Week 7 Homework

Probability Model and Data Analysis

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Homework of Binomial and Poisson RVs

Binomial RVs:

Question 1:

Flip a fair coin two times. Observe each trial whether head or tail facing up after the coin lands. Assume that two trials are independent. The event the head facing up is considered as a success while the event of the tail facing up is considered as a failure. Let X be the number of success.

(1) Find and sketch the PMF of X
(2) Find the expected value $E[X]$
(3) Find and sketch the CDF of X

Solution

To find the Binomial random variable X we need to find the Bernoulli random variable of each trial.

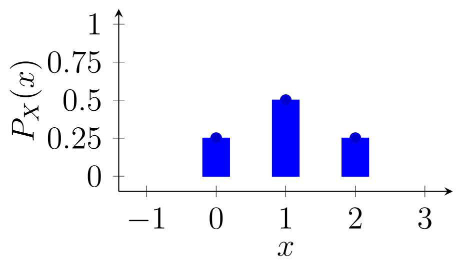
Let C be the random variable of the success event of a Bernoulli trial.

$$P_C[c] = \begin{cases} 0.5 & c = 0 \\ 0.5 & c = 1 \\ 0 & otherwise \end{cases}$$

After we have got the PMF of the Bernoulli trial, we can find the Binomial random variable X where the result implies from PMF of C where $P_C[c = 1]$ is the success event (head facing up) and $P_C[c = 1]$ is the failure event (tail facing up).

$$P_X[x] = \begin{cases} \binom{2}{x}(P_C[c = 1])^x (P_C[c = 0])^{2-x} & x = 0, 1, 2 \\ 0 & otherwise \end{cases}$$

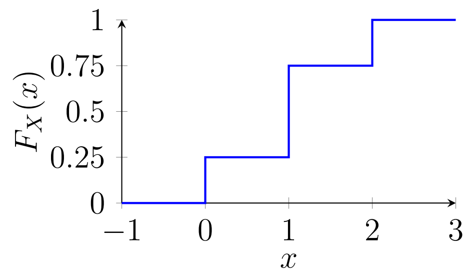
$$\therefore P_X(x) = \begin{cases} 0.25 & x = 0, \\ 0.5 & x = 1, \\ 0.25 & x = 2, \\ 1 & \text{otherwise.} \end{cases}$$



$\therefore$ The expected value $E[X] = np = (2)(0.5) = 1$

After we have got the PMF, we can now find the CDF of the Binomial random variable X.

$$\therefore F_X(x) = \begin{cases} 0 & x < 0, \\ 0.25 & 0 \leq x < 1, \\ 0.75 & 1 \leq x < 2, \\ 1 & x \geq 2 \end{cases}$$



Poisson RVs:

Question 2:

The number of cars pulling into the parking garage every sixty minutes can be described as a Poisson process. If, on average, 5 cars enter the garage every sixty minutes, what is the probability that *at most* 1 car will arrive in the next hour?

Solution

We all know that 60 minutes equal to 1 hour.
$\therefore T = 1$ (Time period is 1 hour)
$\therefore \lambda = 5$ (Average rate is 5 cars)
$\therefore \alpha = (\lambda)(T) = (5)(1) = 5$ (Poisson random variable)

$$\begin{aligned}
\therefore P_X[x \leq 1] &= \sum_{n=0}^{1} P_X[x = n] \\
&= \sum_{n=0}^{1} \frac{\alpha^x e^{-\alpha}}{x!}; \alpha = 5 \\
&= \frac{(5^0)(e^{-5})}{0!} + \frac{(5^1)(e^{-5})}{1!} \\
&= (5^0 + 5^1)(e^{-5}) \\
&= (6)(e^{-5}) \\
&\approx 0.04043
\end{aligned}$$

Answer

$\therefore$ The approxiamate probability that at most 1 car will arrive in the next hour is 0.04043