# Week 3 Homework

**Probability Model and Data Analysis**

**Software Engineering Program,**

**Department of Computer Engineering,**

**School of Engineering, KMITL**

67011352 Theepakorn Phayonrat

# Homework of Law of Tatal Probability and Bayes' Theorem

## Context

The local government of Bangkok wants to interpret ATK test results from people with symptoms and people who have been in close contact with people who are known to be infected. It is found that there in a 70% of a test population having COVID-19 infection. For the ATK test results, there are not 100% accuracy. The result may be positive" (a person may test "positive" and not be infected), or the result may be "false negative" (a person may test "negative" and however be infected). From the ATKs that are used, it is found that the probability that a person tests positive if s/he has COVID (a true positive result) is 0.95, i.e,

$P(Positive \mid COVID) = 0.95$, and

the probability that a person tests negative if she is not infected, or healthy (a true negative result) is 0.90, i.e,

$P(Negative \mid healthy) = 0.90$.

## Question 1

Find the Probability that a test the person is healthy if s/he receives a negative test result. Give the answer in a porpotion in a porpotion in % by rounding the 2 decimal places.

## Solution

From the context given earlier, we know that
$\oplus$ is an event when a test person get a positive test result.
$\ominus$ is an event when a test person get a negative test result.
$C$ is an event when a test person is infected with COVID
$H$ is an event when a test person is healthy

And from the context given earlier, we also know that
$P[C] = 0.70$
$P[H] = 1 - P[C] = 1 - 0.70 = 0.30$

$$\therefore P[\ominus] = P[\ominus \mid C]P[C] + P[\ominus \mid H]P[H] \quad P[\oplus] = P[\oplus \mid C]P[C] + P[\oplus \mid H]P[H]$$

$$= (0.05)(0.70) + (0.90)(0.30) \qquad = (0.95)(0.70) + (0.10)(0.30)$$

$$= 0.035 + 0.27 \qquad\qquad\qquad = 0.665 + 0.03$$
$$= 0.305 \qquad\qquad\qquad\qquad = 0.695$$

$$\therefore P[H \mid \ominus] = \frac{P[\ominus \mid H]P[H]}{P[\ominus]} . \text{ By Bayes' Theorem}$$
$$= \frac{(0.90)(0.30)}{0.305}$$
$$= \frac{0.27}{0.305}$$
$$\approx 0.8852$$
$$= 88.52\%$$

## Answer

$\therefore$ the Probability that a test person is healthy receives a negative test result is in porpotion of 88.52%

## Question 2

Find the Probability that a test person has COVID if s/he receives a positive test result. Give the answer in a porpotion in % by rounding to 2 decimal places.

## Solution

From the context given earlier, we know that

$$\therefore \mathrm{P[C \mid \oplus]} = \frac{P[\oplus \mid C]P[C]}{P[\oplus]}. \text{ By Bayes' Theorem}$$

$$= \frac{(0.95)(0.70)}{0.695}$$

$$= \frac{0.665}{0.695}$$

$$\approx 0.9568$$

$$= 95.68\%$$

## Answer

$\therefore$ the Probability that a test person has COVID receives a positive test result is in porpotion of $95.68\%$