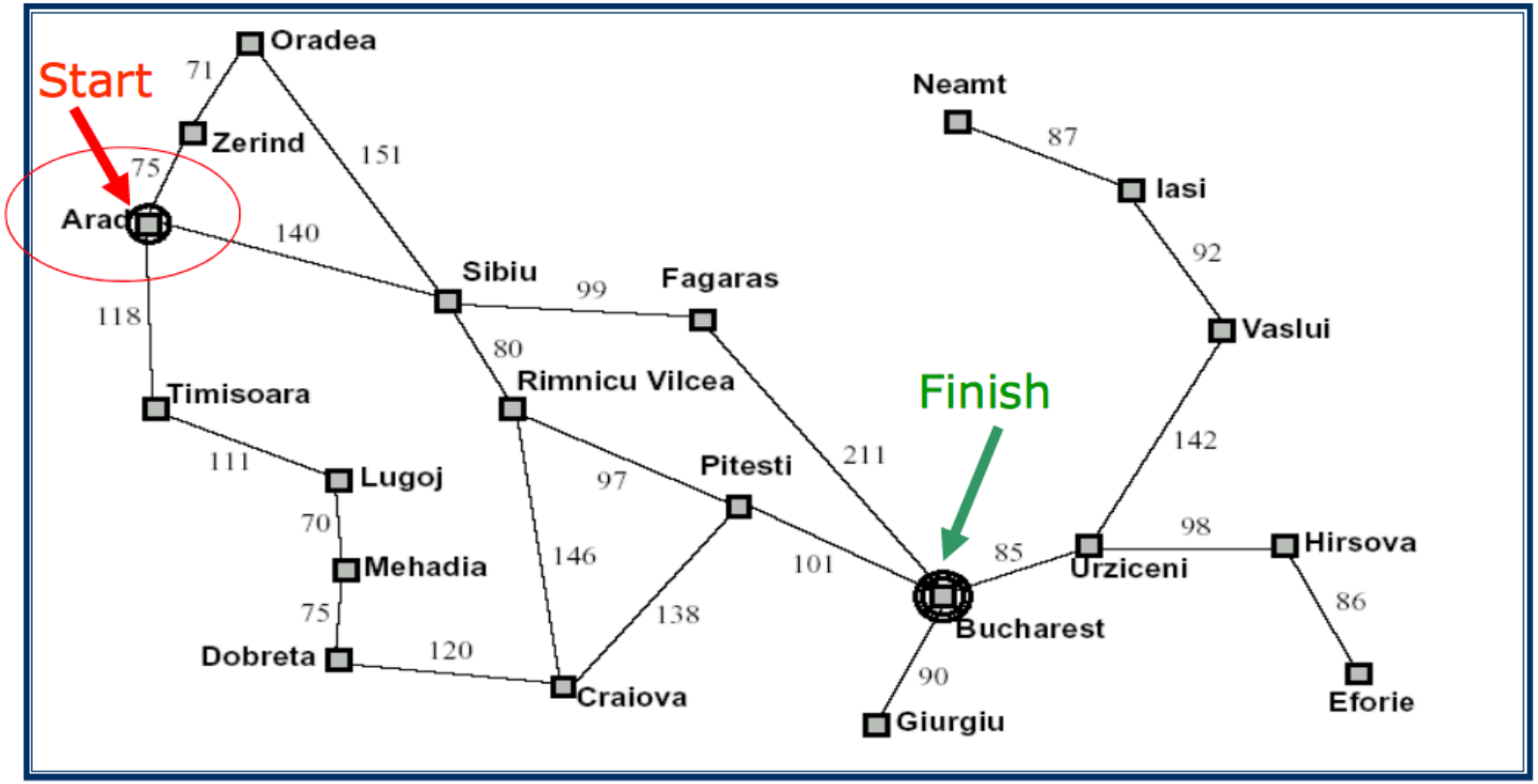
Path Cost

- How much it costs to take a particular sequence of actions
- A *solution* to a problem is an *action sequence* that leads from the initial state to the goal state.

Note: The initial state and the successor function define the ***state space*** which is the set of all states reachable from the initial state (it forms a directed network or graph)

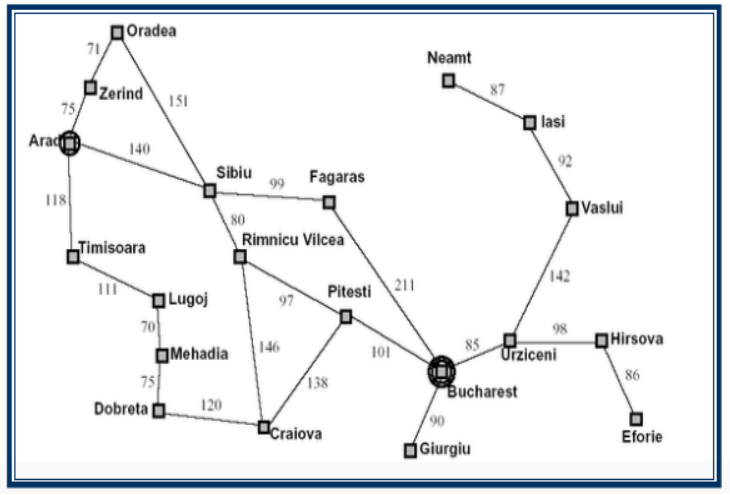
The complexity of a problem depends on the size of the state space.

PROBLEM FORMULATION - ROMANIA



PROBLEM FORMULATION - ROMANIA

- Initial State -> Arad
- Operator
  - driving between cities
  - state space consists of all 20 cities in the graph
- Goal Test
  - is the current state (city) Bucharest?
  - a solution is a path from the initial to the goal state
- Path cost is a function of time/distance/risk/petrol/...



Q: What is the neighbourhood of Arad?

PROBLEM FORMULATION: 8-PUZZLE

5	4	
6	1	8
7	3	2

Initial State

1	4	7
2	5	8
3	6	

Goal State

http://mypuzzle.org/sliding

PROBLEM FORMULATION: 8-PUZZLE

- Initial State
  - specifies the location of each of the eight tiles and the blank in one of the nine squares
- Operators
  - blank tile moves left, right, up or down
- Goal Test
  - the current state matches a certain state
- Path Cost
  - each move of the blank costs 1

