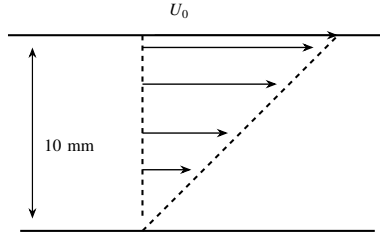


- 4) Which of the following is a quasi-linear partial differential equation?
- $\frac{\partial^2 u}{\partial t^2} + u^2 = 0$
 - $\left(\frac{\partial u}{\partial t}\right)^2 + \frac{\partial u}{\partial x} = 0$
 - $\left(\frac{\partial u}{\partial t}\right)^2 - \left(\frac{\partial u}{\partial x}\right)^2 = 0$
 - $\left(\frac{\partial u}{\partial t}\right)^4 - \left(\frac{\partial u}{\partial x}\right)^3 = 0$
- 5) Let $P(x)$ and $Q(x)$ be the polynomials of degree 5, generated by Lagrange and Newton interpolation methods respectively, both passing through given six distinct points on the xy - plane. Which of the following is correct?
- $P(x) = Q(x)$
 - $P(x) - Q(x)$ is a polynomial of degree 1
 - $P(x) - Q(x)$ is a polynomial of degree 2
 - $P(x) - Q(x)$ is a polynomial of degree 3
- 6) The Laurent series of $f(x) = 1/(z^3 - z^4)$ with center at $z = 0$ in the region $|z| > 1$ is
- $\sum_{n=0}^{\infty} z^{n-3}$
 - $-\sum_{n=0}^{\infty} \frac{1}{z^{n+4}}$
 - $\sum_{n=0}^{\infty} z^n$
 - $\sum_{n=0}^{\infty} \frac{1}{z^n}$
- 7) The value of the surface integral $\iint \bar{F} \cdot n ds$ over the sphere Γ given by $x^2 + y^2 + z^2 = 1$ where $\bar{F} = 4x\hat{i} - z\hat{k}$, and n denotes the outward unit normal, is
- π
 - 2π
 - 3π
 - 4π
- Q.8 - Q.11 carry two marks each
- 8) A diagnostic test for a certain disease is 90% accurate. That is, the probability of a person having (*respectively, not having*) the disease tested positive (*respectively, negative*) is 0.9. Fifty percent of the population has the disease. What is the probability that a randomly chosen person has the disease given that the person tested negative?
- 9) Let $M = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$. Which of the following is correct?
- Rank of M is 1 and M is not diagonalizable
 - Rank of M is 2 and M is diagonalizable
 - 1 is the only eigenvalue and M is not diagonalizable
 - 1 is the only eigenvalue and M is diagonalizable
- 10) Let $f(x) = 2x^3 - 3x^2 + 69, -5 \leq x \leq 5$. Find the point at which f attains the global maximum.
- 11) Calculate $\int_{c_1} \bar{F} \cdot dr - \int_{c_2} dr$, where $c_1 : \bar{r}(t, t^2)$ and $c_2 : \bar{r}(t, \sqrt{t})$, t varying from 0 to 1 and $\bar{F} = xy\hat{j}$.

B.Fluid Mechanics

Q.1-Q.9 carry one mark each.

- 1) In the parallel-plate configuration shown, steady-flow of an incompressible Newtonian fluid is established by moving the top plate with a constant speed, $U_0 = 1\text{ m/s}$. If the force required on the top plate to support this motion is 0.5 per unit area (in m_2) of the plate then the viscosity of the fluid between the plates is _____ N-s/m^2



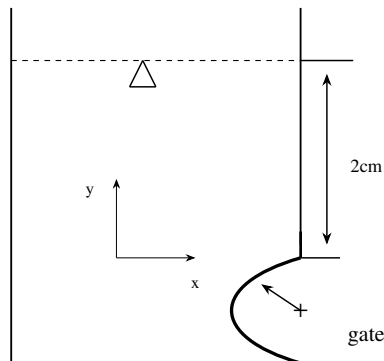
- 2) For a newly designed vehicle by some students, volume of fuel consumed per unit distance travelled ($q_f \text{ in } \text{m}^3/\text{m}$) depends upon the viscosity (μ) and density (ρ) of the fuel and, speed (U) and size (L) of the vehicle as

$$q_f = C \frac{\rho U^2 L}{\mu^3}$$

where C is a constant. The dimensions of the constant C are

- a) $M^0 L^0 T^0$ b) $M^2 L^{-1} T^{-1}$ c) $M^2 L^{-5} T^{-1}$ d) $M^{-2} L^1 T^1$

- 3) A semicircular gate of radius 1 m is placed at the bottom of a water reservoir as shown in the figure below. The hydrostatic force per unit width of the cylindrical gate in y-direction is _____ kN . The gravitational acceleration, $g = 9.8\text{ m/s}^2$ and density of the water = 1000 kg/m^3 .



- 4) Velocity vector in m/s for a 2-D flow is given in Cartesian coordinate (x, y) as $\vec{V} = \left(\frac{x^2}{4} \hat{i} - \frac{xy}{2} \hat{j} \right)$. Symbols bear usual meaning. At a point in the flow, the x-component

and y-component of the acceleration vector are given as $1m/s^2$ and $-0.5m/s^2$, respectively. The velocity magnitude at that points is _____ m/s .

- 5) If $\phi(x, y)$ is velocity potential and $\psi(x, y)$ is stream function for a 2-D, steady, incompressible and irrotational flow, which of the followings is correct?

a) $\left(\frac{dy}{dx}\right)_{\phi=const} = -\frac{1}{\left(\frac{dy}{dx}\right)_{\psi=const}}$

c) $\left(\frac{dy}{dx}\right)_{\phi=const} = \frac{1}{\left(\frac{dy}{dx}\right)_{\psi=const}}$

b) $\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} = 0$

d) $\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = 0$