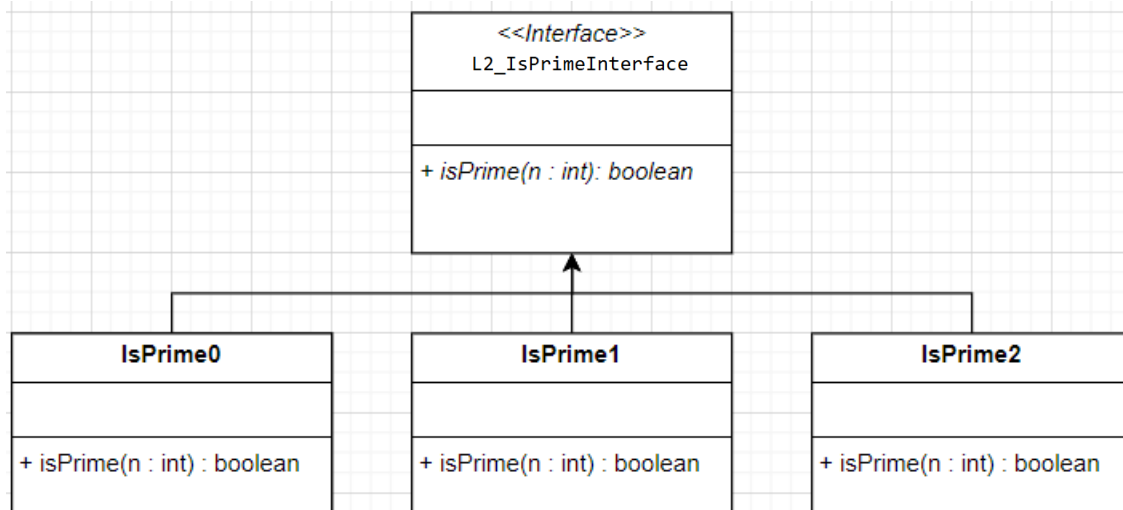


Objective(s):

- To practice on analyzing algorithms' runtime

Task 1: Implement IsPrime0, IsPrime1 and IsPrim2. (in `.\Lab02\pack`)

```

package pack;

public interface L2_IsPrimeInterface {
    boolean isPrime(int n);
}
  
```

```

// IsPrime2
public boolean isPrime(int n) {
    if (n == 1) return false;
    if (n <= 3) return true;
    if ((n%2 == 0) || (n%3 == 0))
        return false;
    int m = (int)Math.sqrt(n);
    for (int i = 5; i <= m; i += 6) {
        if (n % i == 0) return false;
        if (n % (i+2) == 0) return false;
    }
    return true;
}
  
```

```

// IsPrime0
public boolean isPrime(int n) {
    if (n == 1) return false;
    if (n <= 3) return true;
    int m = n/2;
    for (int i = 2; i <= m; i++) {
        if (n % i == 0) return false;
    }
    return true;
}
  
```

```

// IsPrime1
public boolean isPrime(int n) {
    if (n == 1) return false;
    if (n <= 3) return true;
    int m = (int)Math.sqrt(n);
    for (int i = 2; i <= m; i++) {
        if (n % i == 0) return false;
    }
    return true;
}
  
```

The method `isPrime0(n)` takes any positive integer and returns true if it is a prime, false otherwise. The method run through all integer from 2 to $n/2$ and check if n is divisible by any of them.

There are two more methods, `isPrime1(n)` and `isPrime2(n)`. The method `isPrime1(n)` is similar to `isPrime0(n)` but only run from 2 to \sqrt{n} . The method `isPrime2(n)` improves upon `isPrime1(n)` by take out anything divisible by 2 and 3 and not going to test divisibility of number that are multiple of 2 and 3.

For testing, we can use the following program:

```
private static void testIsPrime012() {
    int N = 100;
    int count = 0;
    L2_IsPrimeInterface obj = new IsPrime0();
    for (int n = 1; n < N; n++) {
        if (obj.isPrime(n)) count++;
    }
    System.out.println("Pi (" + N + ") = " + count);
    count = 0;
    obj = new IsPrime1();
    for (int n = 1; n < N; n++) {
        if (obj.isPrime(n)) count++;
    }
    System.out.println("Pi (" + N + ") = " + count);
    count = 0;
    obj = new IsPrime2();
    for (int n = 1; n < N; n++) {
        if (obj.isPrime(n)) count++;
    }
    System.out.println("Pi (" + N + ") = " + count);
}
```

Remark : There are 25 prime numbers between 2 to 100. Check the correctness of your implementation.