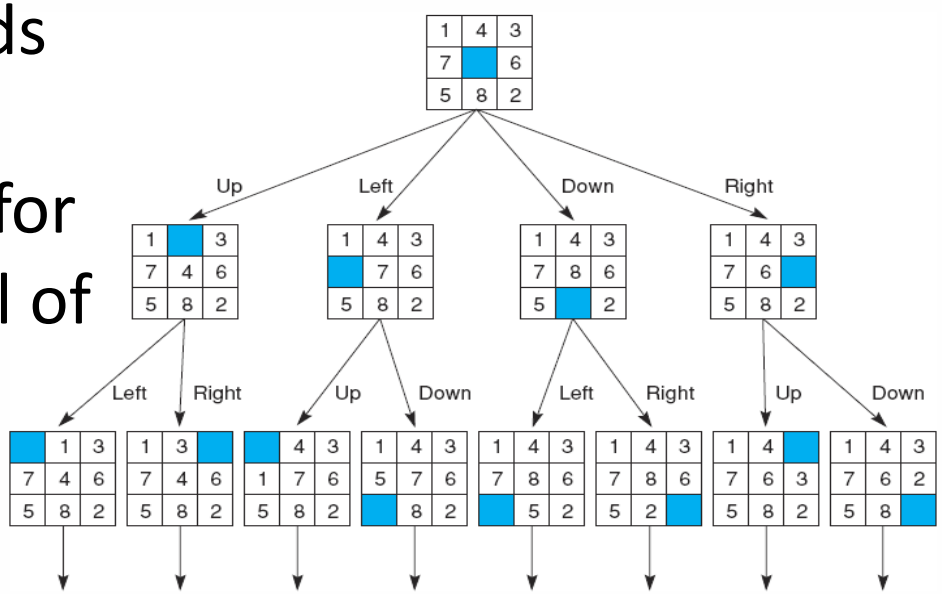
5	4	
6	1	8
7	3	2

1	4	7
2	5	8
3	6	

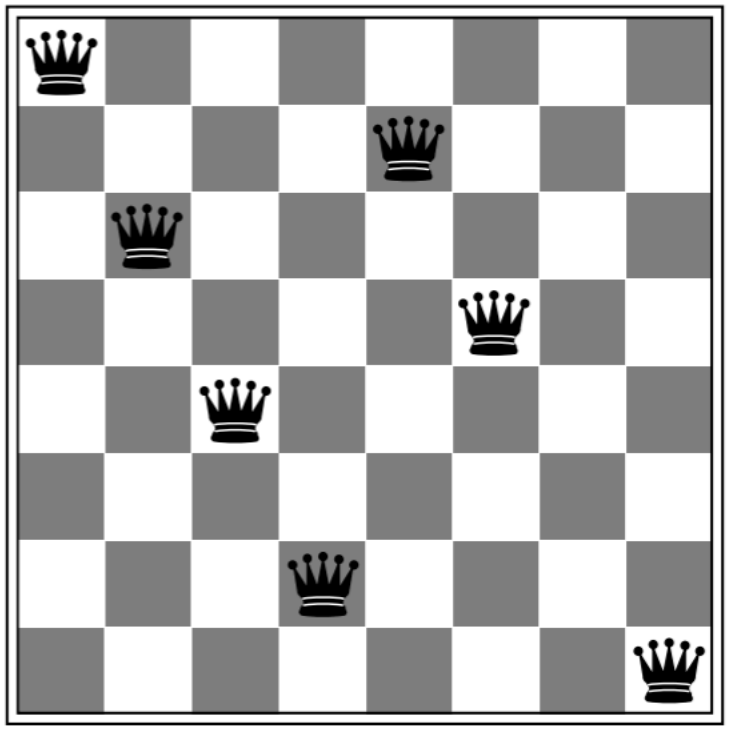
Q: How big is the state space

PROBLEM FORMULATION: 8-PUZZLE

- The number of actions/operators depends on how they are formulated
  - 4 possible moves could be specified for each of the 8 tiles, resulting in a total of **4*8=32** operators.
  - On the other hand, **4** moves for the "blank" square could be specified instead so only 4 operators are needed.
  - => Formulation shift can greatly simplify a problem!



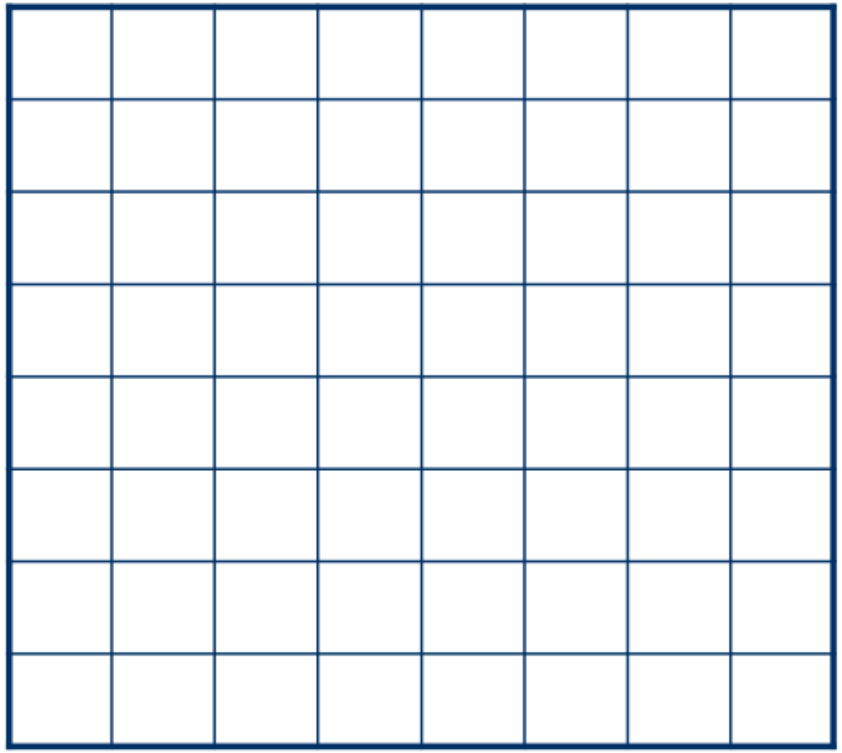
PROBLEM FORMULATION: 8-QUEEN

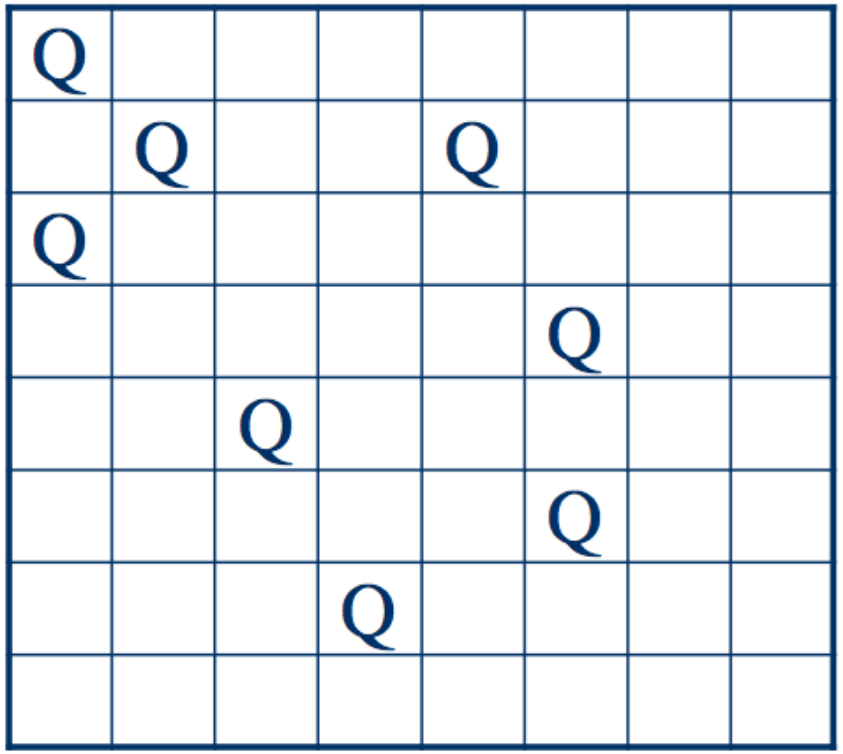


- Initial state
  - An empty 8X8 board
- Operator
  - Add a queen into a cell on the board
- States
  - Any arrangement of 0 to 8 queens on the board
- Goal state
  - A valid arrangement of 8 queens on board (none attacked)

PROBLEM FORMULATION:8-QUEEN

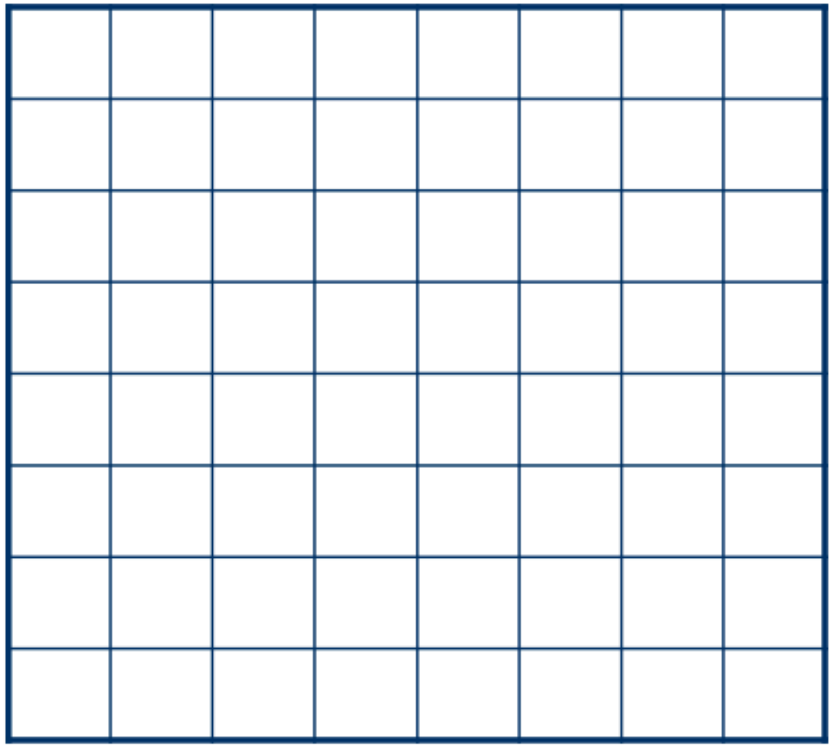
- 1st Formulation: add a queen to any empty square

