9.4.21

EE24BTECH11050 - Pothuri Rahul

Question:
In a bank,principal increases continuously at the rate of 5% per year. An amount of rupees 1000 is deposited with this bank. How much will it worth after 10 years $\left(e^{0.5} = 1.648\right)$

Solution:

variable	description
P	Principle at any time t
t	time in years
C	primary arbitrary constant
C_1	secondary arbitrary constant
P_0	initial principle amount

TABLE 0: Variables Used

Solution: Let P be the principle at any time t. According to the given problem, Rate of change in principle can be given as

$$\frac{dP}{dt} = \left(\frac{5}{100}\right) \times P \tag{0.1}$$

$$\frac{dP}{dt} = \left(\frac{P}{20}\right) \tag{0.2}$$

Seperating the variables in the equation (0.2), We get

$$\frac{dP}{P} = \frac{dt}{20} \tag{0.3}$$

On integrating both sides

$$\int \frac{dP}{P} = \int \frac{dt}{20} \tag{0.4}$$

$$logP = \frac{t}{20} + C \tag{0.5}$$

$$P = e^{\frac{t}{20}+C} \tag{0.6}$$

$$P = e^{\frac{t}{20}}.e^{C} \tag{0.7}$$

$$P = e^{\frac{t}{20}}.C_1 \tag{0.8}$$

Given, at time t=0,P_0=1000 then, from (0.8)

$$1000 = C_1 \tag{0.9}$$

Principle can be given as

$$P = 1000 \times e^{\frac{t}{20}} \tag{0.10}$$

At time t=10,Principle can be given as

$$P = 1000 \times e^{\frac{10}{20}} \tag{0.11}$$

$$P = 1000 \times e^{0.5} \tag{0.12}$$

$$P = 1000 \times 1.648 \tag{0.13}$$

$$P = 1648 \tag{0.14}$$

Logic used for programming:-

Method of finite differences: This method is used to find the approximate solution of the given differential equation by using the values of the function at discrete points.
From the defination of derivative of a function

$$\frac{dy}{dx} \approx \frac{y(x+h) - y(x)}{h} \tag{0.15}$$

by rearranging the terms, we get the function

$$y(x+h) = y(x) + h \times \frac{dy}{dx} \tag{0.16}$$

$$P(t+h) = P(t) + h \times \frac{P}{20} \tag{0.17}$$

Let (t_0, P_0) be points on the curve,

$$t_1 = t_0 + h \tag{0.18}$$

$$P_1 = P_0 + h \times \frac{P}{20} \tag{0.19}$$

On generalising the above equations,

$$t_{n+1} = t_n + h \tag{0.20}$$

$$P_{n+1} = P_n + h \times \frac{P}{20} \tag{0.21}$$

Where h is a very small division (example 0.1),We need iterate this algorithm by taking $P_0 = 1000$ and $t = 0$, till $t_n = 10$. Then we get the principle amount after 10 years. If we plot all the points (t, P), we get the function P varying with t, i.e P vs T graph.

Finding the solution of this equation using the Z-Transform: By using the z-transform method we can convert the differential equation into a linear equation in Z-domain,after finding the solution in z-domin,inverse of it is the solution of the given differential equation.
The differential equation for this question is,

$$\frac{dP}{dt} = \frac{P}{20} \tag{0.22}$$

from (0.21),

$$P_{n+1} = P_n + h \times \frac{P_n}{20} \tag{0.23}$$

$$P_{n+1} = P_n(1 + \frac{h}{20}) \tag{0.24}$$

Applying Z-tranform on both sides,We get,

$$Z\left(P_{n+1}\right) = Z\left(P_n(1 + \frac{h}{20})\right) \tag{0.25}$$

$$Z(P_n + 1) = (1 + \frac{h}{20})Z(P_n) \tag{0.26}$$

Let,

$$Z\left(P_n\right) = P\left(z\right) \tag{0.27}$$

Then,

$$Z\left(P_{n+1}\right) = zP(z) - zP_0 \tag{0.28}$$

Now,

$$zP(z) - zP_0 = P(z)(1 + h/20) \tag{0.29}$$

$$P(z)\left[z - \left(1 + \frac{h}{20}\right)\right] = zP_0 \tag{0.30}$$

$$\tag{0.31}$$

$$P(z) = P_0\left[\frac{z}{z - \left(1 + \frac{h}{20}\right)}\right] \tag{0.32}$$

By inversing, we get

$$P_n = P_0 \times \left(1 + \frac{h}{20}\right)^n \tag{0.33}$$

We know that,

$$1 + \frac{h}{20} \approx e^{\frac{h}{20}} \tag{0.34}$$

then,

$$P_n = P_0\left(e^{\frac{h}{20}}\right)^n \tag{0.35}$$

$$P_n = P_o e^{\frac{nh}{20}} \tag{0.36}$$

As h is the small division of time and n are the total no.of divisions, nh turns to be t at that point,Then

$$P\left(t\right) = P_0 e^{\frac{t}{20}} \tag{0.37}$$

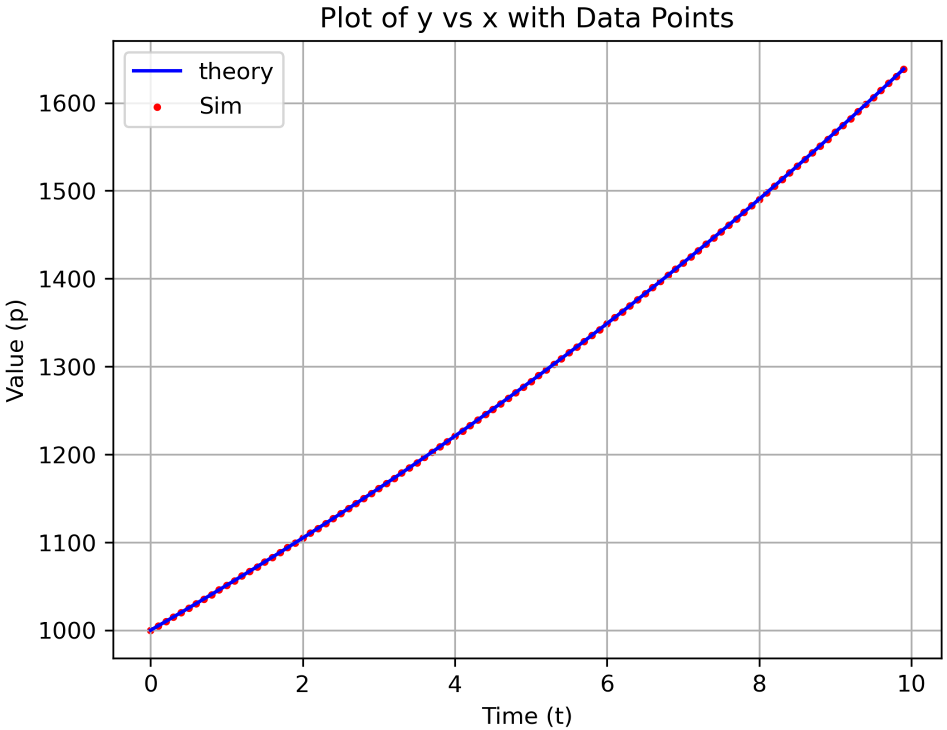
Plot of y vs x with Data Points
theory
Sim
1000
1100
1200
1300
1400
1500
1600
Value (p)
0
2
4
6
8
10
Time (t)

Fig. 0.1: Plot