Task 2: run the program with isPrime0, isPrime1, and isPrime2. Record your result into the following table.

Running-time table					
n	numPrime(n)	time (milliseconds)			
		Lab's isPrime0	isPrime0	isPrime1	isPrime2
100,000	9592	353	296	6	2
200,000	17984	1,283	1100	12	5
300,000	25997	2,792	2384	17	6
400,000	33860	4,820	4149	25	9
500,000	41538	7,370	6374	34	12
600,000	49098	15,580	9037	44	16
700,000	56543	24,557	12183	54	19
800,000	63951	31,716	15779	65	23
900,000	71274	39,964	20002	77	27
1,000,000	78498	48,785	24193	89	31

```

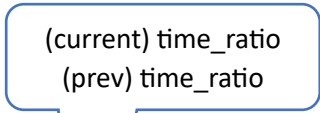
public static void bench_isPrime(L2_IsPrimeInterface obj) {
    int your_cpu_factor = 1; /* increase by 10 times */
    int N = 100;
    int count = 0;
    for (N = 100_000; N <= 1_000_000 * your_cpu_factor; N+= 100_000 * your_cpu_factor) {
        count = 0;
        long start = System.currentTimeMillis();
        for (int n = 1; n < N; n++) {
            if (obj.isPrime(n)) count++;
        }
        long time = (System.currentTimeMillis() - start);
        System.out.println(N + "\t" + count + "\t" + time);
        // System.out.printf("%s\t %s\t %s",String.format("%,d",N),
String.format("%,d",count), String.format("%,d",time));
    }
}

```

Taks 3 : Analyze whether time increased on isPrime0 is linear.

When running algorithms on different input sizes, it's important to understand how the runtime grows. This helps us understand the behavior of the growth rate, and how it compares to the expected Big O complexity.

Complete the table below.



Running-Time Analysis							
n	Data size ratio	Lab's (example) isPrime0	Time Ratio (compared to n)	(Time) Increased Factor	your isPrime0	Time Ratio (compared to n)	(Time) Increased Factor
100,000	n	353	1.0000	-	296	1.0000	-
200,000	2n	1,283	3.6345	3.6345	1100	3.7162	3.7162
300,000	3n	2,792	7.9095	2.1762	2384	8.0541	2.1672
400,000	4n	4,820	13.6543	1.7263	4149	14.0169	1.7404
500,000	5n	7,370	20.4413	1.5290	6374	21.5338	2.1762
600,000	6n	15,580	44.1359	2.1140	9037	30.5304	1.5363
700,000	7n	24,557	69.5665	1.5762	12183	41.1588	1.3481
800,000	8n	31,716	89.8470	1.2519	15779	53.3074	1.2952
900,000	9n	39,964	113.2124	1.2601	20002	67.5743	1.2676
1,000,000	10n	48,785	138.2011	1.2207	24193	81.7331	1.2095

What Should We Expect?

1. Should growth per step stabilize?

- Yes, for linear algorithms → growth per 100k should stay ~constant.
- If it increases, the algorithm may be $O(n \log n)$ or $O(n^2)$.

2. Should increase factor converge?

- Yes — this reflects the asymptotic behavior.
- In this case, it approaches ~ 1.2 — a sign of super-linear growth.

3. Does Machine Matter?

If we run the same algorithm on different machines (e.g., Windows vs macOS):

