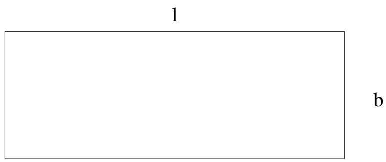
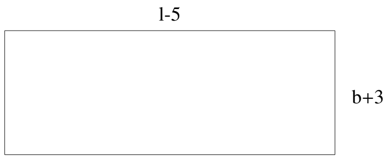
3.5.4.5

EE24BTECH11050 - Pothuri Rahul

Question :
The area of the rectangle gets reduced by 9 square units if its length is reduced by 5 units and breadth is increased by 3 units. If we increase the length by 3 units and breadth by 2 units, The area increases by 67 square units. Find the dimenssions of the reactangle. **Solution:**

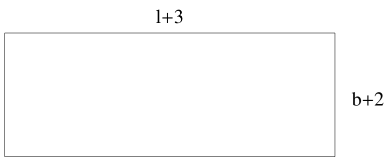


Let us consider l ,b as length and breadth of the rectangle.
Then Area = $l \times b$
Given, Area reduced by 9 units when length reduced by 5 units and breadth increased by 3 units.



$$(l-5)\times(b+3) = lb-9 \tag{0.1}$$
$$3l-5b-15+lb = lb-9 \tag{0.2}$$
$$3l-5b = 6 \tag{0.3}$$

Also given that area increased by 67 sq units when length incresed by 3 units and breadth increased by 67 units.



$$(l+3) \times (b+2) = lb + 67 \tag{0.4}$$
$$2l + 3b + 6 + lb = lb + 67 \tag{0.5}$$
$$2l + 3b = 61 \tag{0.6}$$

The required pair of linear equations in two variables are (0.3) and (0.6) .
We can get the dimensions of the rectangle i.e., length (l) and breadth (b) by solving the above pair of equations .
Let us represent the above equations in matrix form i.e., Ax = b form

$$\begin{pmatrix} 3 & -5 \\ 2 & 3 \end{pmatrix} \times \begin{pmatrix} l \\ b \end{pmatrix} = \begin{pmatrix} 6 \\ 61 \end{pmatrix} \tag{0.7}$$

Where

$$A = \begin{pmatrix} 3 & -5 \\ 2 & 3 \end{pmatrix} \tag{0.8}$$

To solve this , Let us decompose the matrix A into LU , where L is lower triangular matrix(With all diagonal elements 1) and U is upper triangular matrix.
1. Initialization: - Start by initializing **L** as the identity matrix $\mathbf{L} = \mathbf{I}$ and **U** as a copy of **A**.
2. Iterative Update: - For each pivot $k = 1, 2, \ldots, n$: - Compute the entries of U using the first update equation. - Compute the entries of L using the second update equation.
3. Result: - After completing the iterations, the matrix **A** is decomposed into $\mathbf{L} \cdot \mathbf{U}$, where **L** is a lower triangular matrix with ones on the diagonal, and **U** is an upper triangular matrix.

1. Update for $U_{k,j}$ (Entries of U)

For each column $j \geq k$, the entries of U in the k-th row are updated as:

$$U_{k,j} = A_{k,j} - \sum_{m=1}^{k-1} L_{k,m} \cdot U_{m,j}, \quad \text{for } j \geq k.$$

This equation computes the elements of the upper triangular matrix **U** by eliminating the lower triangular portion of the matrix.

2. Update for $L_{i,k}$ (Entries of L)

For each row $i > k$, the entries of L in the k-th column are updated as:

$$L_{i,k} = \frac{1}{U_{k,k}} \left(A_{i,k} - \sum_{m=1}^{k-1} L_{i,m} \cdot U_{m,k} \right), \quad \text{for } i > k.$$

This equation computes the elements of the lower triangular matrix **L**, where each entry in the column is determined by the values in the rows above it.
Using a code we get L,U as

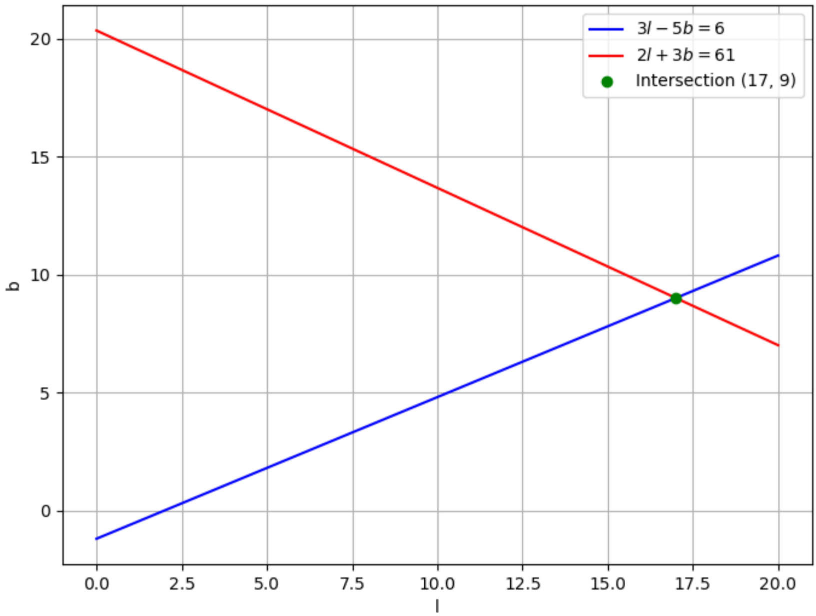
$$L = \begin{pmatrix} 1 & 0 \\ \frac{2}{3} & 1 \end{pmatrix} \tag{0.9}$$

$$U = \begin{pmatrix} 3 & -5 \\ 0 & \frac{19}{3} \end{pmatrix} \tag{0.10}$$

Now Ax = B can be written as LUx = B.
Let the matrix y = Ux.
Then, Ly = B.
As L is lower triangular y can be calculated by forward substitution. Now Ux = y. As U is upper triangular matrix x can be calculated by backward substitution. That results that

$$x = \begin{pmatrix} 17 \\ 9 \end{pmatrix} \tag{0.11}$$

Therefore length l=17 units and breadth b=9 units .
Graphical Approach : We can represent the system of equations we got i.e., (0.3) and (0.6) as the pair of stright lines in l-b plane . The point where these lines are getting intersected gives the values of (l,b) .


$3l - 5b = 6$
$2l + 3b = 61$
Intersection (17, 9)
b
l
0
5
10
15
20
0.0
2.5
5.0
7.5
10.0
12.5
15.0
17.5
20.0