# 9.4.21

EE24BTECH11050 - Pothuri Rahul

**Question:**
In a bank,principal increases continuously at the rate of 5% per year. An amount of rupees 1000 is deposited with this bank. How much will it worth after 10 years $\left(e^{0.5} = 1.648\right)$

**Solution:**

| variable | description |
|---|---|
| $P$ | Principle at any time t |
| $t$ | time in years |
| $C$ | primary arbitrary constant |
| $C_1$ | secondary arbitrary constant |
| $P_0$ | initial principle amount |

TABLE 0: Variables Used

**Solution:** Let P be the principle at any time t. According to the given problem, Rate of change in principle can be given as

$$\frac{dP}{dt} = \left(\frac{5}{100}\right) \times P \tag{0.1}$$

$$\frac{dP}{dt} = \left(\frac{P}{20}\right) \tag{0.2}$$

Seperating the variables in the equation (0.2), We get

$$\frac{dP}{P} = \frac{dt}{20} \tag{0.3}$$

On integrating both sides

$$\int \frac{dP}{P} = \int \frac{dt}{20} \tag{0.4}$$

$$logP = \frac{t}{20} + C \tag{0.5}$$

$$P = e^{\frac{t}{20}+C} \tag{0.6}$$

$$P = e^{\frac{t}{20}}.e^{C} \tag{0.7}$$

$$P = e^{\frac{t}{20}}.C_1 \tag{0.8}$$

Given, at time t=0,$P_0$=1000 then, from (0.8)

$$1000 = C_1 \tag{0.9}$$

Principle can be given as

$$P = 1000 \times e^{\frac{t}{20}} \tag{0.10}$$

At time t=10,Principle can be given as

$$P = 1000 \times e^{\frac{10}{20}} \tag{0.11}$$

$$P = 1000 \times e^{0.5} \tag{0.12}$$

$$P = 1000 \times 1.648 \tag{0.13}$$

$$P = 1648 \tag{0.14}$$

**Logic used for programming:-**

**Method of finite differences:** This method is used to find the approximate solution of the given differential equation by using the values of the function at discrete points.
From the defination of derivative of a function

$$\frac{dy}{dx} \approx \frac{y(x+h) - y(x)}{h} \tag{0.15}$$

by rearranging the terms, we get the function

$$y(x+h) = y(x) + h \times \frac{dy}{dx} \tag{0.16}$$

$$P(t+h) = P(t) + h \times \frac{P}{20} \tag{0.17}$$

Let $(t_0, P_0)$ be points on the curve,

$$t_1 = t_0 + h \tag{0.18}$$

$$P_1 = P_0 + h \times \frac{P}{20} \tag{0.19}$$

On generalising the above equations,

$$t_{n+1} = t_n + h \tag{0.20}$$

$$P_{n+1} = P_n + h \times \frac{P}{20} \tag{0.21}$$

Where h is a very small division (example 0.1),We need iterate this algorithm by taking $P_0 = 1000$ and $t = 0$, till $t_n = 10$. Then we get the principle amount after 10 years. If we plot all the points $(t, P)$, we get the function P varying with t, i.e P vs T graph.

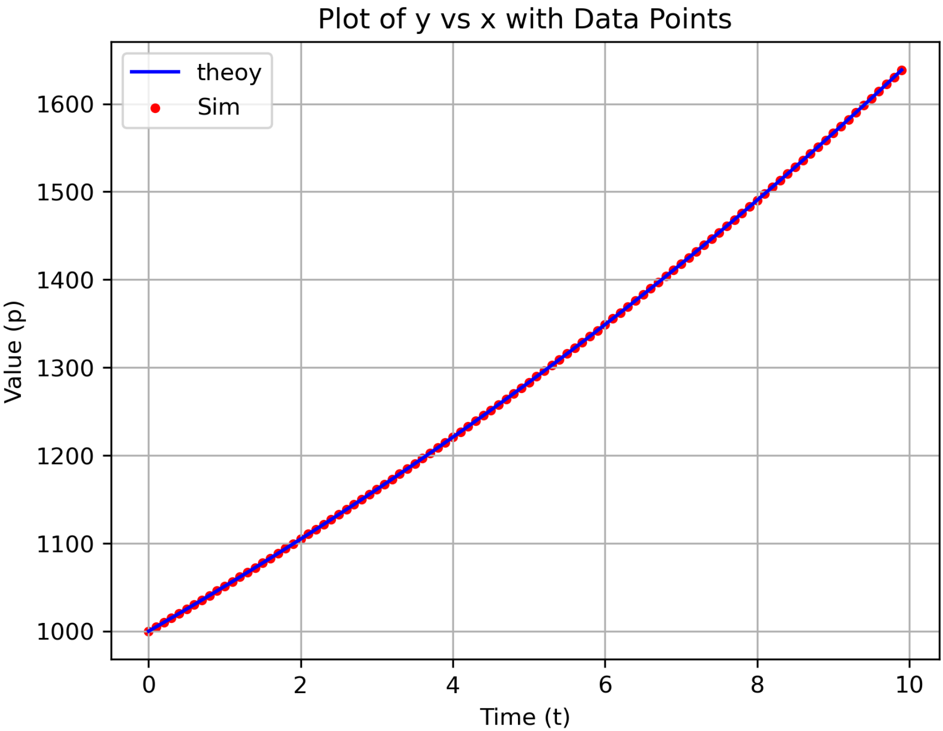
Plot of y vs x with Data Points
theoy
Sim
1000
1100
1200
1300
1400
1500
1600
Value (p)
0
2
4
6
8
10
Time (t)

Fig. 0.1: Plot