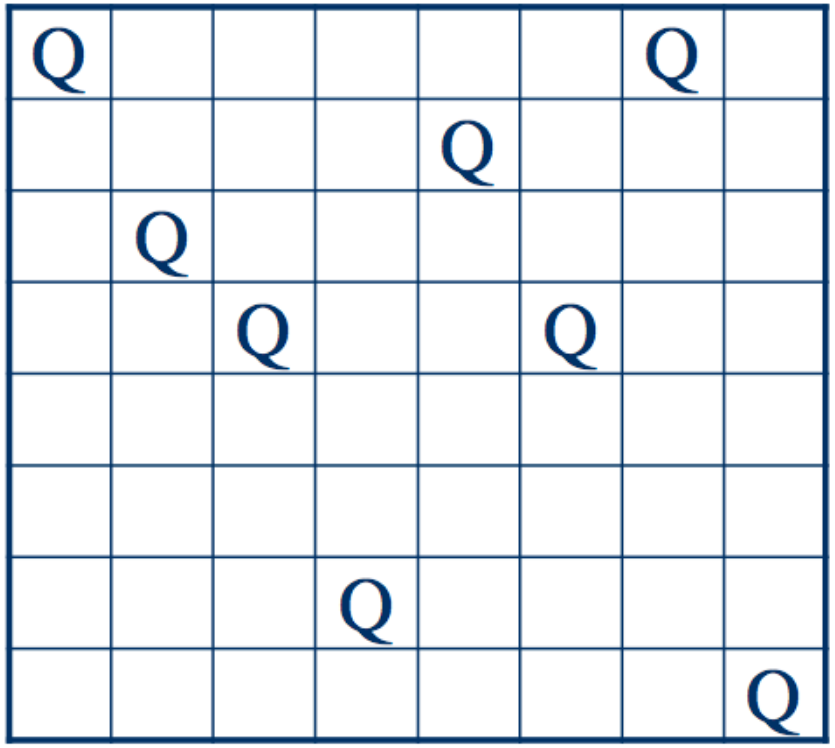




PROBLEM FORMULATION:8-QUEEN

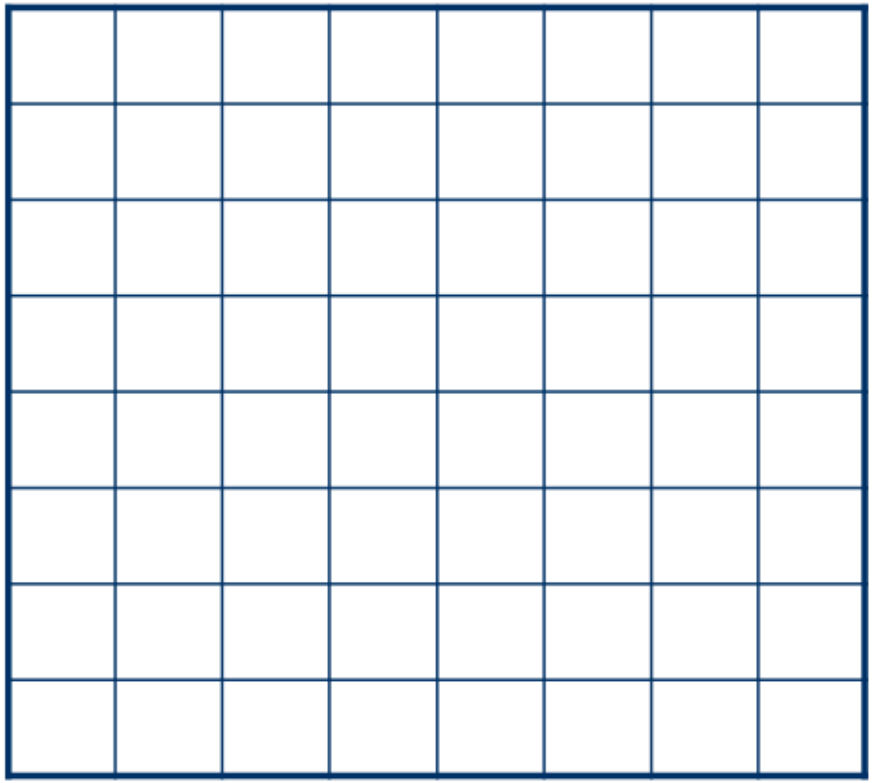
- 2nd Formulation: add a queen to any square in the leftmost empty column

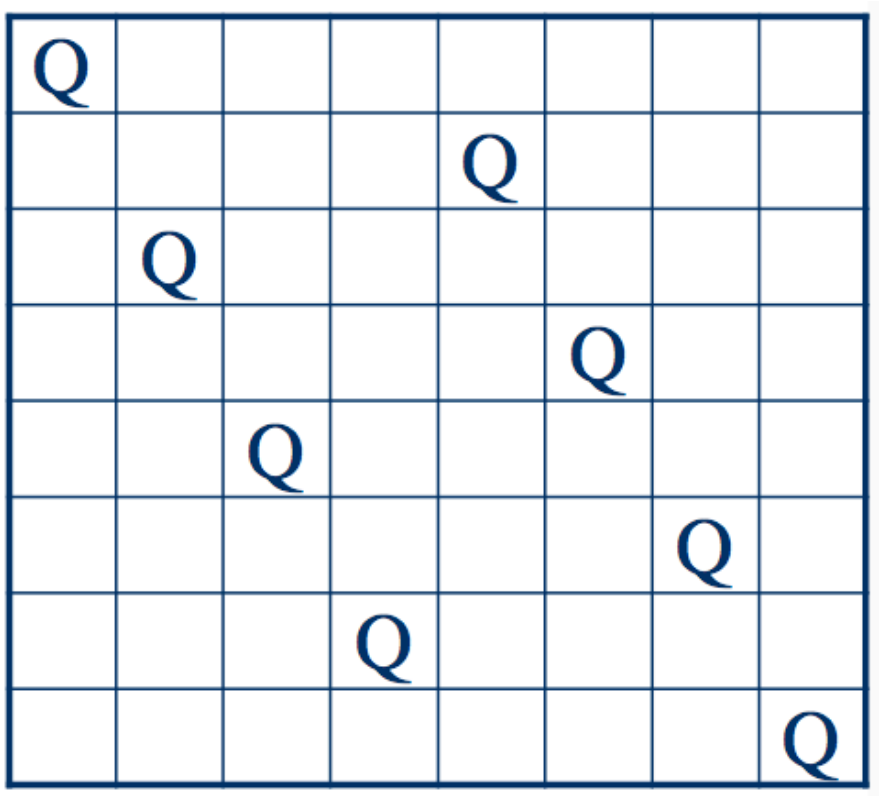




PROBLEM FORMULATION:8-QUEEN

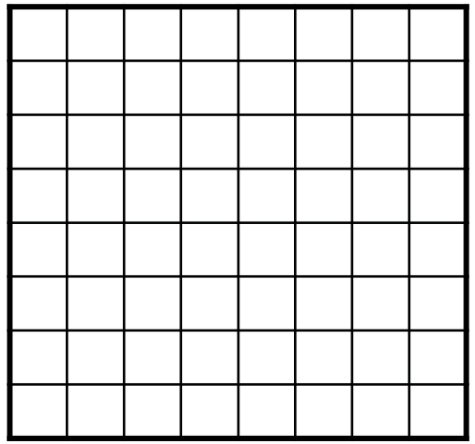
- 3rd Formulation: add a queen to any square in the leftmost empty column such that it is not attacked by any other queen





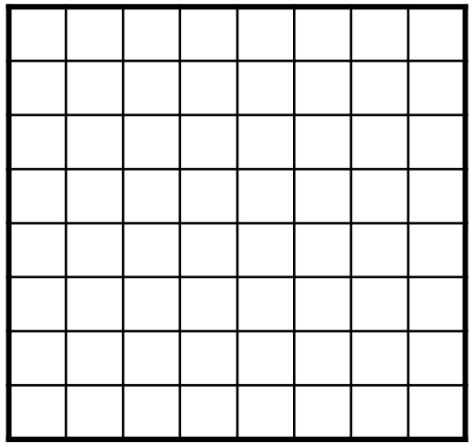
PROBLEM FORMULATION: 8-QUEEN (1ST FORMULATION)

- 1st level: 1 root node (empty board)
- 2nd level: **64** nodes
- 3rd level: **63** nodes for each of the node on the 2nd level
- …



PROBLEM FORMULATION: 8-QUEEN (2ND FORMULATION)

- 1st level: 1 root node (empty board)
- 2nd level: **8** nodes
- 3rd level: **8** nodes for each of the node on the 2nd level
- …
- Smaller tree