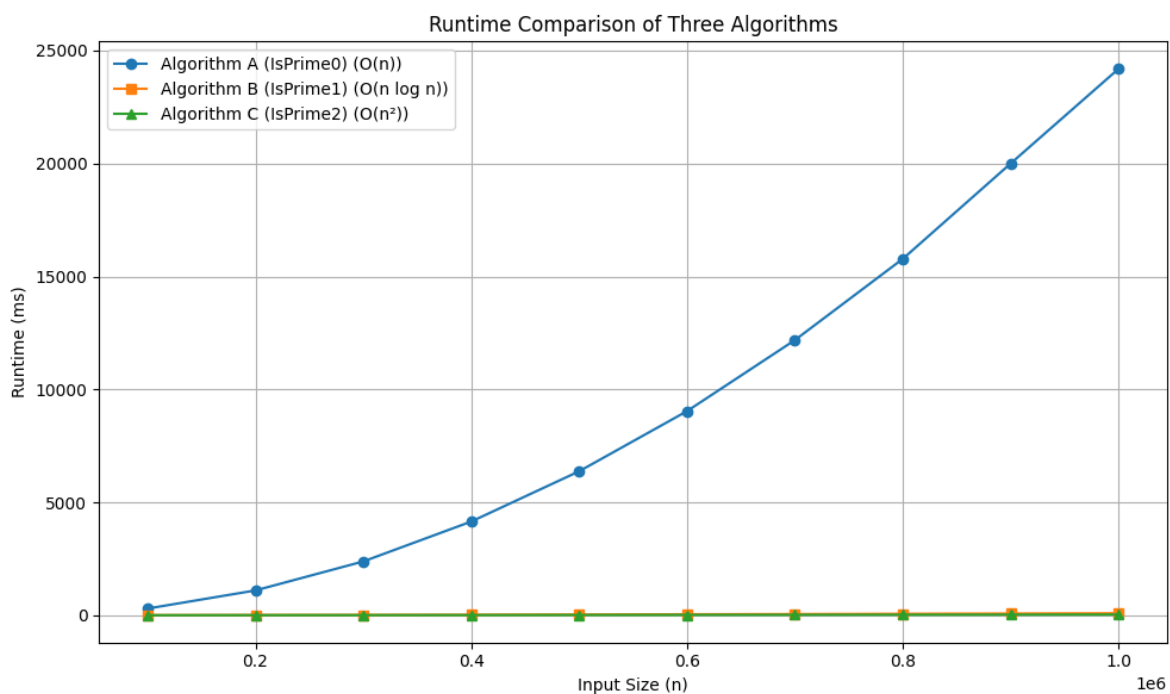
- Raw runtimes will differ due to hardware/OS differences.
- But the time ratios and increase factors should follow the same pattern.

