Week 6 Homework

Probability Model and Data Analysis

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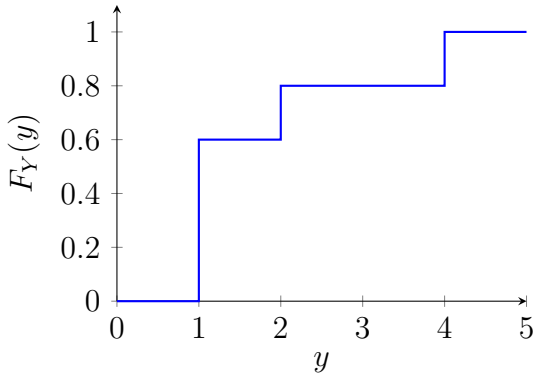
Homework of CDF of the discrete random variable and Bernoulli RV

Question 1

Use the CDF $F_Y[y]$ to find the following probabilities:



(1) $P[Y < 1]$ (2) $P[Y \leq 1]$
(3) $P[Y > 2]$ (4) $P[Y \geq 2]$
(5) $P[Y = 1]$ (6) $P[Y = 3]$

Solution

From the CDF graph given earlier, we can analyse that

$$\mathrm{F}_Y(y) = \begin{cases} 0 & y < 1, \\ 0.6 & 1 \leq y < 2, \\ 0.8 & 2 \leq y < 4, \\ 1 & y \geq 4 \end{cases} \qquad \therefore P_Y(y) = \begin{cases} 0.6 & y = 1, \\ 0.2 & y = 2, \\ 0 & y = 3, \\ 0.2 & y = 4, \\ 0 & \text{otherwise.} \end{cases}$$

Answer

Therefore, we can now find the answer of the questions above.

(1) $P[Y < 1] = 0$ (2) $P[Y \leq 1] = 0 + 0.6 = 0.6$
(3) $P[Y > 2] = 0 + 0.2 = 0.2$ (4) $P[Y \geq 2] = 0.2 + 0 + 0.2 = 0.4$
(5) $P[Y = 1] = 0.6$ (6) $P[Y = 3] = 0$

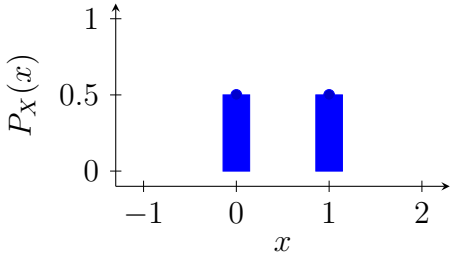
Question 2

Flip a coin and let it land on the table, then observe whether head or tail facing up after the coin lands. The event of the head facing up is considered as a success while the event of tail facing up is considered as failure.

Let X be the random variable of the success event.
1. Find and sketch the PMF of X.
2. Find the expected value of $E[X]$
3. Find and sketch the CDF of X.

Answer

$$P_X(x) = \begin{cases} 0.5 & x = 0, \\ 0.5 & x = 1, \\ 0 & \text{otherwise.} \end{cases}$$



$\therefore E[X] = P[X = 0](0) + P[X = 1](1) = 0.5(0) + 0.5(1) = 0 + 0.5 = 0.5$

$$F_X(x) = \begin{cases} 0 & x < 0, \\ 0.5 & 0 \leq x < 1, \\ 1 & x \geq 1 \end{cases}$$