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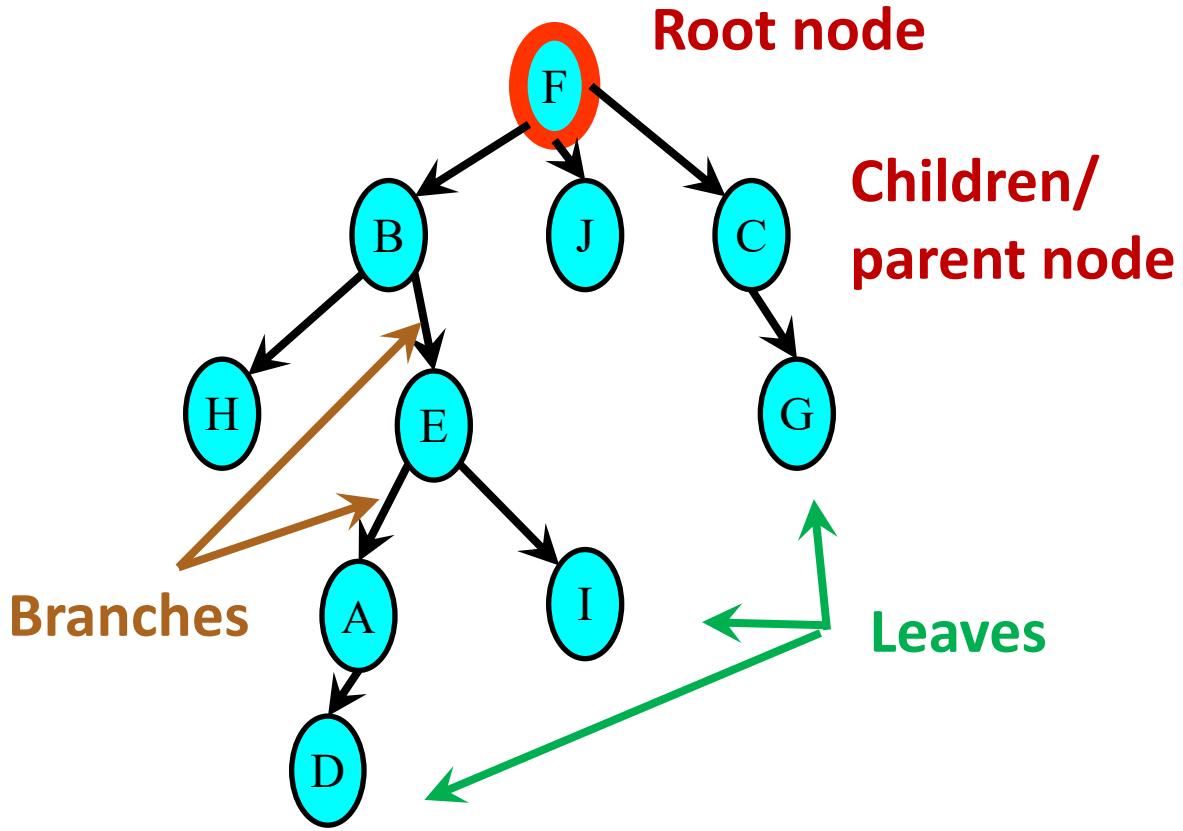
    - Simplest exhaustive search
    - Breadth first search, depth first search, Uniform cost search
  - Informed search algorithms (4th Session)
    - Use of heuristics that apply domain knowledge
    - A* algorithm, Minimax algorithm

SEARCHING THE STATE SPACE

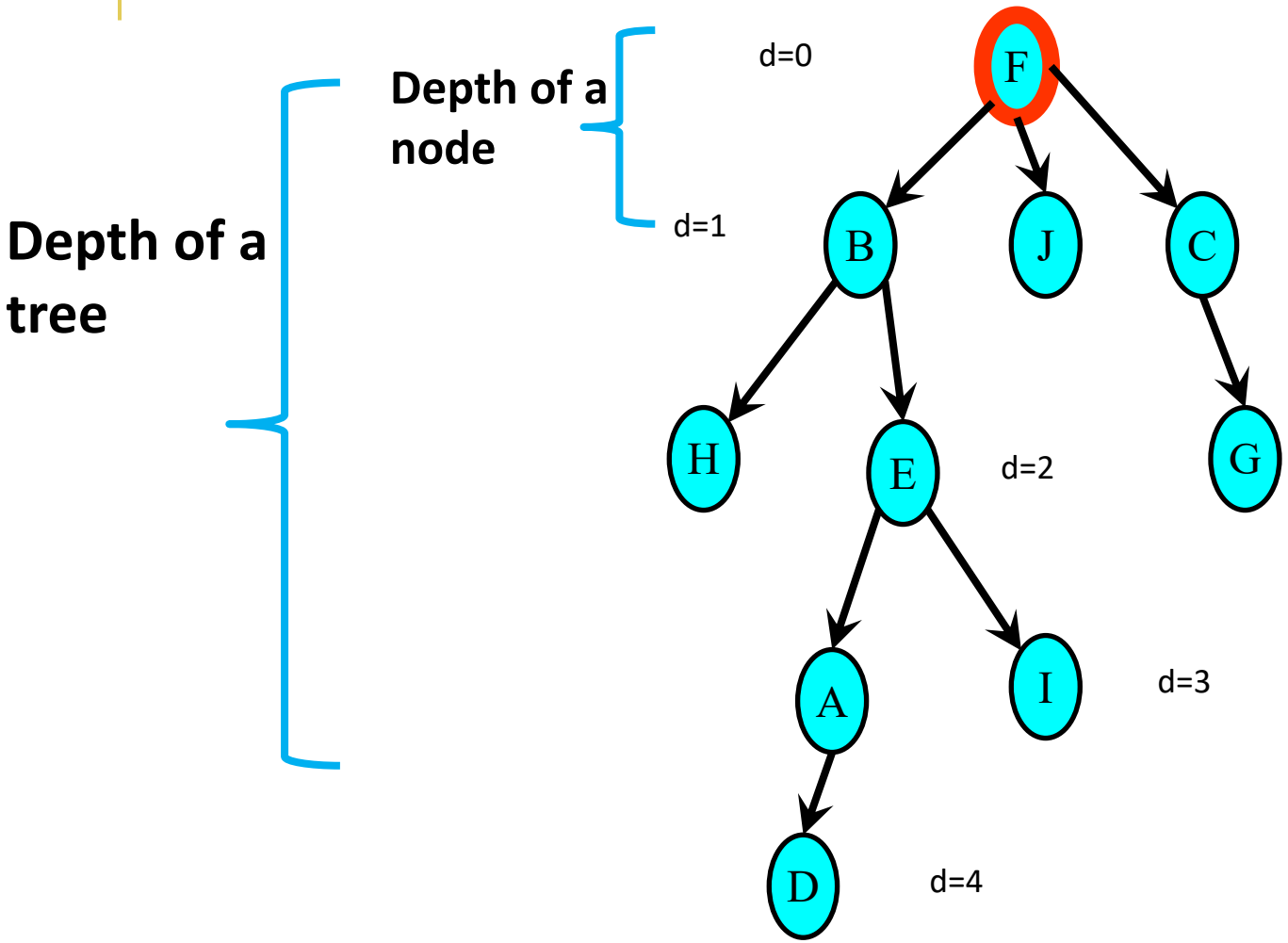
- ***State space*** is the set of all states reachable from the initial state
- Representing the state space provides a powerful tool for measuring the structure and complexity of problems
- Problem solving is a process of searching the state space for a path to a solution
- The choice of which state to expand is determined by the search strategy(Session 3 to 4)

TREES - TERMINOLOGY

- Nodes
  - Root node
  - Children/parent of nodes
  - Leaves
- Branches
- **Average branching factor**
  - average number of branches of the nodes in the tree



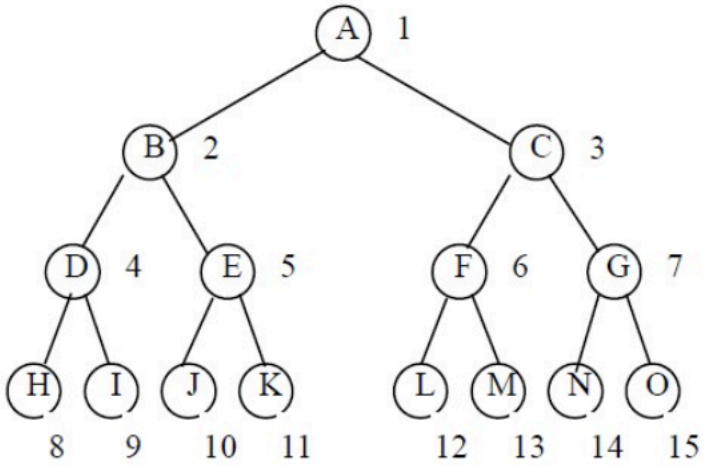
TREES – TERMINOLOGY (2)



