# 9-9.5-5

EE24BTECH11050 - Pothuri Rahul

**Question**:
Find the area bounded by the curves $(x-1)^2 + y^2 = 1$ and $x^2 + y^2 = 1$.
**Solution:**

| variable | discription |
|---|---|
| $x_1$ | first intersection point |
| $x_2$ | second intersection point |
| $V, u, f$ | Parameters of conic |
| $h, m$ | Parameters of line |

TABLE 0: Variables Used

The equation of a conic in Matrix form is

$$\mathbf{x}^\top \mathbf{V}\mathbf{x} + 2\mathbf{u}^\top \mathbf{x} + f = 0 \tag{0.1}$$

The equation of line is

$$\mathbf{x} = \mathbf{h} + \kappa \mathbf{m} \tag{0.2}$$

The conic parameters for the two circles can be expressed as

$$V_1 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \; u_1 = \begin{pmatrix} -1 \\ 0 \end{pmatrix}, \; f_1 = 0, \tag{0.3}$$

$$V_2 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \; u_2 = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \; f_2 = -1. \tag{0.4}$$

On substituting from (0.3) in (9.1.4.1), we obtain

$$\begin{pmatrix} 1+\mu & 0 & -1 \\ 0 & 1+\mu & 0 \\ -1 & 0 & -\mu \end{pmatrix} = 0 \tag{0.5}$$

yielding

$$\mu = -1. \tag{0.6}$$

Substituting (0.3) in (9.1.3.1), we obtain

$$x^T \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} x + 2 \begin{pmatrix} -1 \\ 0 \end{pmatrix}^T \begin{pmatrix} x \\ y \end{pmatrix} + 1 = 0 \tag{0.7}$$

$$\Rightarrow (-2)(x-1) = 1 \quad \Rightarrow x = \frac{1}{2}. \tag{0.8}$$

Therefore the intersection of the two circles is a line with parameters

$$m = \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \ h = \begin{pmatrix} \frac{1}{2} \\ 0 \end{pmatrix}. \tag{0.9}$$

The intersection parameters of the chord with the first circle in is obtained by substituting line equation in the conic equation.

$$\kappa_i = \pm \frac{\sqrt{3}}{2}. \tag{0.10}$$

Hence the points of intersection are obtained from (0.2) as

$$x_1 = \left( \frac{1}{2}, \frac{\sqrt{3}}{2} \right), \quad x_2 = \left( \frac{1}{2}, -\frac{\sqrt{3}}{2} \right). \tag{0.11}$$

The desired area of the region is given as

$$2 \left( \int_0^{1/2} \sqrt{1-(x-1)^2} dx + \int_0^{1/2} \sqrt{1-x^2} dx \right) \tag{0.12}$$

$$= 2 \left[ \frac{1}{2} (x-1) \sqrt{1-(x-1)^2} + \frac{1}{2} \sin^{-1}(x-1) \right]_0^{1/2} + 2 \left[ \frac{1}{2} x \sqrt{1-x^2} + \frac{1}{2} \sin^{-1}(x) \right]_0^{1/2} \tag{0.13}$$

$$= \frac{2\pi}{3} - \frac{\sqrt{3}}{2}. \tag{0.14}$$

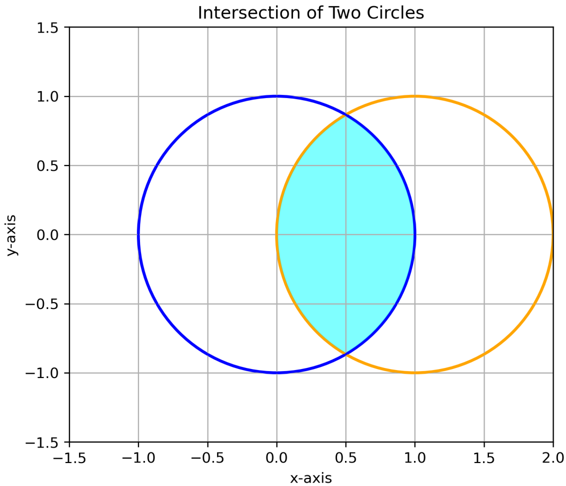


Fig. 0.1: Intersection of the circles.