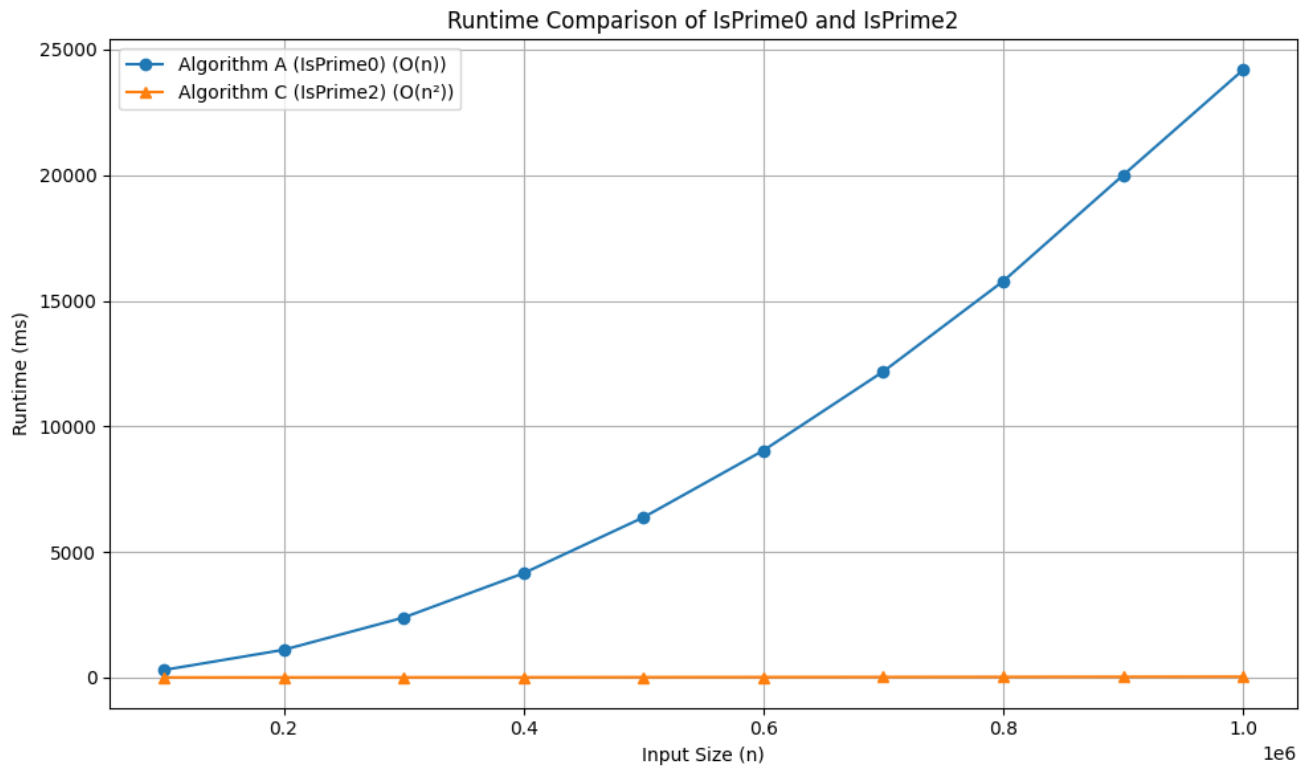
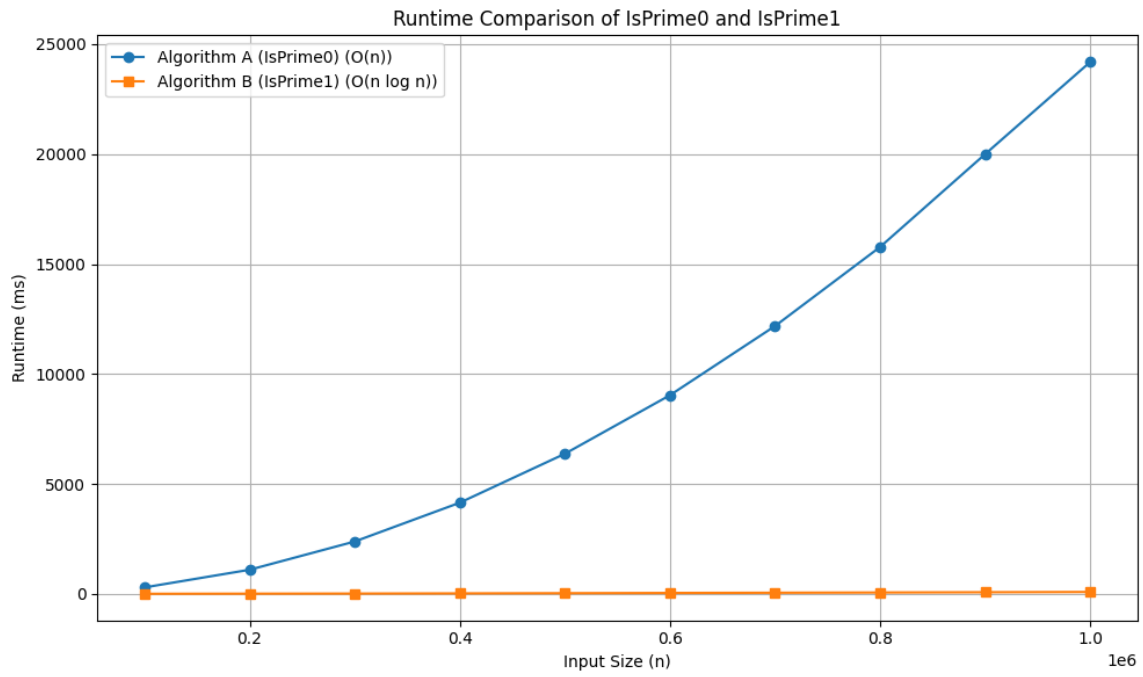
Time ratio stays the same, even if absolute values change.