- The depth, d, of a node is just the number of edges (branches) away from the root node
- The depth of a tree is the depth of the deepest node
  - in this case, depth=4

TREE SIZE

- Branching factor b=2 (binary tree)

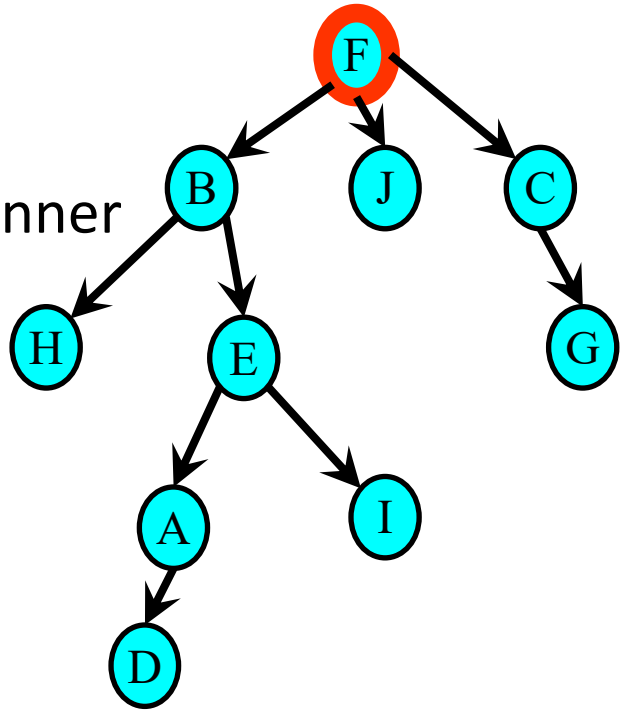


Depth d	# Nodes at d = 2^d	# Nodes in a tree = $2^{d+1}-1$
0	1	1
1	2	3
2	4	7
3	8	15
4	16	31
5	32	63

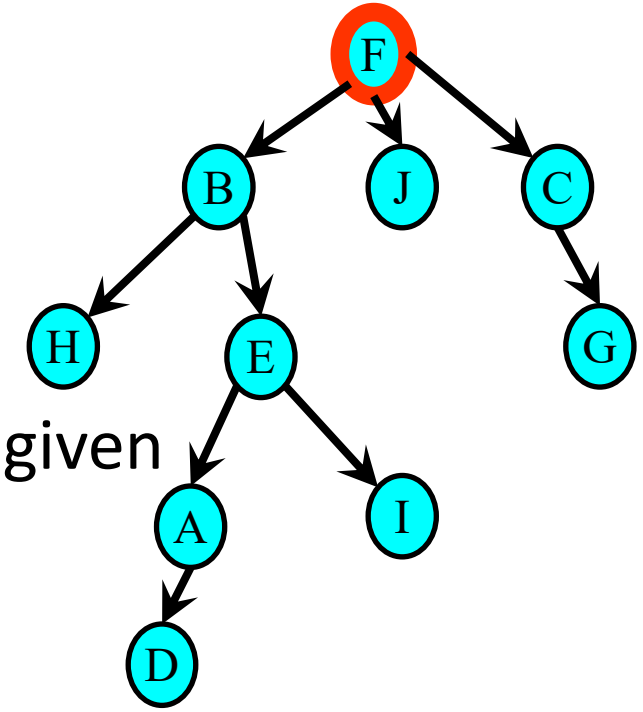
- Why we discuss the size of the tree?

PROBLEM REPRESENTATION

- States – nodes
  - Possible states of problem, defined in some suitable manner
- Initial State - root node
  - The starting state of the problem
- State Space – all nodes in the tree
  - The set of all states reachable from the initial state

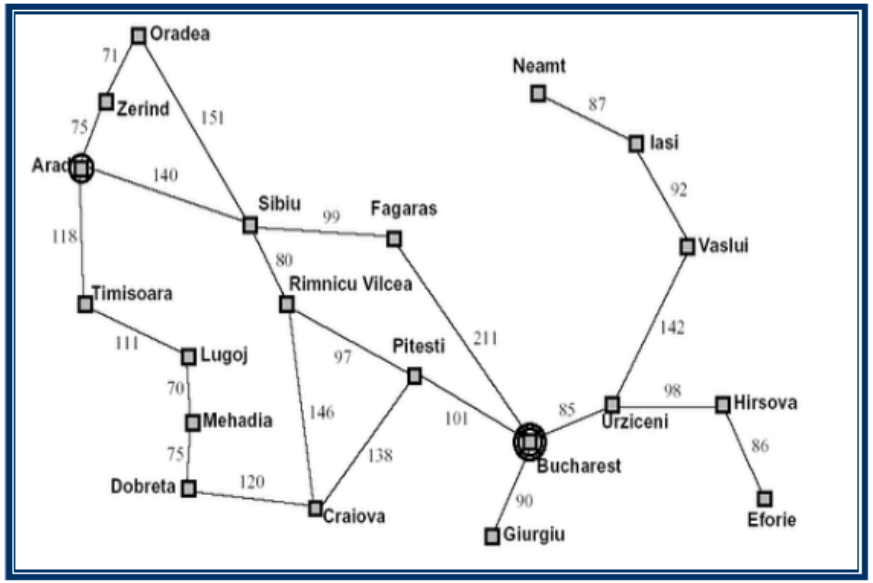


PROBLEM REPRESENTATION

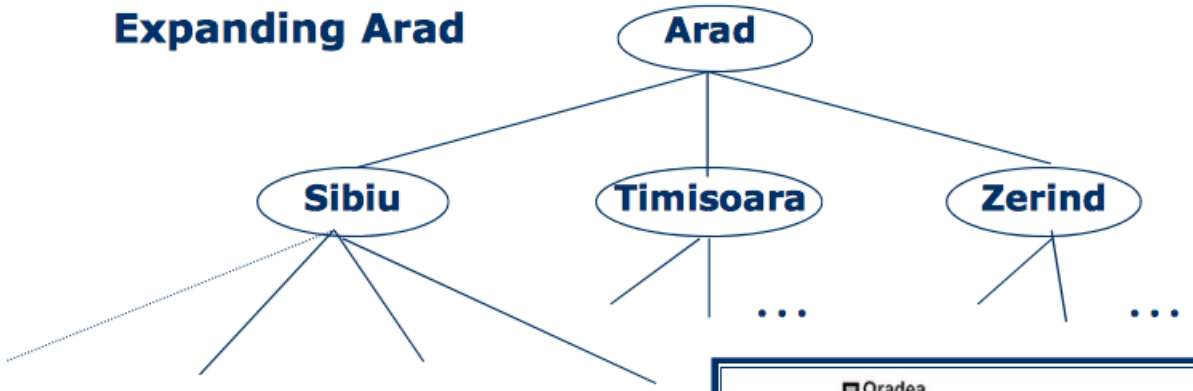


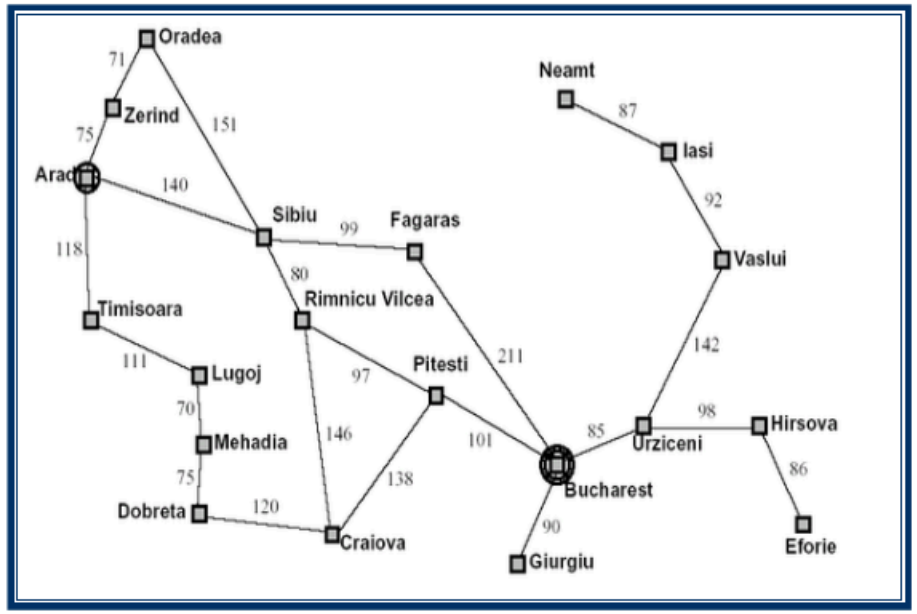
- Neighborhood
  - The set of all possible states reachable from a given state
  - Branching factor: number of neighborhoods
- Operator(s) – branches
  - A set of actions that moves the problem from one state to another

