



Week 2 Homework

Probability Model and Data Analysis

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Homework of Conditional Probability

Question 1

With respect of Example 1.16, consider the priori probability model

$$P[rr] = 0.01, P[ra] = 0.01, P[ar] = 0.01, P[aa] = 0.97.$$

Let B denote the event that the first chip tested image rejected and similarly, let denote A the event that the second circuit is a failure.

Find $P[A^c|B]$ and $P[A^c|B^c]$

Solution

From the information given by the question, we know that

$$\begin{aligned} P[A] &= P[rr] + P[ar] \\ &= 0.01 + 0.01 \\ &= 0.02 \end{aligned}$$

$$\begin{aligned} P[A^c] &= P[aa] + P[ar] \\ &= 0.01 + 0.01 \\ &= 0.02 \end{aligned}$$

$$\begin{aligned} P[B] &= P[rr] + P[ra] \\ &= 0.01 + 0.01 \\ &= 0.02 \end{aligned}$$

$$\begin{aligned} P[B^c] &= P[aa] + P[ar] \\ &= 0.97 + 0.01 \\ &= 0.98 \end{aligned}$$

$$\begin{aligned} P[A^c B] &= P[ra] \\ &= 0.01 \end{aligned}$$

$$\begin{aligned} P[A^c B^c] &= P[aa] \\ &= 0.97 \end{aligned}$$

$$\begin{aligned} \therefore P[A^c|B] &= \frac{P[A^c B]}{P[B]} \\ &= \frac{0.01}{0.02} = \frac{1}{2} = 0.5 \end{aligned}$$

$$\begin{aligned} \therefore P[A^c|B^c] &= \frac{P[A^c B^c]}{P[B^c]} \\ &= \frac{0.97}{0.98} \end{aligned}$$

Question 2

Two manufacturing plants produce similar parts. Plant 1 produces 1,000 parts, 100 of which are defective. Plant 2 produces 2,000 parts, 150 of which are defective. A part is selected at random and found to be defective. What is the probability that it came from plant 1? (Hint: Let B be the event that "the part selected is defective," and let A be the event that "the part selected came from plant 1." Find $P[A|B]$)

Solution

From the information given by the question, we know that

$$\begin{aligned} P[A] &= \frac{\text{parts produced from plant 1}}{\text{all part from both plants}} \\ &= \frac{100}{3000} \end{aligned}$$

$$\begin{aligned} P[B] &= \frac{\text{all defected parts}}{\text{all parts from both plants}} \\ &= \frac{250}{3000} \end{aligned}$$

$$\begin{aligned} P[AB] &= \frac{\text{parts from plant 1 and defected}}{\text{all parts from both plants}} \\ &= \frac{100}{3000} \end{aligned}$$

$$\begin{aligned} \therefore P[A|B] &= \frac{P[AB]}{P[B]} \\ &= \frac{\frac{100}{3000}}{\frac{250}{3000}} = \frac{100}{250} = \frac{2}{5} = 0.4 \end{aligned}$$

Answer

\therefore The probability of getting parts produced in plant 1 where randomly inspect where the part is defective is equal to 0.4