**Topics: Normal distribution, Functions of Random Variables**

1. The time required for servicing transmissions is normally distributed with *μ* = 45 minutes and *σ* = 8 minutes. The service manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?
2. 0.3875
3. 0.2676
4. 0.5
5. 0.6987

from scipy import stats

from scipy.stats import norm

# Find Z-Scores at X=50; Z = (X - µ) / σ

Z=(50-45)/8

Z

0.625

# Find probability P(X>50) = 1-stats.norm.cdf(abs(z\_score))

1-stats.norm.cdf(abs(0.625))

0.26598552904870054

# OR Find probability P(X<=50); p\_value=stats.norm.cdf(abs(z\_score))

p\_value=stats.norm.cdf(abs(0.625))

p\_value

0.7340144709512995

# P(X>50) = 1 - P(X<=50)

1-0.734

0.266

1. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean *μ* = 38 and Standard deviation *σ* =6. For each statement below, please specify True/False. If false, briefly explain why.
2. More employees at the processing center are older than 44 than between 38 and 44.
3. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

from scipy import stats

from scipy.stats import norm

# A. More employees at the processing center are older than 44 than between 38 and 44.

# p(X>44); Employees older than 44 yrs of age

1-stats.norm.cdf(44,loc=38,scale=6)

0.15865525393145707

# p(38<X<44); Employees between 38 to 44 yrs of age

stats.norm.cdf(44,38,6)-stats.norm.cdf(38,38,6)

0.3413447460685429

# B. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

# P(X<30); Employees under 30 yrs of age

stats.norm.cdf(30,38,6)

0.09121121972586788

# No. of employees attending training program from 400 nos. is N\*P(X<30)

400\*stats.norm.cdf(30,38,6)

36.484487890347154

1. If *X1* ~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2) are *iid* normal random variables, then what is the difference between 2 *X*1 and *X*1 + *X*2? Discuss both their distributions and parameters.

Both are normal with mean 2μ, but the variance of the sum is 2σ2 rather than 4σ2.

E(2X1) = 2E(X1) = 2μ and E(X1 + X2) = E(X1) +E(X2) = μ + μ whereas

Var(2X1) = 4Var(X1) =4σ2 and Var(X1 + X2) = Var(X1) + Var(X2) = 2σ2

Think of 2X1 = X1+X1 as the sum of two perfectly dependent normal random variables whereas X1 + X2 is the sum of two independent normal random variables.

(OR)

As we know that if X ∼N(μ1, σ1^2 ), and Y ∼N(μ2, σ2^2 ) are two independent random variables then X + Y ∼N(μ1 + μ2, σ1^2 + σ2^2 ) , and X − Y ∼N(μ1 − μ2, σ1^2 + σ2^2 ) .Similarly if Z = aX + bY , where X and Y are as defined above,

i.e Z is linear combination of X and Y , then Z ∼N(aμ1 + bμ2, a^2σ1^2 + b^2σ2^2 )

1. Let X ~ N(100, 202). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.
2. 90.5, 105.9
3. 80.2, 119.8
4. 22, 78
5. 48.5, 151.5
6. 90.1, 109.9

**from** scipy **import** stats

**from** scipy.stats **import** norm

stats**.**norm**.**interval(0.99,100,20)

(48.48341392902199, 151.516586070978)

1. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit1 ~ N(5, 32) and Profit2 ~ N(7, 42) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45
2. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.

Rupee ranges in between [9.9 to 98.1] Crore Rupees, 95% of the time for the Annual Profit of the Company.

1. Specify the 5th percentile of profit (in Rupees) for the company

The 5TH Percentile of profit for the company is 17 Crore Rupees

1. Which of the two divisions has a larger probability of making a loss in a given year?

The Division #2 (Profit2 ~ N(7, 42) ) has a larger probability of making a loss in a given year