9.4.21

EE24BTECH11050 - Pothuri Rahul

Question:
In a bank,principal increases continuously at the rate of 5% per year. An amount of rupees 1000 is deposited with this bank. How much will it worth after 10 years $\left(e^{0.5} = 1.648\right)$

Solution:

variable	description
P	Principle at any time t
t	time in years
C	primary arbitrary constant
C_1	secondary arbitrary constant
P_0	initial principle amount

TABLE 0: Variables Used

Solution: Let P be the principle at any time t. According to the given problem, Rate of change in principle can be given as

$$\frac{dP}{dt} = \left(\frac{5}{100}\right) \times P \tag{0.1}$$

$$\frac{dP}{dt} = \left(\frac{P}{20}\right) \tag{0.2}$$

Seperating the variables in the equation (0.2), We get

$$\frac{dP}{P} = \frac{dt}{20} \tag{0.3}$$

On integrating both sides

$$\int \frac{dP}{P} = \int \frac{dt}{20} \tag{0.4}$$

$$logP = \frac{t}{20} + C \tag{0.5}$$

$$P = e^{\frac{t}{20}+C} \tag{0.6}$$

$$P = e^{\frac{t}{20}}.e^{C} \tag{0.7}$$

$$P = e^{\frac{t}{20}}.C_1 \tag{0.8}$$

Given, at time t=0,P_0=1000 then, from (0.8)

$$1000 = C_1 \tag{0.9}$$

Principle can be given as

$$P = 1000 \times e^{\frac{t}{20}} \tag{0.10}$$

At time t=10,Principle can be given as

$$P = 1000 \times e^{\frac{10}{20}} \tag{0.11}$$
$$P = 1000 \times e^{0.5} \tag{0.12}$$
$$P = 1000 \times 1.648 \tag{0.13}$$
$$P = 1648 \tag{0.14}$$

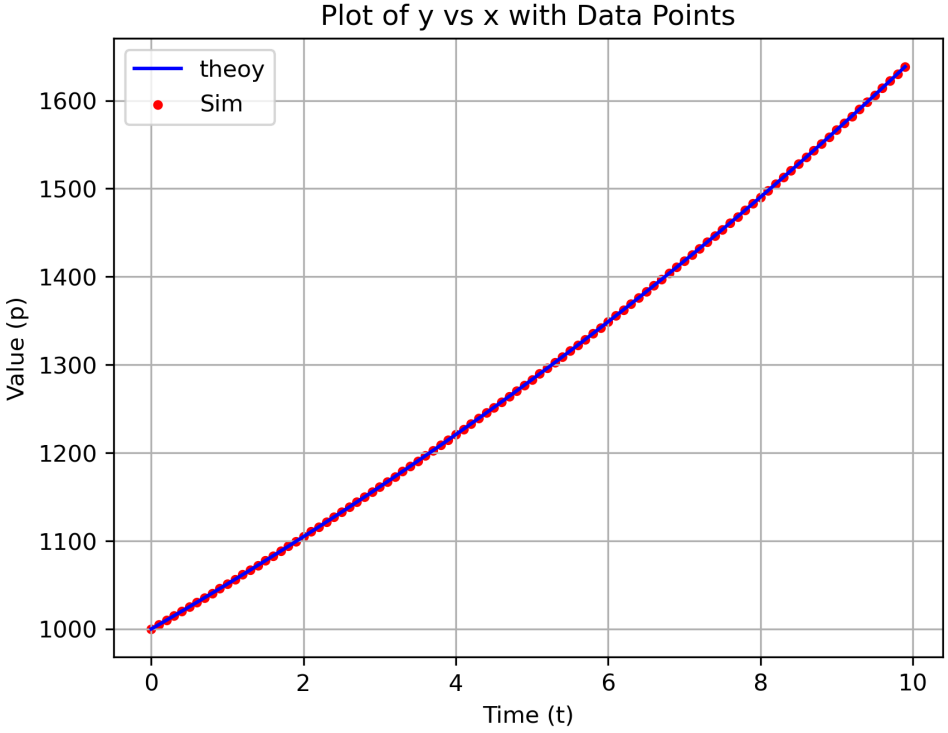

Fig. 0.1: Plot