SEARCH TREE – ROMANIA

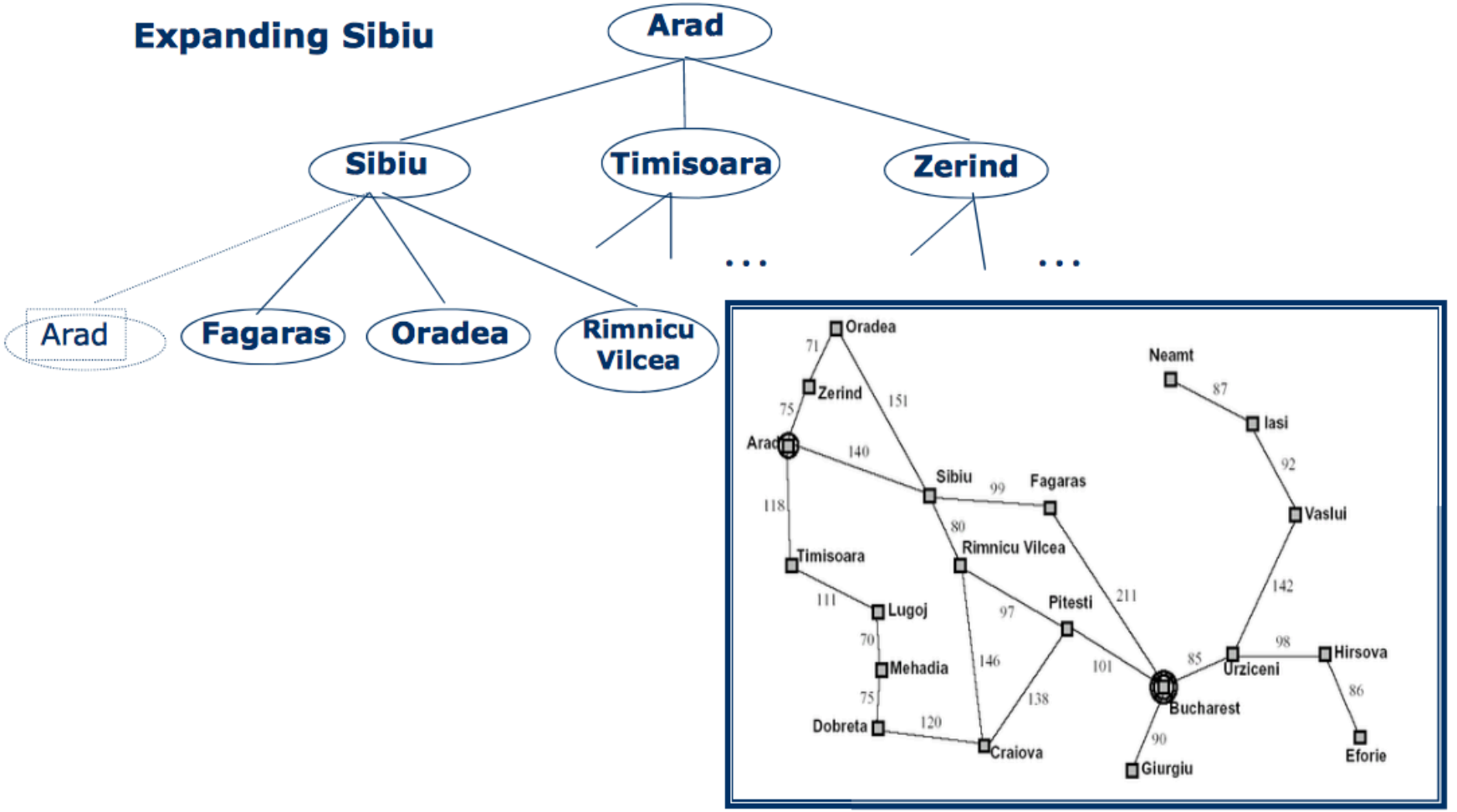


SEARCH TREE – ROMANIA

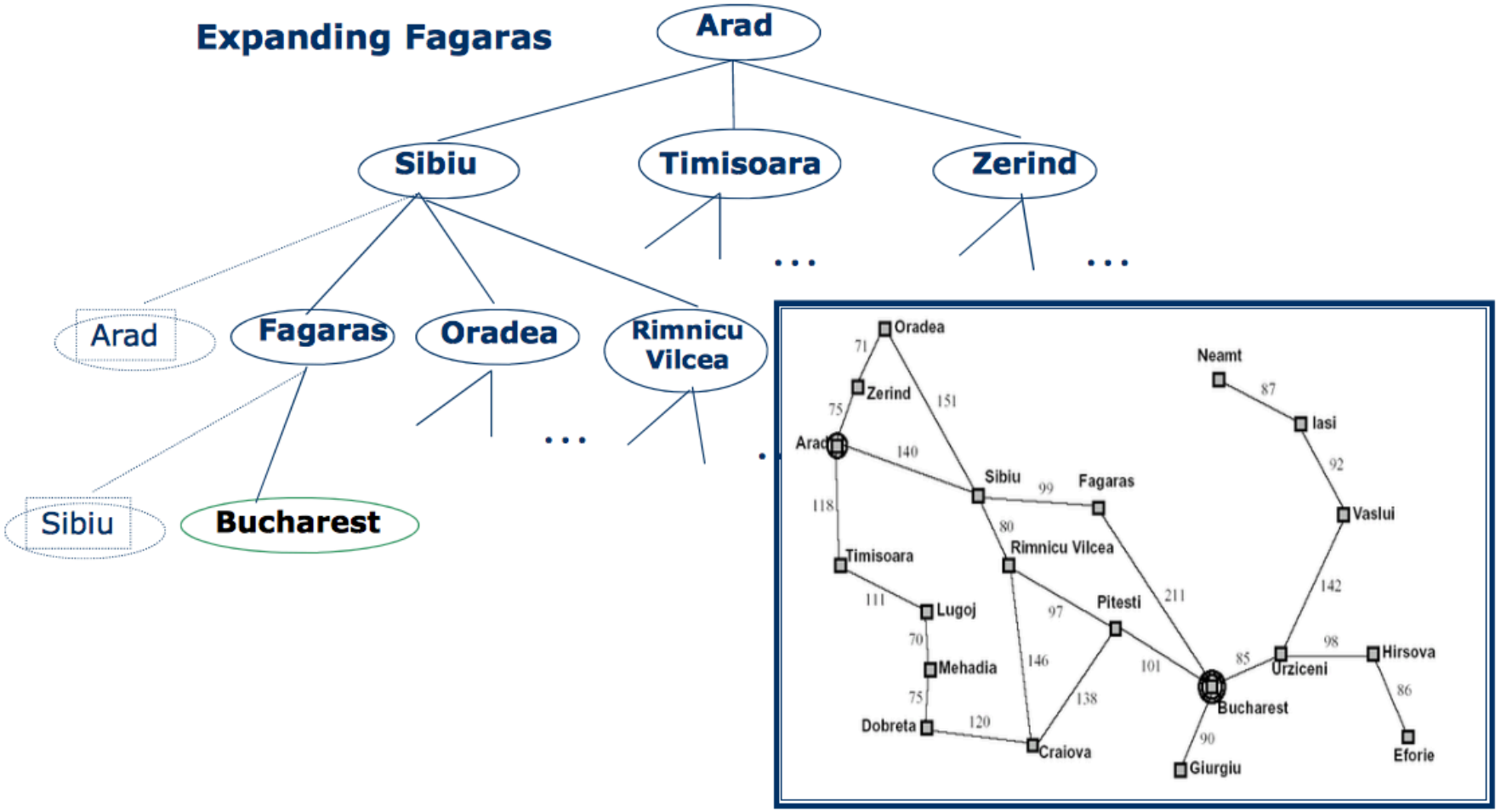




SEARCH TREE – ROMANIA

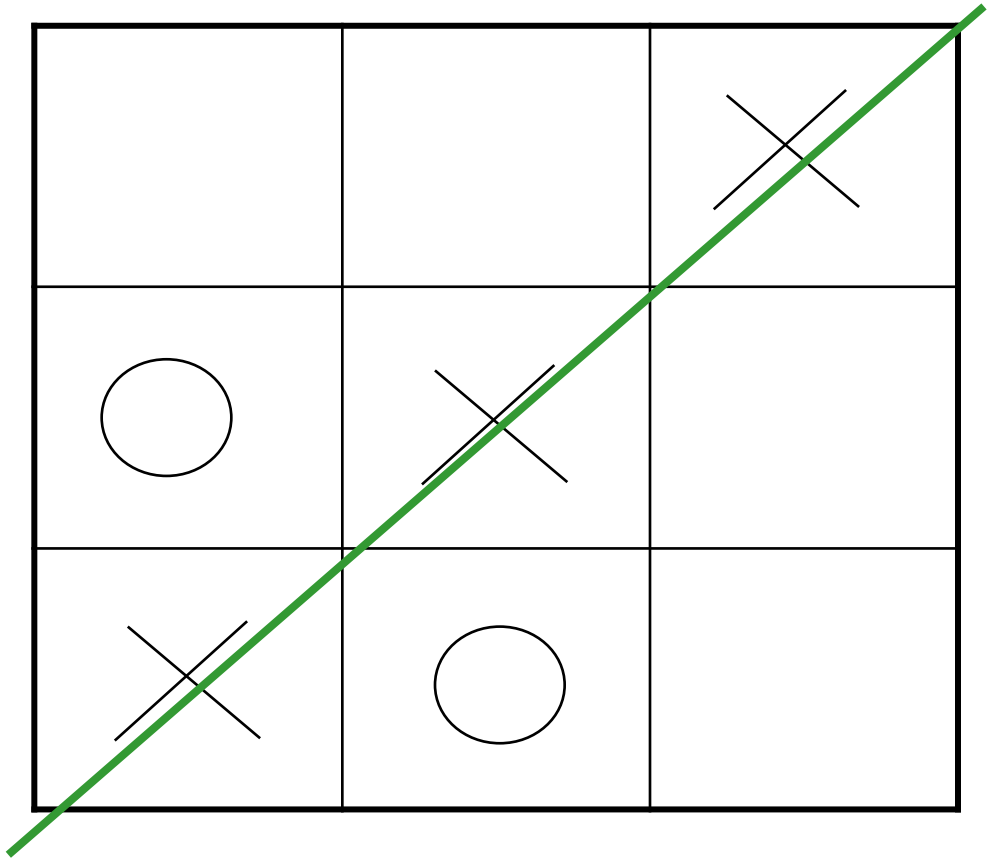


