

9.4.21

EE24BTECH11050 - Pothuri Rahul

QUESTION: Find the equation of the curve passing through the points $(0, \frac{\pi}{4})$ whose differential equation is $\sin x \cos y dx + \cos x \sin y dy = 0$

Solution: Given differential equation is

$$\sin x \cos y dx + \cos x \sin y dy = 0 \quad (0.1)$$

By rearranging the terms in (0.1)

$$\sin x \cos y dx = -\cos x \sin y dy \quad (0.2)$$

$$\tan x dx = -\tan y dy \quad (0.3)$$

Integrating both sides of (0.3),

$$\int \tan x dx = \int -\tan y dy \quad (0.4)$$

$$-\ln \cos x = -(-\ln \cos y) + C \quad (0.5)$$

$$-\ln \cos x = (\ln \cos y) + C \quad (0.6)$$

Where C is the integration constant, To find C, Lets use the given condition that the curve passes through $(0, \frac{\pi}{4})$

By substituting given point in (0.5),

$$-\ln 1 = \ln \frac{1}{\sqrt{2}} + C \quad (0.7)$$

$$0 = \ln \frac{1}{\sqrt{2}} + C \quad (0.8)$$

$$C = -\ln \frac{1}{\sqrt{2}} \quad (0.9)$$

$$C = \ln \sqrt{2} \quad (0.10)$$

By substituting (0.10) in (0.6) ,

$$-\ln \cos x = \ln \cos y + \ln \sqrt{2} \quad (0.11)$$

$$\ln \cos y = -\ln \cos x - \ln \sqrt{2} \quad (0.12)$$

$$\ln \cos y = -(\ln(\cos x \sqrt{2})) \quad (0.13)$$

$$\cos y = \frac{1}{\cos x \sqrt{2}} \quad (0.14)$$

$$y = \cos^{-1} \left(\frac{1}{\cos x \sqrt{2}} \right) \quad (0.15)$$

Logic used for programming:-

Method of finite differences: This method is used to find the approximate solution of the given differential equation by using the values of the function at discrete points.

From the definition of derivative of a function

$$\frac{dy}{dx} \approx \frac{y(x+h) - y(x)}{h} \quad (0.16)$$

by rearranging the terms, we get the function

$$y(x+h) = y(x) + h \times \frac{dy}{dx} \quad (0.17)$$

For this question,

$$\frac{dy}{dx} = -\frac{\tan x}{\tan y} \quad (0.18)$$

By substituting in (0.17),

$$y(x+h) = y(x) - h \times \frac{\tan x}{\tan y} \quad (0.19)$$

Let (t_0, P_0) be points on the curve,

$$x_1 = x_0 + h \quad (0.20)$$

$$y_1 = y_0 - h \times \frac{\tan x_n}{\tan y_n} \quad (0.21)$$

On generalising the above equations,

$$x_{n+1} = x_n + h \quad (0.22)$$

$$y_{n+1} = y_n - h \times \frac{\tan x_n}{\tan y - n} \quad (0.23)$$

Where h is a very small division (example 0.1), We need iterate this algorithm by taking $y_0 = \frac{\pi}{4}$ and $x = 0$. If we plot all the points (x, y) , we get the function y varying with x, i.e y vs x graph.

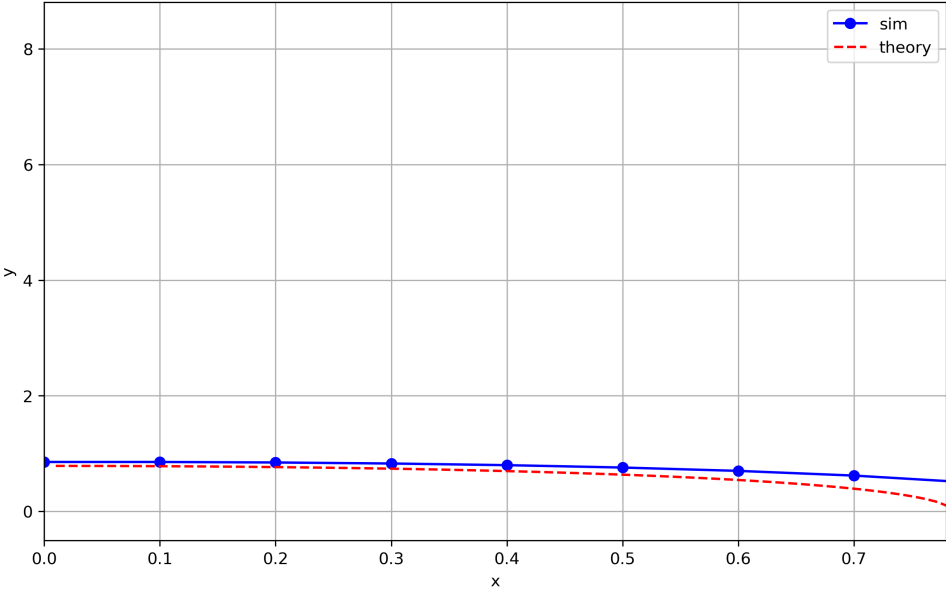


Fig. 0.1: Plot