QF603 Quantitative Analysis of Financial Market
Homework1

Zhou Shen

08/31/2021

1 Q1

$$P(A_D|B_D) = \frac{P(A_D B_D)}{P(B_D)} = \frac{10\%}{15\%} \approx 66.67\%$$

The probability that Bond A defaults given that Bond B defaults is 67%.

2 Q2

X1 can be arranged as:

$$X1 = [-0.3, -0.1, 0.1, 0.2, 0.4, 0.5, 2.0].$$

So the median of X1 is the 4th number 0.2.

3 Q3

The mean of X1 is calculated by

$$mean(X1) = \frac{(-0.3 + (-0.1) + 0.1 + 0.2 + 0.4 + 0.5 + 2.0)}{7} = 0.4.$$

4 Q4

The variance of X1 is calculated by

$$Var(X1) = \frac{\sum_{i:i=1}^{7} (X1_i - mean(X1))^2}{7} \approx 0.49.$$

5 Q5

5.1 B1

The probability of an underperformer to beat market 3 years in a row is

$$P(3B|Under) = P(B|under)^3 = 0.1^3 = 0.10\%.$$

The probability of an in-line performer to beat market 3 years in a row is

$$P(3B|Inline) = P(B|Inline)^3 = 0.5^3 = 12.50\%.$$

The probability of an outperformer to beat market 3 years in a row is

$$P(3B|Out) = P(B|Out)^3 = 0.55^3 \approx 16.64\%.$$

5.2 B2

The unconditional probability of observing the manager beat the market 3 years in a row given prior beliefs is

$$\begin{aligned} P(3B) &= P(3B|Under)P(Under) + P(3B|Inline)P(Inline) + P(3B|Out)P(Out) \\ &\approx 12.09\%. \end{aligned}$$

5.3 B3

The probability of manager to be an underperformer is

$$\begin{aligned} P(Under|3B) &= \frac{P(3B|Under)P(Under)}{P(3B)} \\ &\approx 0.041\%. \end{aligned}$$

The probability of manager to be an in-line performer is

$$\begin{aligned} P(Inline|3B) &= \frac{P(3B|Inline)P(Inline)}{P(3B)} \\ &\approx 93.08\%. \end{aligned}$$

The probability of manager to be an outperformer is

$$\begin{aligned} P(Out|3B) &= \frac{P(3B|Out)P(Out)}{P(3B)} \\ &\approx 6.88\%. \end{aligned}$$

5.4 B4

First, we calculate $P(4B)$:

$$\begin{aligned} P(4B) &= P(4B|Under)P(Under) + P(4B|Inline)P(Inline) + P(4B|Out)P(Out) \\ &\approx 6.08\%. \end{aligned}$$

The probability of manager to be an underperformer is

$$\begin{aligned} P(Under|4B) &= \frac{P(4B|Under)P(Under)}{P(4B)} \\ &\approx 0.0082\%. \end{aligned}$$

The probability of manager to be an in-line performer is

$$\begin{aligned} P(Inline|4B) &= \frac{P(4B|Inline)P(Inline)}{P(4B)} \\ &\approx 92.47\%. \end{aligned}$$

The probability of manager to be an outperformer is

$$\begin{aligned} P(Out|4B) &= \frac{P(4B|Out)P(Out)}{P(4B)} \\ &\approx 7.52\%. \end{aligned}$$