SEARCH TREE – ROMANIA



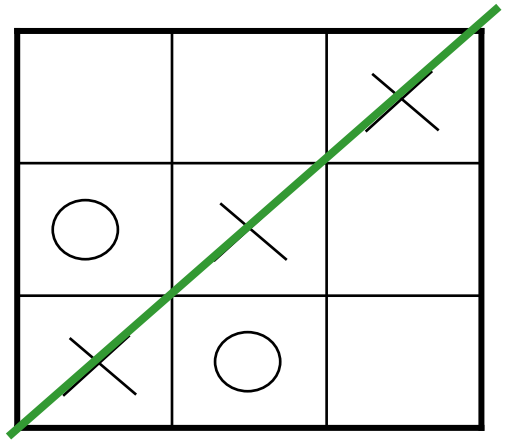
SEARCH TREE – TIC-TAC-TOE

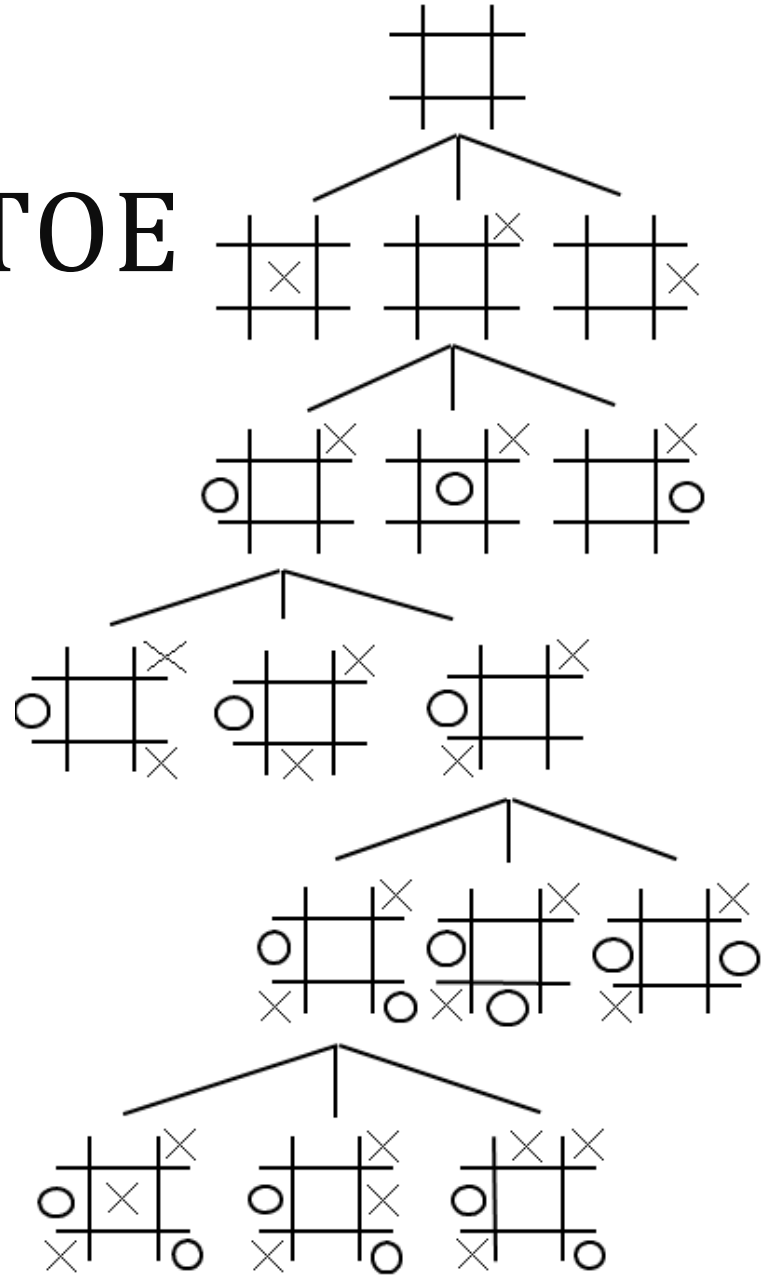
- Noughts and Crosses (Tic-Tac-Toe)