Key Takeaways

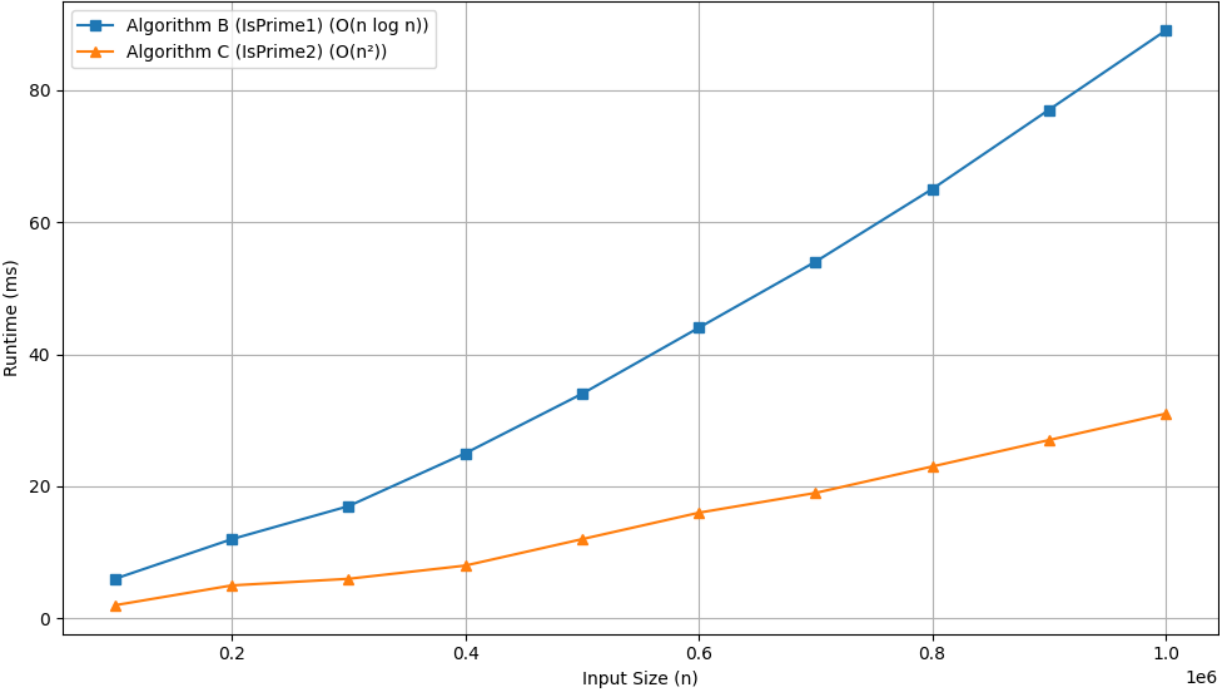
- Time ratio and growth analysis help determine algorithm complexity.
- Growth trends and increase factors help distinguish between $O(n)$, $O(n \log n)$, and $O(n^2)$.
- As input size grows, patterns converge, even on different machines.

Task 4: Plot 2 runtime graphs your isPrime0's vs. your isPrime1's and isPrime1's vs. isPrime2's





Runtime Comparison of IsPrime1 and IsPrime2



The following code might be helpful.

```
import matplotlib.pyplot as plt

# Sample input sizes
input_sizes = [100_000, 200_000, 300_000, 400_000, 500_000, 600_000, 700_000,
800_000, 900_000, 1_000_000]

# Sample runtimes in milliseconds (replace with real data)
runtime_A = [100, 200, 300, 400, 500, 600, 700, 800, 900, 1000]      #
O(n)
runtime_B = [130, 290, 470, 680, 920, 1190, 1490, 1810, 2150, 2500]  #
O(n log n)
runtime_C = [90, 350, 780, 1400, 2200, 3200, 4400, 5800, 7400, 9200]  #
O(n^2)

# Plotting
plt.figure(figsize=(10, 6))
plt.plot(input_sizes, runtime_A, marker='o', label='Algorithm A (O(n))')
plt.plot(input_sizes, runtime_B, marker='s', label='Algorithm B (O(n log n))')
plt.plot(input_sizes, runtime_C, marker='^', label='Algorithm C (O(n^2))')

plt.title('Runtime Comparison of Three Algorithms')
plt.xlabel('Input Size (n)')
plt.ylabel('Runtime (ms)')
plt.grid(True)
plt.legend()
plt.tight_layout()
#export to PNG/PDF using plt.savefig('filename.png')
plt.show()
```

Submission: this pdf. L2_IsPrime0.java L2_IsPrime1.java and L2_IsPrime2.java

Due Date: TBA