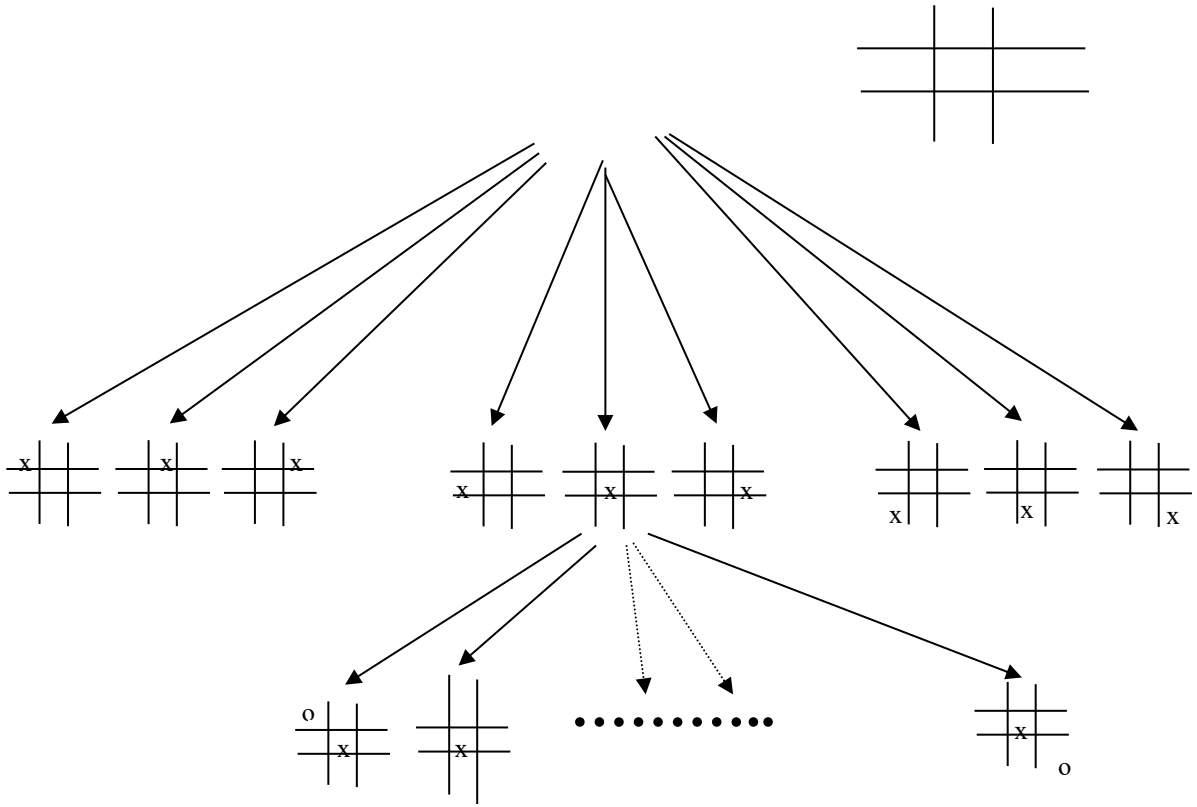
SEARCH TREE – TIC-TAC-TOE

- Nodes: states of problem
- Root node: initial state of the problem
- Branches: moves by operator
- Branching factor: number of neighbours



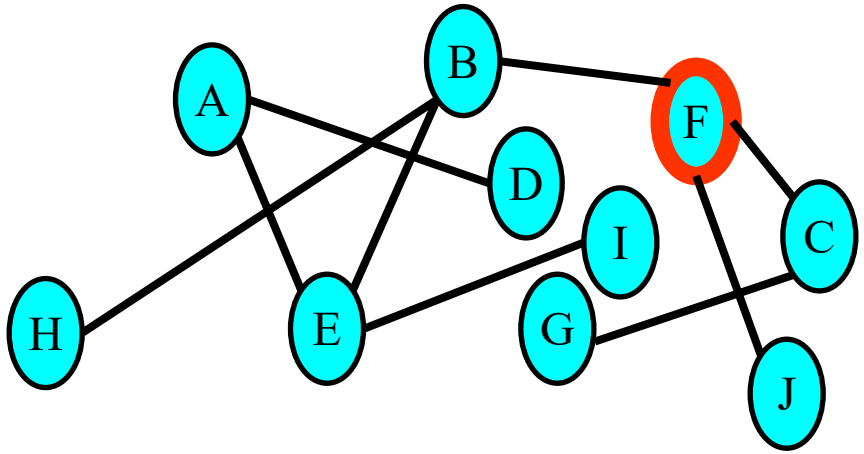


FULL SEARCH TREE- TIC-TAC-TOE



FINDING GOALS IN TREES

- Does the following tree contain a node “I”?
- Easily read from the graph



- so why the big deal about search?

FINDING GOALS IN TREES: REALITY

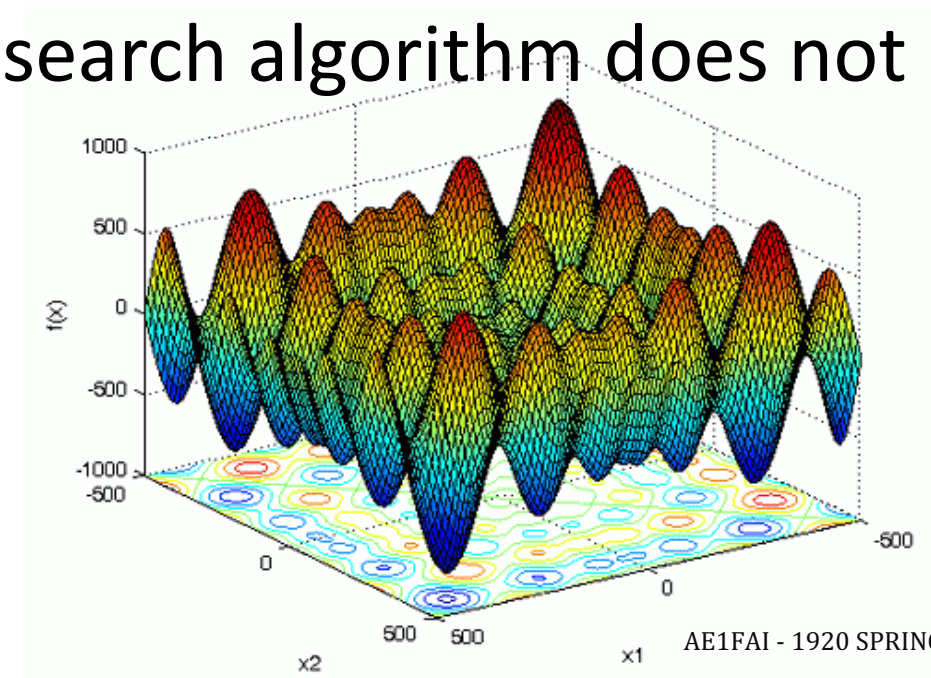
- Does the tree under the following root contain a node “G”?

- All you get to see at first is tne root node
  - and a guarantee that it is a tree
- The rest is up to you to discover during the process of search -> discover/create “on the fly”

WHY IS GOAL SEARCH NOT TRIVIAL?

- Because the tree is not given as a pretty picture “on a piece of paper”

- At the start of the search, the search algorithm does not know
  - the size of the tree
  - the shape of the tree
  - the depth of the goal states



WHY IS GOAL SEARCH NOT TRIVIAL?

- How big can a search tree be?
  - say there is a constant branching factor b
  - and one goal exists at depth d
  - search tree which includes a goal can have b^d different branches in the tree (worst case)
- Examples:
  - b=2,d=10: $b^d = 2^{10} = 1024$
  - b = 10, d = 10: $b^d = 10^{10} = 10{,}000{,}000{,}000$

EVALUATING A SEARCH

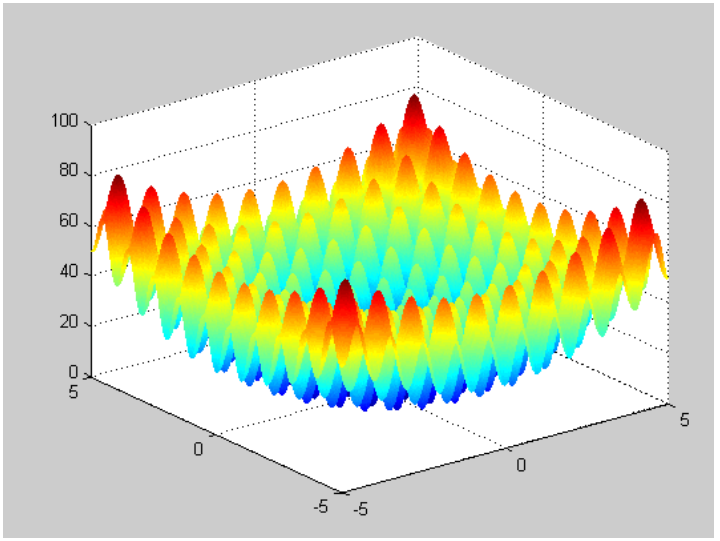
- Completeness
  - Is the strategy **guaranteed to find** a solution?
- Time Complexity
  - **How long** does it take to find a solution?
- Space Complexity
  - How much **memory** does it take to perform the search?
- Optimality
  - Does the strategy find the **optimal solution** where there are several solutions?

PROPERTIES OF SEARCH

- We will say a search method is "*complete*" if it has both the following properties:
  - if a **goal exists** then the search will always **find it**
  - if **no goal exists** then the search will eventually finish and be able to say for **sure** that no goal exists
- We only look at complete search methods

HOW GOOD IS A SOLUTION?

- Does our search method actually find a solution?
- Is it a good solution?
  - Path Cost
  - Search Cost (Time and Memory)
- Does it find the optimal solution?
  - But what is optimal?



$$x_i \in [-5.12, 5.12]$$

SUMMARY

- Problem formulation
- Representation
  - Problem
  - State space
  - Search tree
- Properties of search
- Evaluation criteria for search

FURTHER READING

- AIMA Chapter 3.1-3.2
- Self study slides: fundamental issues in AI
- Next week: AIMA Chapter 3.3-3.4